<u>CRF - ab</u> (Fixed Effects Model)

$$Y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk} \qquad \varepsilon_{ijk} \overset{iid}{\sim} N(0,\sigma_e^2) \qquad \begin{array}{l} i=1,\ldots,a \\ j=1,\ldots,b \\ k=1,\ldots,r \end{array}$$

Source	d.f.	SS	MS	E(MS)
Factor A		$\sum_i\sum_j\sum_k\left(\bar{Y}_{i\bullet\bullet}-\bar{Y}_{\bullet\bullet\bullet}\right)^2$	*MSA*	$\sigma_e^2+\dfrac{\sum\sum\sum\ \alpha_i^2}{a-1}$
Factor B		$\sum_i\sum_j\sum_k\left(\bar{Y}_{\bullet j\bullet}-\bar{Y}_{\bullet\bullet\bullet}\right)^2$	*MSB*	$\sigma_e^2+\dfrac{\sum\sum\sum\ \beta_i^2}{b-1}$
Interaction (A*B)		$\sum_i\sum_j\sum_k\left(\bar{Y}_{ij\bullet}-\bar{Y}_{i\bullet\bullet}-\bar{Y}_{\bullet j\bullet}+\bar{Y}_{\bullet\bullet\bullet}\right)^2$	*MS(AB)*	$\sigma_e^2+\dfrac{\sum\sum\sum\ (\alpha\beta)_{ij}^2}{(a-1)(b-1)}$
Error		$\sum_i\sum_j\sum_k\left(Y_{ijk}-\bar{Y}_{ij\bullet}\right)^2$	*MSE*	σ_e^2
Total	*rab-1*	$\sum_i\sum_j\sum_k\left(Y_{ijk}-\bar{Y}_{\bullet\bullet\bullet}\right)^2$		

<u>CRF - ab</u> (Mixed Effects Model: A Fixed, B Random)

$$Y_{ijk} = \mu + \alpha_i + b_j + (ab)_{ij} + \varepsilon_{ijk} \qquad \begin{array}{l} \varepsilon_{ijk} \overset{iid}{\sim} N(0,\sigma_e^2) \\ b_j \overset{iid}{\sim} N(0,\sigma_b^2) \end{array} \qquad \begin{array}{l} i=1,\ldots,a \\ j=1,\ldots,b \\ k=1,\ldots,r \end{array}$$

Source	d.f.	SS	MS	E(MS)
Factor A		$\sum_i\sum_j\sum_k\left(\bar{Y}_{i\bullet\bullet}-\bar{Y}_{\bullet\bullet\bullet}\right)^2$	*MSA*	$\sigma_e^2+r\sigma_{ab}^2+rb\dfrac{\sum\sum\sum\ \alpha_i^2}{a-1}$
Factor B		$\sum_i\sum_j\sum_k\left(\bar{Y}_{\bullet j\bullet}-\bar{Y}_{\bullet\bullet\bullet}\right)^2$	*MSB*	$\sigma_e^2+r\sigma_{ab}^2+ra\sigma_b^2$
Interaction (A*B)		$\sum_i\sum_j\sum_k\left(\bar{Y}_{ij\bullet}-\bar{Y}_{i\bullet\bullet}-\bar{Y}_{\bullet j\bullet}+\bar{Y}_{\bullet\bullet\bullet}\right)^2$	*MS(AB)*	$\sigma_e^2+r\sigma_{ab}^2$
Error		$\sum_i\sum_j\sum_k\left(Y_{ijk}-\bar{Y}_{ij\bullet}\right)^2$	*MSE*	σ_e^2
Total	*rab-1*	$\sum_i\sum_j\sum_k\left(Y_{ijk}-\bar{Y}_{\bullet\bullet\bullet}\right)^2$		

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Statistical Model for Nested Factors

The factors in the nested design hierarchy can be fixed or random factors. The design for glucose standards in Example 7.3 would have all random factors if days, runs, and replicate serum preparations were considered random samples of their respective populations.

