

```

PROC GLM DATA=tubes;
  CLASS voltage exhaust;
  MODEL pressure = voltage exhaust voltage*exhaust;
  RANDOM exhaust voltage*exhaust / TEST;
  RUN;

```

The GLM Procedure

| Class | Levels | Values |
|---------|--------|-------------|
| voltage | 3 | 120 170 220 |
| exhaust | 3 | 60 90 120 |

Number of Observations Used 18

Dependent Variable: pressure

| Source | DF | Sum of Squares | Mean Square | F Value | Pr > F |
|-----------------|----|----------------|-------------|---------|--------|
| Model | 8 | 6532.444444 | 816.555556 | 50.34 | <.0001 |
| Error | 9 | 146.000000 | 16.222222 | | |
| Corrected Total | 17 | 6678.444444 | | | |

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|-----------------|----|-------------|-------------|---------|--------|
| voltage | 2 | 140.777778 | 70.388889 | 4.34 | 0.0479 |
| exhaust | 2 | 6044.111111 | 3022.055556 | 186.29 | <.0001 |
| voltage*exhaust | 4 | 347.555556 | 86.888889 | 5.36 | 0.0174 |

| Source | Type III Expected Mean Square |
|-----------------|--|
| voltage | Var(Error) + 2 Var(voltage*exhaust) + Q(voltage) |
| exhaust | Var(Error) + 2 Var(voltage*exhaust) + 6 Var(exhaust) |
| voltage*exhaust | Var(Error) + 2 Var(voltage*exhaust) |

Tests of Hypotheses for Mixed Model Analysis of Variance

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|---------|----|-------------|-------------|---------|--------|
| voltage | 2 | 140.777778 | 70.388889 | 0.81 | 0.5065 |
| exhaust | 2 | 6044.111111 | 3022.055556 | 34.78 | 0.0030 |
| Error | 4 | 347.555556 | 86.888889 | | |

Error: MS(voltage*exhaust)

| Source | DF | Type III SS | Mean Square | F Value | Pr > F |
|------------------|----|-------------|-------------|---------|--------|
| voltage*exhaust | 4 | 347.555556 | 86.888889 | 5.36 | 0.0174 |
| Error: MS(Error) | 9 | 146.000000 | 16.222222 | | |

```

PROC MIXED DATA=tubes;
  CLASS voltage exhaust;
  MODEL pressure = voltage ;
  RANDOM exhaust voltage*exhaust / TYPE=VC;
  RUN;

```

For PROC MIXED the statistical model is: $Y = X * \text{Beta} + Z * \text{Gamma} + \text{Epsilon}$

where

Y is the vector of observed data,
 Beta is an unknown vector of fixed effects with known model matrix X,
 Gamma is an unknown vector of random effects with known model matrix Z, and
 Epsilon is an unknown random error vector.

and the

MODEL statement generates one or more columns in the model matrix X, and the
 RANDOM statement generates one or more columns in the model matrix Z.

The Mixed Procedure

Model Information

| | |
|---------------------------|---------------------|
| Data Set | WORK.TUBES |
| Dependent Variable | pressure |
| Covariance Structure | Variance Components |
| Estimation Method | REML |
| Residual Variance Method | Profile |
| Fixed Effects SE Method | Model-Based |
| Degrees of Freedom Method | Containment |

| Class | Levels | Values |
|---------|--------|-------------|
| voltage | 3 | 120 170 220 |
| exhaust | 3 | 60 90 120 |

Dimensions

| | |
|-----------------------|----|
| Covariance Parameters | 3 |
| Columns in X | 4 |
| Columns in Z | 12 |

Number of Observations Used 18

Iteration History

| Iteration | Evaluations | -2 Res Log Like | Criterion |
|-----------|-------------|-----------------|------------|
| 0 | 1 | 139.10271542 | |
| 1 | 1 | 106.90677851 | 0.00000000 |

Convergence criteria met.

Covariance Parameter Estimates

| Cov Parm | Estimate |
|-----------------|----------|
| exhaust | 489.19 |
| voltage*exhaust | 35.3333 |
| Residual | 16.2222 |

Fit Statistics

| | |
|--------------------------|-------|
| -2 Res Log Likelihood | 106.9 |
| AIC (smaller is better) | 112.9 |
| AICC (smaller is better) | 115.1 |
| BIC (smaller is better) | 110.2 |

Type 3 Tests of Fixed Effects

| Effect | Num DF | Den DF | F Value | Pr > F |
|---------|--------|--------|---------|--------|
| voltage | 2 | 4 | 0.81 | 0.5065 |