

CS 295A/395D: Artificial Intelligence

Elementary Game Theory

Prof. Emma Tosch

30 March 2022



The University of Vermont

Agenda

Review decision theory

Elementary game theory

- Optimum vs. optimal solutions
- Strategies
- Vocabulary

Logistics

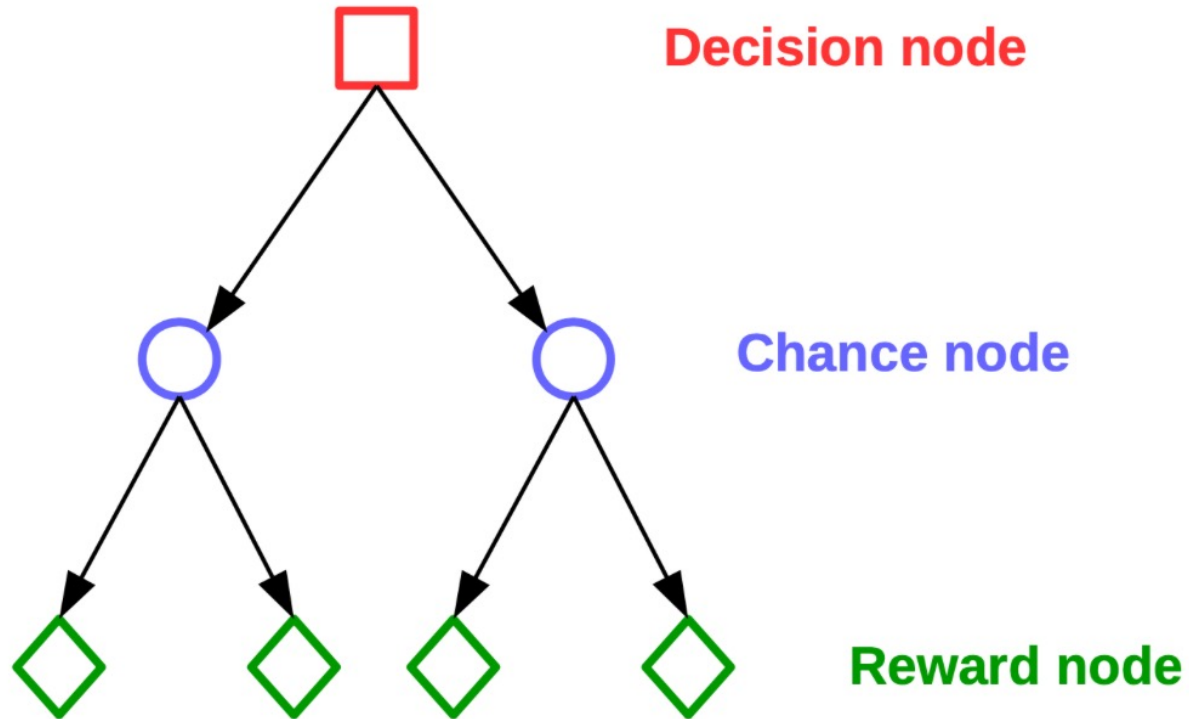
- BB theory assignment out today
- Exam through game theory
 - Temporal reasoning pushed to next unit
 - Exam next Friday (April 8)
- Next unit: temporal reasoning and program synthesis
 - Removed machine learning

