

Split-plot Design Examples

Assign ...

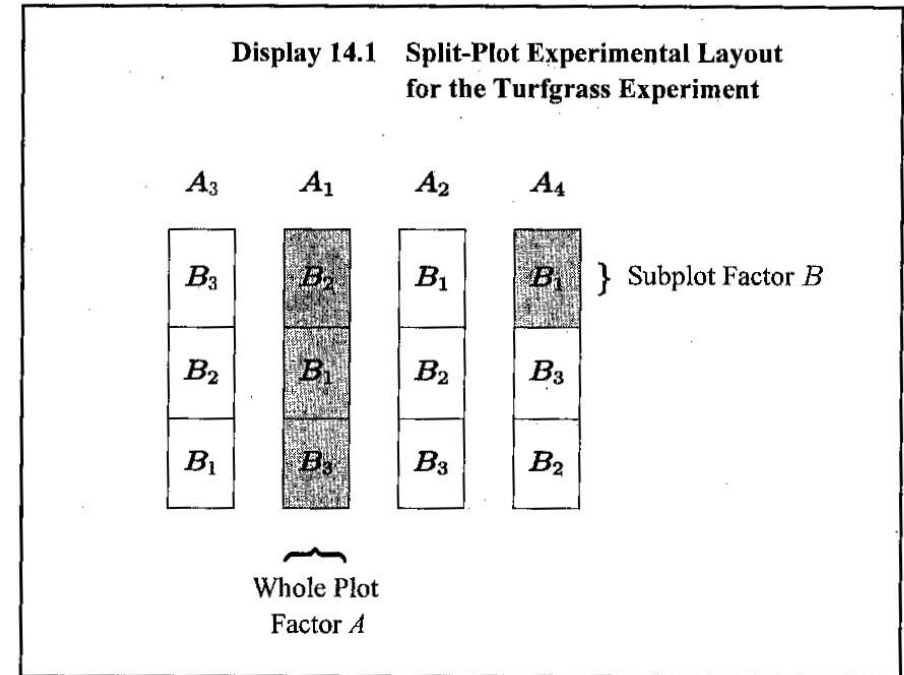
- Type of fabric to an entire bolt of cloth, pre-treatment to individual pieces of fabric
- Batches of initial concentrations (in a drug assay study), dilutions to plates
- Temperature to a growth chamber, nutrients to individual petri dishes within the chamber
- Temperature to a growth chamber, diets to individual cages within the chamber
- diets to each pen, hormone treatments can be applied to each animal
- Temperature to an oven, components to specified baking times
- Crop varieties to plots, irrigation treatments to a section or subplot
- Crop varieties to plots, treatments (pest management) to subplots

Example 14-1

DV: chlorophyll content (mg/g) of grass for golf course (Creeping Bent Grass)

IVs:

- Nitrogen source (4 forms of nitrogen fertilizer)
- Years of thatch buildup (2, 5, 8 yrs) – variable added to an existing RCB design



Source: Dr. I. Pepper, Department of Soil and Water Science, University of Arizona.

An Altered Randomization for Split-
One of 2 blocks (or replications) is shown

Split Plot Designs

$$Y_{ijk} = \mu + \alpha_i + \rho_k + d_{ik} + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

- ρ_k : effect of the k^{th} level of the block ($k = 1, \dots, r$)
- α_i : effect of the i^{th} level of **Whole Plot** (WP) factor A ($i = 1, \dots, a$)
- β_j : effect of the j^{th} level of **Sub Plot** (SP) factor B ($j = 1, \dots, b$)
- $(\alpha\beta)_{ij}$: interaction effect

$$d_{ik} \stackrel{iid}{\sim} N(0, \sigma_d^2) : \text{Whole Plot (WP) random error (Error}_1)$$

$$\varepsilon_{ijk} \stackrel{iid}{\sim} N(0, \sigma_e^2) : \text{Sub Plot (SP) random error (Error}_2)$$

Source	d.f.	MS	E(MS)
Blocks	$r-1$	$MS(Block)$	
A	$a-1$	MSA	$\sigma_e^2 + b\sigma_d^2 + rb\theta_a^2$
Error₁ (WP)	$(a-1)(r-1)$	MSE_1	$\sigma_e^2 + b\sigma_d^2$
B	$b-1$	MSB	$\sigma_e^2 + rab\theta_b^2$
AB	$(a-1)(b-1)$	$MS(AB)$	$\sigma_e^2 + r\theta_{ab}^2$
Error₂ (SP)	$a(b-1)(r-1)$	MSE_2	σ_e^2
Total	$abr - 1$		

The name of the Split-Plot (prefix) reflects the design associated with the WP experimental units.

Randomization

- The first factor levels are randomly assigned to the WP EUs according to the WP design (CRD, RBD, LSDx)
- The second factor levels are randomly assigned sub-units within each WP EU according to the rules for a RBD.

3 types of split-plot designs (with r replications of each design)

CRD		RBD		LSDx	
Source	d.f.	Source	d.f.		
		Blocks	$r-1$	Rows	$a-1$
				Cols	$a-1$
A	$a-1$	A	$a-1$	A	$a-1$
Error₁ (WP)	$a(r-1)$	Error₁ (WP)	$(a-1)(r-1)$	Error₁ (WP)	$(a-1)(a-2)$
SubTotal(WP)	$ar - 1$		$ar - 1$		$aa - 1$
B	$b-1$	B	$b-1$	B	$b-1$
AB	$(a-1)(b-1)$	AB	$(a-1)(b-1)$	AB	$(a-1)(b-1)$
Error₂ (SP)	$a(r-1)(b-1)$	Error₂ (SP)	$a(r-1)(b-1)$	Error₂ (SP)	$a(a-1)(b-1)$
SubTotal(SP)	$ar(b-1)$		$ar(b-1)$		$aa(b-1)$
Total	$abr - 1$		$abr - 1$		$aar - 1$

A Randomized Block split-plot design with r replications

- Factors A & B in a CRF-ab on Whole units
- Factor C on the Sub-units
- r replications of the design (i.e., r blocks)