Binomial Probability Model: *n* independent trials with 2 outcomes (*S/F*) and *p*=P(*S*)
Y = # of successes in *n* trials

Y ~ *Binomial*(3,*p*) = # of affected individuals in a sample of size *n*=3

	y				
p	0	1	2	3	**sum**
0.2	0.512	0.348	0.096	0.008	1.0
0.6	0.064	0.288	0.432	0.216	1.0

Bernoulli Trials: a series of independent trials with 2 outcomes (*S/F*) and *p*=P(*S*) is constant
Y~*Bernoulli*(*p*) ←→ *Binomial*(*1*,*p*)

y	P(*Y=y*)		
1	*p*	$f(y) = p^y (1-p)^{1-y}$ $y = 0,1$	a function of *y* for a fixed *p*
0	*(1-p)*	$L(p) = p^y (1-p)^{1-y}$ $0 \le p \le 1$	a function of *p* for a fixed *y*

	Disease	No Disease	
Exposed	15	85	100
Not	10	90	100
	25	175	

Disease Odds Ratio (DOR)

$$= \frac{O(D|E)}{O(D|\bar{E})} = \frac{\frac{P(D|E)}{[1 - P(D|E)]}}{\frac{P(D|\bar{E})}{[1 - P(D|\bar{E})]}}$$

$$= \frac{\frac{15}{100} \Big/ \frac{85}{100}}{\frac{10}{100} \Big/ \frac{90}{100}} = \frac{15 * 90}{10 * 85}$$

Exposure Odds Ratio (EOR)

$$EOR = \frac{O(E|D)}{O(E|\bar{D})} = \frac{\frac{P(E|D)}{[1 - P(E|D)]}}{\frac{P(E|\bar{D})}{[1 - P(E|\bar{D})]}}$$

When the probability of the disease is small, the Odds Ratio (OR) gives a good approximation to the Relative Risk (RR).

Sampling Distribution for ***ln(OR)***

- $\ln(\hat{O}R) = \ln\left(\frac{a \cdot d}{b \cdot c}\right)$

- Approximately normal with …

$$SE = \sqrt{\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{1}{d}}$$

Bernoulli Trials: Y~*Bernoulli*(*p*) ←→ *Binomial*(*1,p*)

y	$P(Y=y)$		
1	p	$f(y) = p^y (1-p)^{1-y}$ $\quad y = 0,1$	a function of y for a fixed p
0	$(1-p)$	$L(p) = p^y (1-p)^{1-y}$ $\quad 0 \le p \le 1$	a function of p for a fixed y

Likelihood function for a sample of n Bernoulli trials

$$L(p) = \prod_{i=1}^{n} p^{y_i}(1-p)^{1-y_i}$$
$$= \exp\left[(\ln p)^{\sum y_i} + (\ln(1-p))^{\sum(1-y_i)}\right]$$
$$= \exp\left[\sum_{i=1}^{n}\left\{y_i \ln\left(\frac{p}{1-p}\right) + \ln(1-p)\right\}\right]$$

1-parameter (no-intercept) Model

Reparametrize the Likelihood function using:

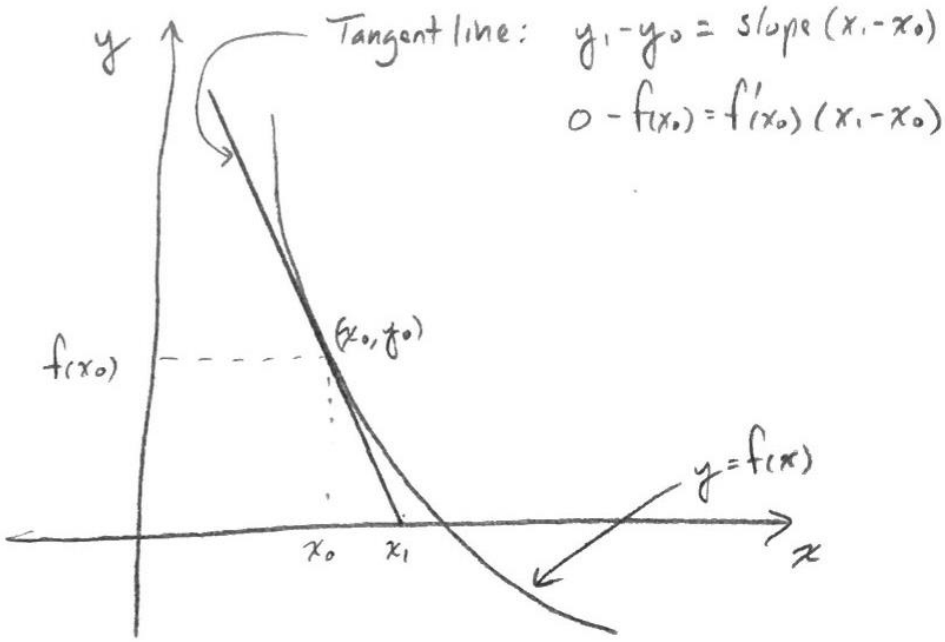
$$\ln\left(\frac{p}{1-p}\right) = \beta x_i \quad \rightarrow \quad (1-p) = \frac{1}{1+\exp(\beta x_i)}$$

$$L(\beta) = \exp\left[\sum_{i=1}^{n}\left\{y_i(\beta x_i) + \ln\left(\frac{1}{1+\exp(\beta x_i)}\right)\right\}\right]$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Newton's Method



1-parameter (no-intercept) Model

Likelihood: $L(\beta) = \exp\left[\sum_{i=1}^{n}\left\{y_i(\beta x_i) + \ln\left(\frac{1}{1+\exp(\beta x_i)}\right)\right\}\right]$

Log Likelihood: $l(\beta) = \sum_{i=1}^{n}\left[y_i \beta x_i - \ln(1+\exp(\beta x_i))\right]$

$\frac{\mathrm{d}l}{\mathrm{d}\beta} = 0$ → Solve for the MLE of β

Let $f(\beta) = \frac{\mathrm{d}l}{\mathrm{d}\beta} = \sum_{i=1}^{n}\left[y_i x_i - \frac{x_i \exp(\beta_n x_i)}{1+\exp(\beta_n x_i)}\right]$

$$\hat{\beta}_{n+1} = \hat{\beta}_n - \frac{f(\hat{\beta}_n)}{f'(\hat{\beta}_n)}$$

- In R, you will see these iterations indicated as Fisher Scoring iterations.
- Rather than Sums of Squares, you will see Deviance values for models.
- Rather than residuals (observed – predicted), you will see Deviance Residuals