Recap: Decision Theory

We can express taking actions in a world with uncertainty via *decision trees*

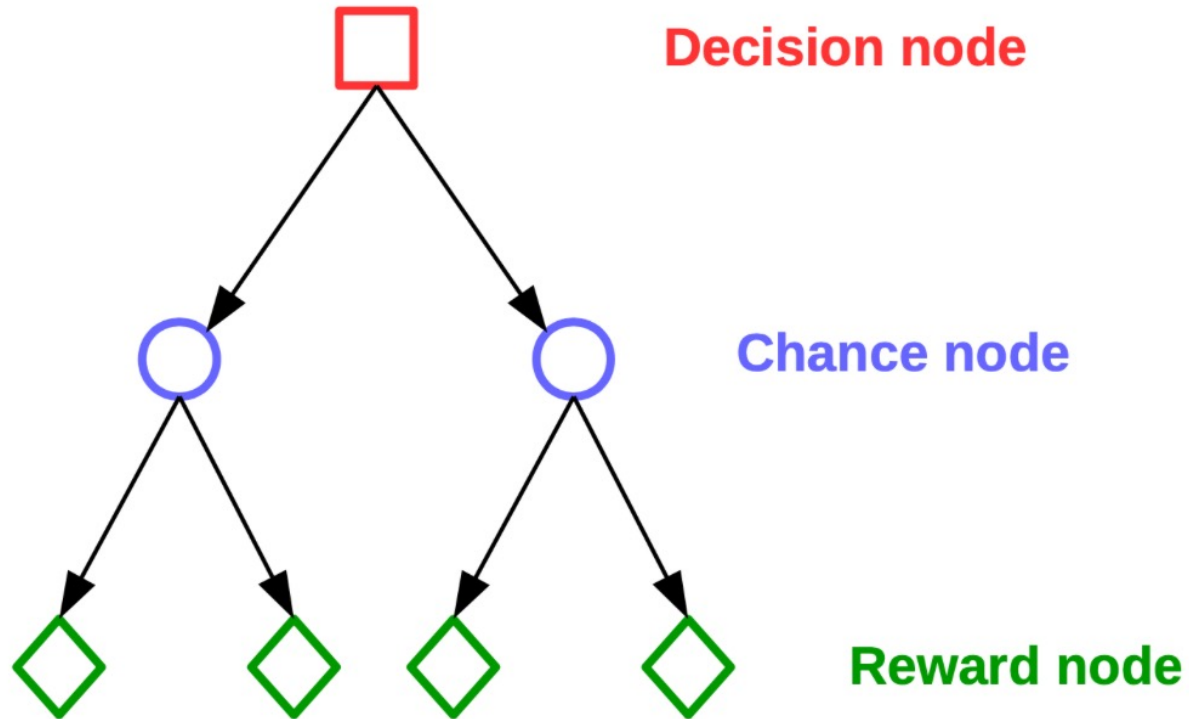
Decision trees are temporally-ordered nodes where each level corresponds to alternating:

- **Decision nodes** – state of the system; outgoing edges represent different actions
- **Chance nodes** – probability distributions over outcomes; outgoing edges represent reachable states with some probability
- **Reward nodes** – utility obtained from following the path



Note: "decision tree" also refers to a classification algorithm in machine learning and is completely different from the type of decision tree we will talk about here.

Recap: Maximize expected utility



$$EU[a | e_1, e_2, \dots] = \sum_{s'} P(S_{t+1} = s' | a, e_1, e_2, \dots) U(s')$$

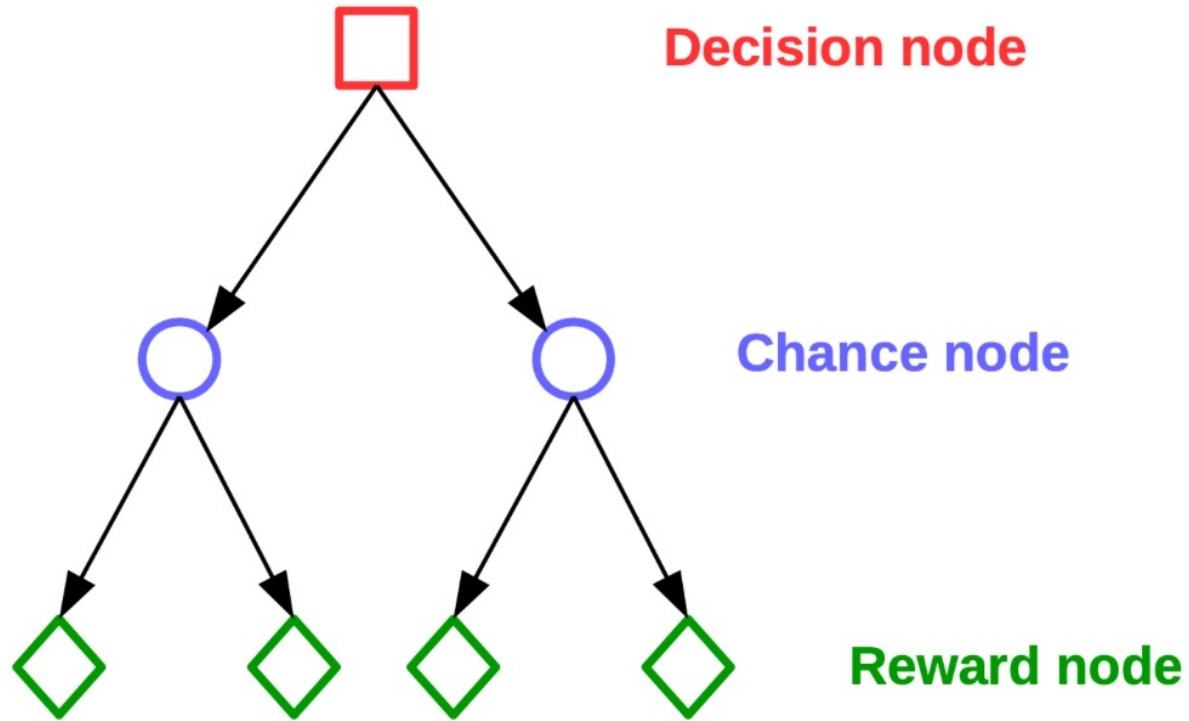
Best action is the action a that maximizes

