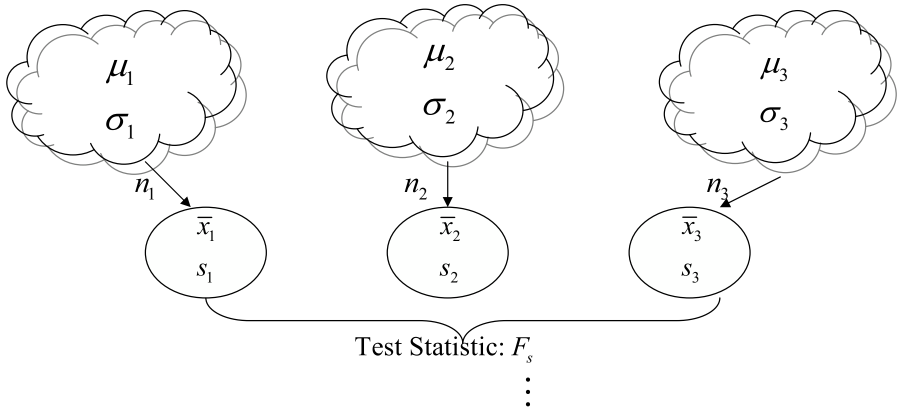


Test $H_0: \mu_1 = \mu_2$ at the α=.05 *l.o.s.*
$H_0: \mu_1 = \mu_3$ at the α=.05 *l.o.s.*
$H_0: \mu_2 = \mu_3$ at the α=.05 *l.o.s.*

⇨ α=P(type I error) for each test $\left(=\alpha_{\text{Comparison-Wise}}\right)$

P(at least one type I error) >>α $\left(=\alpha_{\text{Experiment-Wise}}\right)$

<u>AN</u><u>alysis</u> **<u>O</u>**<u>f</u> **<u>VA</u>**<u>riance</u> **(ANOVA)**

$H_0: \mu_1 = \mu_2 = \mu_3$

H_a : not all means are equal

$$F_s = \frac{\text{variation } \textit{between} \text{ samples}}{\text{variation } \textit{within} \text{ samples (pooled)}}$$

ANalysis Of VAriance

	Source	**d.f.**	**SS**	**MS**	**F**
(Between Groups)	Treatment	*t-1*	$\sum_{i=1}^{t}\sum_{j=1}^{n_i}\left(\bar{Y}_{i\bullet} - \bar{Y}_{\bullet\bullet}\right)^2$	*SST / (t-1)*	*MST / MSE*
(Within Groups)	Error	*N-t*	$\sum_{i=1}^{t}\sum_{j=1}^{n_i}\left(Y_{ij} - \bar{Y}_{i\bullet}\right)^2$	*SSE / (N-t)*	
	Total	*N-1*	$\sum_{i=1}^{t}\sum_{j=1}^{n_i}\left(Y_{ij} - \bar{Y}_{\bullet\bullet}\right)^2$		

<u>Statistical Model</u>: $Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \qquad \varepsilon_{ij} \sim N(0, \sigma^2)$

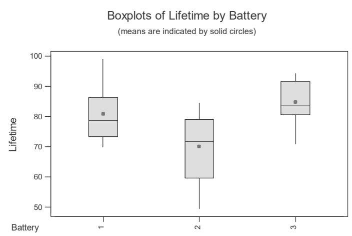
<u>Parameter Estimates</u>:

$\hat{\mu} = \bar{Y}_{\bullet\bullet}$

$\hat{\alpha}_i = \bar{Y}_{i\bullet} - \bar{Y}_{\bullet\bullet}$

$\hat{\varepsilon}_{ij} = Y_{ij} - \bar{Y}_{i\bullet}$

$\hat{\sigma}^2 = MSE$



$$N = n_1 + n_2 + \ldots + n_t$$

$$Y_{ij} = j^{th} \text{ obs. in } i^{th} \text{ treatment group}$$

$$\bar{Y}_{\bullet\bullet} = \sum_{i=1}^{t} \sum_{j=1}^{n_i} Y_{ij} \Big/ N = \text{Grand Mean}$$

$$\bar{Y}_{i\bullet} = \sum_{j=1}^{n_i} Y_{ij} \Big/ n_i \qquad = \text{mean of } i^{th} \text{ trt group}$$

$$SS(Total) = \sum_{i=1}^{t} \sum_{j=1}^{n_i} \left(Y_{ij} - \bar{Y}_{\bullet\bullet} \right)^2$$

