Complete Block Designs

$$Y_{ij} = \mu + \tau_i + \rho_j + \varepsilon_{ij} \qquad \varepsilon_{ij} \overset{iid}{\sim} N(0, \sigma_e^2) \quad i = 1, \ldots, t \quad j = 1, \ldots, r$$

Residuals

$$\hat{\varepsilon}_{ij} = Y_{ij} - \left(\hat{\mu} + \hat{\tau}_i + \hat{\rho}_j\right)$$
$$= Y_{ij} - \left[\overline{Y}_{\bullet\bullet} + \left(\overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet}\right) + \left(\overline{Y}_{\bullet j} - \overline{Y}_{\bullet\bullet}\right)\right]$$
$$= Y_{ij} - \overline{Y}_{i\bullet} - \overline{Y}_{\bullet j} + \overline{Y}_{\bullet\bullet}$$

Decomposition of the *SS(Total)*

$$\underset{total}{\left(Y_{ij} - \overline{Y}_{\bullet\bullet}\right)} = \underset{Trt}{\left(\overline{Y}_{i\bullet} - \overline{Y}_{\bullet\bullet}\right)} + \underset{Block}{\left(\overline{Y}_{\bullet j} - \overline{Y}_{\bullet\bullet}\right)} + \underset{Residual}{\left(Y_{ij} - \overline{Y}_{i\bullet} - \overline{Y}_{\bullet j} + \overline{Y}_{\bullet\bullet}\right)}$$

Source	d.f.	SS	MS	E(MS)
Trt	*t-1*	*SST*	*MST = SST / (t-1)*	$\sigma^2 + \sum\sum \tau_k^2 / (t-1)$
Blocks	*r-1*	*SSR*	*MSR = SSR / (r-1)*	--
Error	*(t-1)(r-1)*	*SSE*	*MSE*	σ^2
Total	*rt - 1*	*SS(Tot)*		

Relative Efficiency (REF)

- Blocking is done to reduce error variance → increase efficiency
- The more variability explained by blocking, the more efficient the RBD

CRF

Source	d.f.	E(MS)
A	*a-1*	$\sigma^2 + b\sigma_a^2$
B	*b-1*	$\sigma^2 + a\sigma_b^2$
Error	*(a-1)(b-1)*	σ^2

RBD

Source	d.f.	E(MS)
Trt	*t-1*	$\sigma^2 + b\theta_t^2$
Blocks	*b-1*	$\sigma^2 + t\sigma_b^2$
Error	*(t-1)(b-1)*	σ^2

Latin Square Designs

$$Y_{ijk} = \mu + \rho_i + \gamma_j + \tau_k + \varepsilon_{ijk} \qquad \varepsilon_{ijk} \overset{iid}{\sim} N(0, \sigma_e^2)$$

Residuals

$$\begin{aligned}\hat{\varepsilon}_{ijk} &= Y_{ijk} - \left(\hat{\mu} + \hat{\rho}_i + \hat{\gamma}_j + \hat{\tau}_k\right)\\ &= Y_{ijk} - \left[\overline{Y}_{\bullet\bullet\bullet} + \left(\overline{Y}_{i\bullet\bullet} - \overline{Y}_{\bullet\bullet\bullet}\right) + \left(\overline{Y}_{\bullet j\bullet} - \overline{Y}_{\bullet\bullet\bullet}\right) + \left(\overline{Y}_{\bullet\bullet k} - \overline{Y}_{\bullet\bullet\bullet}\right)\right]\\ &= Y_{ijk} - \overline{Y}_{i\bullet\bullet} - \overline{Y}_{\bullet j\bullet} - \overline{Y}_{\bullet\bullet k} + 2\overline{Y}_{\bullet\bullet\bullet}\end{aligned}$$