$$EU[a | e_1, e_2, \dots]$$

Sum of the utility of actions taken.

Note: "decision tree" also refers to a classification algorithm in machine learning and is completely different from the type of decision tree we will talk about here.

Recap: Comparing actions



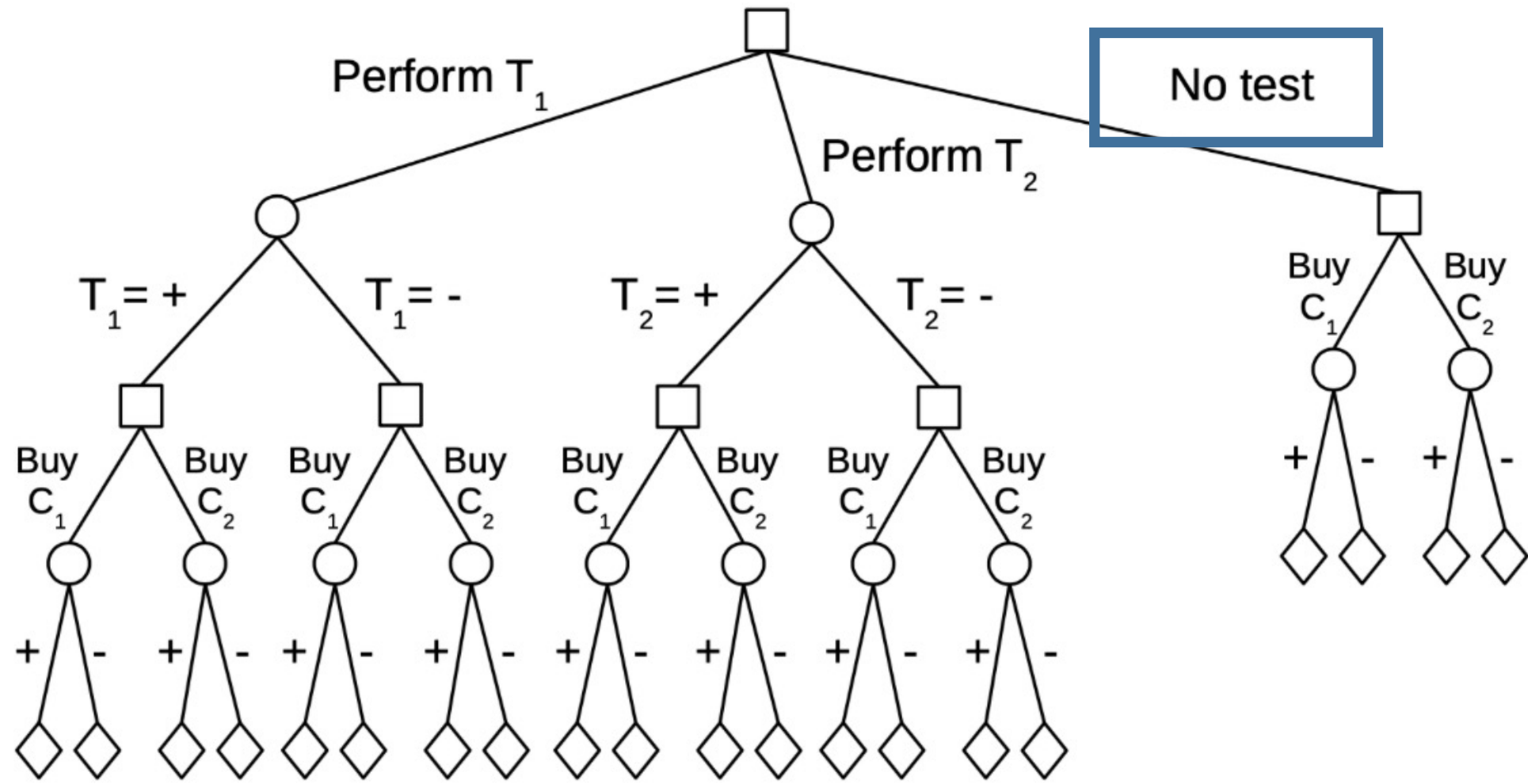