The linear model for a nested design with three random nested factors—A, B within A, and C within B—is

$$y_{ijk} = \mu + a_i + b_{j(i)} + c_{k(ij)} \qquad (7.10)$$

$$i = 1, 2, \ldots, a \qquad j = 1, 2, \ldots, b \qquad k = 1, 2, \ldots, c$$

where a_i is the effect of factor A, $b_{j(i)}$ is the effect of factor B nested within A, and $c_{k(ij)}$ is the effect of factor C nested within B. The subscript $j(i)$ refers to the factor represented by the j subscript nested within the factor represented by the i subscript. The effects a_i, $b_{j(i)}$, and $c_{k(ij)}$ are assumed to be independent random effects with means 0 and variances σ_a^2, $\sigma_{b(a)}^2$, and $\sigma_{c(b)}^2$, respectively.

The computations for the analysis of variance are identical to those for subsampling as shown in Table 5.5. The expectations for the mean squares with all effects random are given in the abbreviated analysis of variance outline shown in Table 7.8.

Table 7.8 Expected mean squares for the analysis of variance of a nested design with three random factors, A, B, and C

Source of Variation	*Degrees of Freedom*	*Mean Square*	*Expected Mean Square*
Total	$abc - 1$		
A	$a - 1$	MSA	$\sigma_{c(b)}^2 + c\sigma_{b(a)}^2 + bc\sigma_a^2$
B within A	$a(b - 1)$	$MS(B/A)$	$\sigma_{c(b)}^2 + c\sigma_{b(a)}^2$
C within B	$ab(c - 1)$	$MS(C/B)$	$\sigma_{c(b)}^2$

$y_{ijk} = \mu + a_i + b_{j(i)} + c_{k(ij)}$ No ε_{ijk} term

Standard Errors for Means $\bar{y}_{i..} = \mu + a_i + \bar{b}_{\cdot(i)} + \bar{c}_{\cdot(i\cdot)}$

The variances for the grand mean for the study $\bar{y}_{...}$ and a day mean $\bar{y}_{i..}$, are

$\bar{y}_{...} = \mu + \bar{a}_{\cdot} + \bar{b}_{\cdot(\cdot)} + \bar{c}_{\cdot(\cdot\cdot)}$

$$\sigma_{\bar{y}_{...}}^2 = \frac{\sigma_{c(b)}^2}{abc} + \frac{\sigma_{b(a)}^2}{ab} + \frac{\sigma_a^2}{a} \quad \text{and} \quad \sigma_{\bar{y}_{i..}}^2 = \frac{\sigma_{c(b)}^2}{bc} + \frac{\sigma_{b(a)}^2}{b} + \sigma_a^2 \qquad (7.12)$$

respectively. The estimates are

$$s_{\bar{y}_{...}}^2 = \frac{MS\ \text{Day}}{abc} = \frac{6.88}{(3)(2)(3)} = 0.38$$

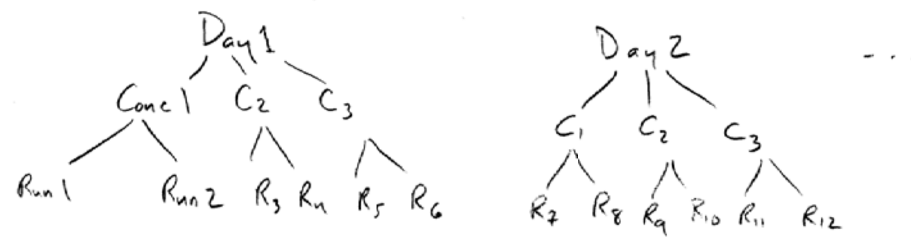
and

$$s_{\bar{y}_{i..}}^2 = \frac{MS(\text{Day})}{bc} \neq \frac{MS(\text{Run/Day})}{bc}$$

Example 7.5 (Calibration/Standardization of a new spectrophotometer)

Factors: Day of Experiment (D) . 1, 2, 3 (Random Effect)
Concentration of Glucose (C) 1, 2, 3 (Fixed Effect)
Machine Run (R) 1, 2 (Random Effect)

* 2 Samples are taken for each Run



$$y_{ijkl} = \mu + \underbrace{\alpha_i}_{\text{Conc}} + \underbrace{d_j}_{\text{Day}} + \underbrace{r_{k(j)}}_{\text{Run}} + \underbrace{(ad)_{ij}}_{C*D} + \underbrace{(ar)_{ik(j)}}_{C*R} + \underbrace{e_{ijkl}}_{\text{Error}}$$

Contrast of interest: $C = \mu_{1..} - \mu_{2..}$

$\hat{C} = \bar{y}_{1...} - \bar{y}_{2...}$