Decomposition of the *SS(Total)*

$$\underbrace{\left(Y_{ijk} - \overline{Y}_{\bullet\bullet\bullet}\right)}_{total} = \underbrace{\left(\overline{Y}_{i\bullet\bullet} - \overline{Y}_{\bullet\bullet\bullet}\right)}_{Rows} + \underbrace{\left(\overline{Y}_{\bullet j\bullet} - \overline{Y}_{\bullet\bullet\bullet}\right)}_{Columns} + \underbrace{\left(\overline{Y}_{\bullet\bullet k} - \overline{Y}_{\bullet\bullet\bullet}\right)}_{Treatment} + \underbrace{\left(Y_{ijk} - \overline{Y}_{i\bullet\bullet} - \overline{Y}_{\bullet j\bullet} - \overline{Y}_{\bullet\bullet k} + 2\overline{Y}_{\bullet\bullet\bullet}\right)}_{Residual}$$

Source	d.f.	SS	MS	E(MS)
Trt	*t-1*	*SST*	*MST = SST / (t-1)*	$\sigma^2 + \underbrace{\sum\sum\sum}_{D} \tau_k^2 / (t-1)$
Rows	*t-1*	*SSR*	*MSR = SSR / (t-1)*	--
Cols	*t-1*	*SSC*	*MSC = SSC /(t-1)*	--
Error		*SSE*	*MSE*	σ^2
Total		*SS(Tot)*		

Randomization

Step 1: Standard Square:

	(1)	(2)	(3)	(4)	(5)
(1)	A	B	C	D	E
(2)	B	A	D	E	C
(3)	C	E	A	B	D
(4)	D	C	E	A	B
(5)	E	D	B	C	A

Step 2: Randomize order of the columns: (3,4,2,5,1)

	(3)	(4)	(2)	(5)	(1)
(1)	C	D	B	E	A
(2)	D	E	A	C	B
(3)	A	B	E	D	C
(4)	E	A	C	B	D
(5)	B	C	D	A	E

Step 3: Randomize order of the rows: (4,1,5,3,2)

(4)	E	A	C	B	D
(1)	C	D	B	E	A
(5)	B	C	D	A	E
(3)	A	B	E	D	C
(2)	D	E	A	C	B

Step 4: Randomize treatment assignments to letters

<u>Latin Square Designs with Replication</u> (same square)

$$Y_{ijkl} = \mu + \rho_i + \gamma_j + \tau_k + \varepsilon_{ijk} + d_{ijkl} \qquad l = 1, \ldots, n$$

<u>Residuals</u>

$$\varepsilon_{ijk} = \text{residual error effect}$$

$$\hat{\varepsilon}_{ijk} = \overline{Y}_{ijk\bullet} - \overline{Y}_{i\bullet\bullet\bullet} - \overline{Y}_{\bullet j\bullet\bullet} - \overline{Y}_{\bullet\bullet k\bullet} + 2\overline{Y}_{\bullet\bullet\bullet\bullet}$$

encompasses any potential interaction effects

$$d_{ijkl} = \text{within cell error effect}$$

$$=$$

Source	d.f.	SS	MS	E(MS)
Trt	*t-1*	*SST*	*MST = SST / (t-1)*	$\sigma^2 + t\sum \tau_k^2/(t-1)$
Rows	*t-1*	*SSR*	*MSR = SSR / (t-1)*	
Cols	*t-1*	*SSC*	*MSC = SSC /(t-1)*	
Error/Resid	*(t-1)(t-2)*	*SSE*	*MSE*	
Within Cell		*SS(WC)*	*MS(WC)*	
Total		*SS(Tot)*		

<u>Latin Square Designs with Replication</u> (*s* different squares)

$$Y_{ijl} = \mu + \kappa_l + \rho_{i(l)} + \gamma_{j(l)} + \tau_k + \varepsilon_{ijl} \qquad i, j, k = 1, \ldots, t \qquad l = 1, \ldots, s$$

Source	d.f.	SS	MS
Trt	*t-1*	*SST*	*MST*
Squares	*s-1*	*SS(squares)*	*MS(squares)*
Rows within Squares		*SSR*	*MSR*
Cols within Squares		*SSC*	*MSC*
Error		*SSE*	*MSE*
Total		*SS(Tot)*	