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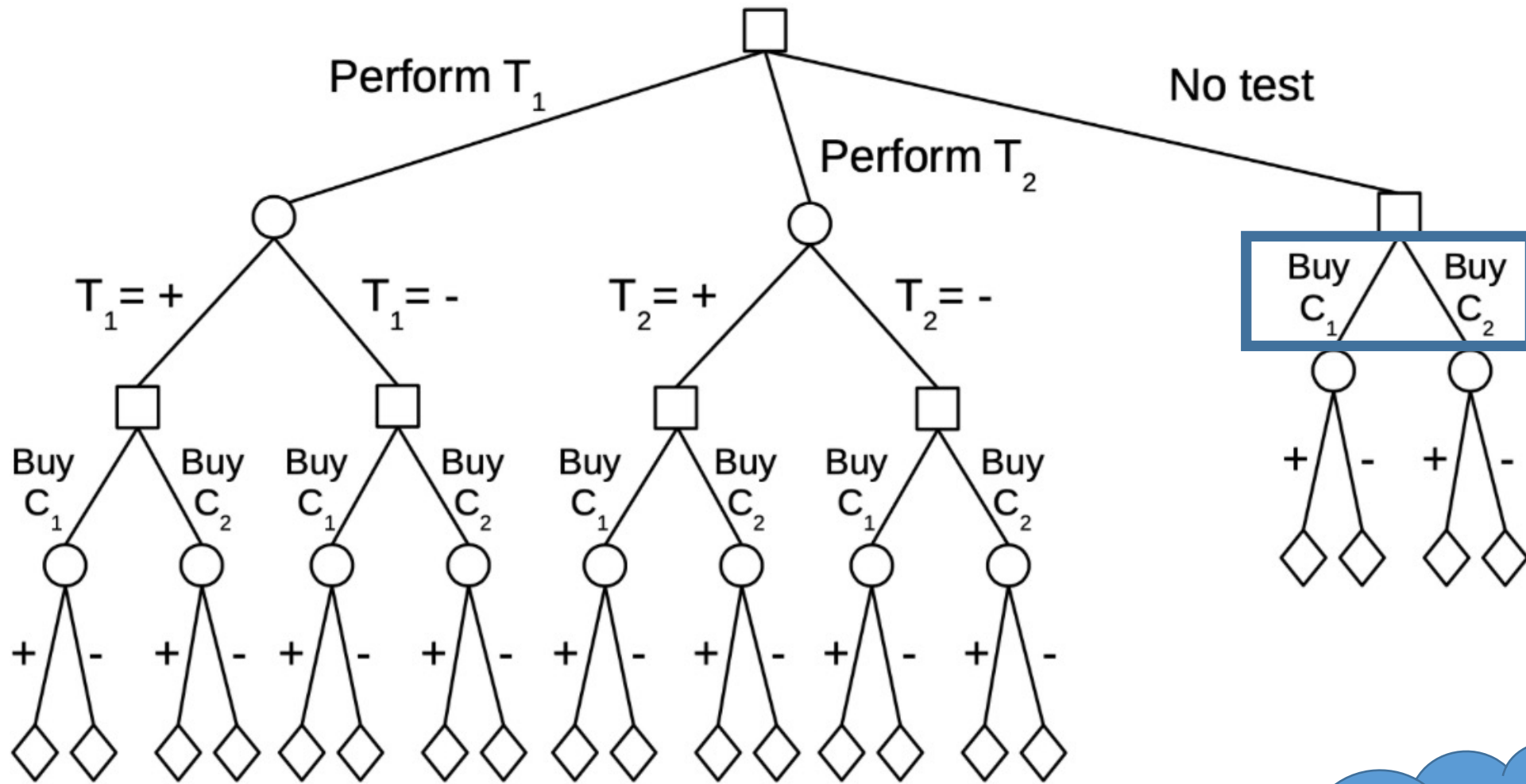
$$EU[a | e_1, e_2, \dots] = \sum_{s'} P(S_{t+1} = s' | a, e_1, e_2, \dots) U(s')$$

Probability mass function – over all sources of uncertainty associated with this action.