Using Only Sample 1 and Sample2

ANOVA

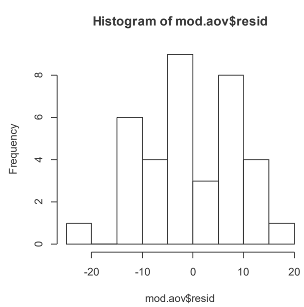
```
> file <- "http://www.uvm.edu/~rsingle/stat211/data/extra/ch6_BatteryLife.txt"
> dat  <- read.table(file, header=T, na.strings=c("NA","."))
> dat.1.2 <- dat[dat$battery %in% 1:2,]
> mod2.aov = aov(lifetime ~ battery, data=dat.1.2)
> summary(mod2.aov)

            Df Sum Sq Mean Sq F value Pr(>F)
battery      1  749.6   749.6   7.312 0.0124 *
Residuals   24 2460.4   102.5
```

T-TEST

```
> t.test(lifetime ~ battery, data=dat2, var.equal=TRUE)

        Two Sample t-test
data:  lifetime by battery
t = 2.7041, df = 24, p-value = 0.01239        [Note: 2.7041^2 = 7.312]
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval: 2.550145, 18.991760
```

Using All 3 Samples

```
> mod3.aov = aov(lifetime ~ as.factor(battery), data=dat)
> summary(mod3.aov)

                   Df Sum Sq Mean Sq F value  Pr(>F)
as.factor(battery)  2   1438   718.8   8.046 0.00142 **
Residuals          33   2948    89.3

  n lifetime.Mean lifetime.SD     SE* Lower95% Upper95%
1 12    80.896667    9.385130  2.7285  75.345   86.448
2 14    70.125714   10.711163  2.5261  64.986   75.265
3 10    84.857000    7.361748  2.9889  78.776   90.938
 *SE uses the pooled estimate of the error variance
  SE = sqrt(MSE/n_i)
```



Post-Hoc Tests / Multiple Comparisons

- **Fisher's Protected T-tests (Least Significant Difference):**

  * Two stage procedure, with protection via overall ANOVA F-test (STOP if overall ***F*** is not significant)

  Critical Value (for difference): $t_{\frac{\alpha}{2}, N-t}$ Note: this is the ordinary T critical value

  Rejection Region: $t_{s(ij)} = \dfrac{\left|\bar{Y}_{i\bullet} - \bar{Y}_{j\bullet}\right|}{\sqrt{MSE(1/n_i + 1/n_j)}} \geq t_{\frac{\alpha}{2}, N-t}$

- **Tukey's HSD Procedure (Studentized Range Test):**

  Critical Value (for difference): $q_\alpha(k, df_{Error})$ for *k* tests using the Studentized Range Distribution

  Rejection Region: $\dfrac{\left|\bar{Y}_{i\bullet} - \bar{Y}_{j\bullet}\right|}{\sqrt{MSE/n}} \geq q_\alpha(k, df_{Error})$

Fisher's LSD

```
pairwise.t.test(dat$lifetime, dat$battery, p.adj = "none") #No adjustment
                                           p.adj = "bonf"
```

Level		Mean
3	A	84.857000
1	A	80.896667
2	B	70.125714

Levels not connected by same letter are significantly different. (unadjusted for Multiple Tests)

Level:Level		Difference	SE.diff	LowerCL	UpperCL	p.value(*unadjusted*)
3	2	14.73129	3.913428	6.76936	22.69322	0.0007*
1	2	10.77095	3.718329	3.20596	18.33595	0.0066*
3	1	3.960330	4.047031	-4.27341	12.19408	0.3349

Tukey's HSD

```
> dat$battery <- as.factor(dat$battery)
> mod.aov <- aov(lifetime ~ battery, data=dat)
> TukeyHSD(mod.aov)
> plot(TukeyHSD(mod.aov))

    Tukey multiple comparisons of means
      95% family-wise confidence level

    Fit: aov(formula = lifetime ~ battery, data = dat)

    $battery
              diff        lwr       upr     p adj
    2-1 -10.770952 -19.894968 -1.646937 0.0177794
    3-1   3.960333  -5.970249 13.890915 0.5953302
    3-2  14.731286   5.128537 24.334035 0.0018431
```