Utility function

- Basic actions: reward
- Actions with subsequent actions with uncertain outcomes: EU of those actions

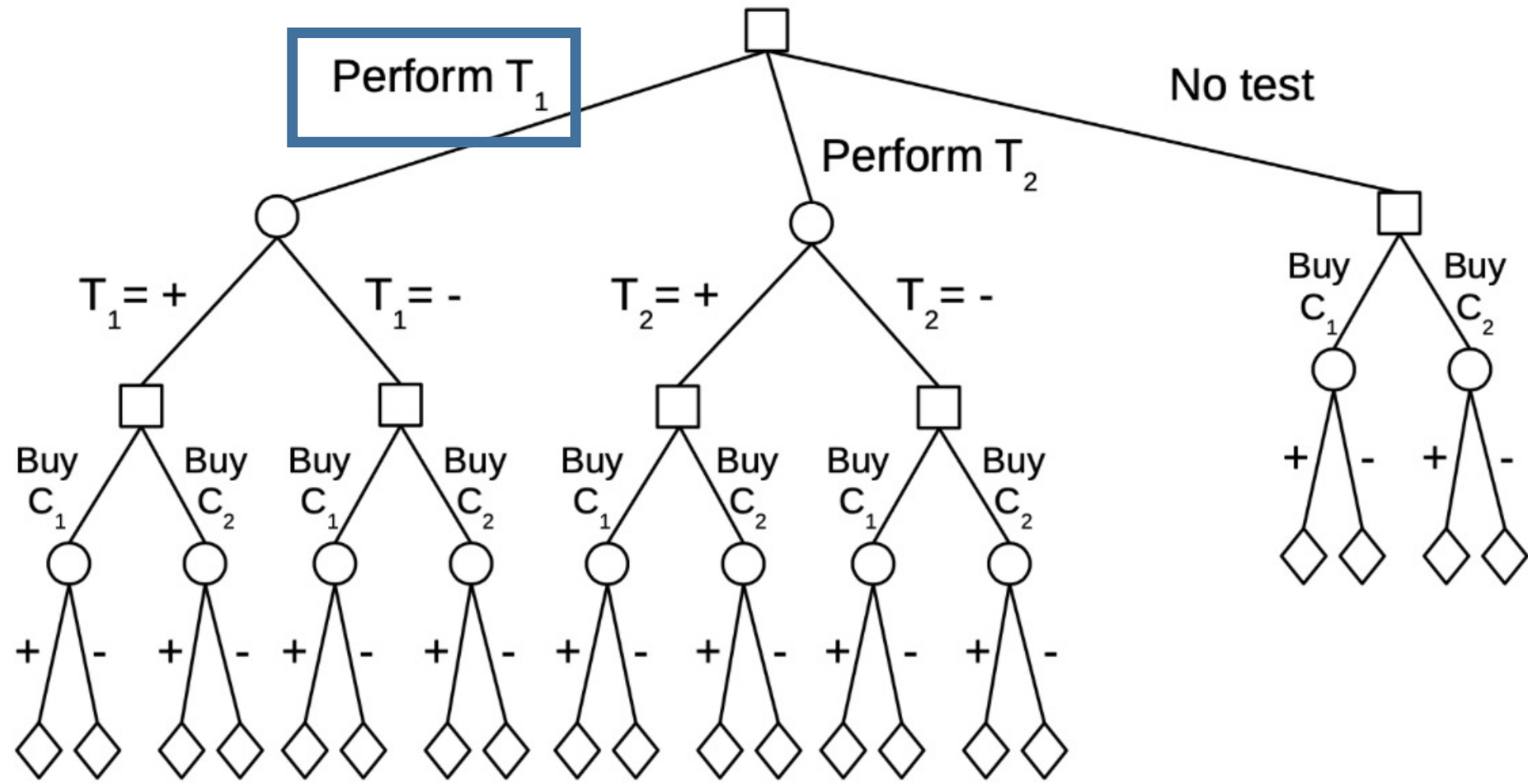


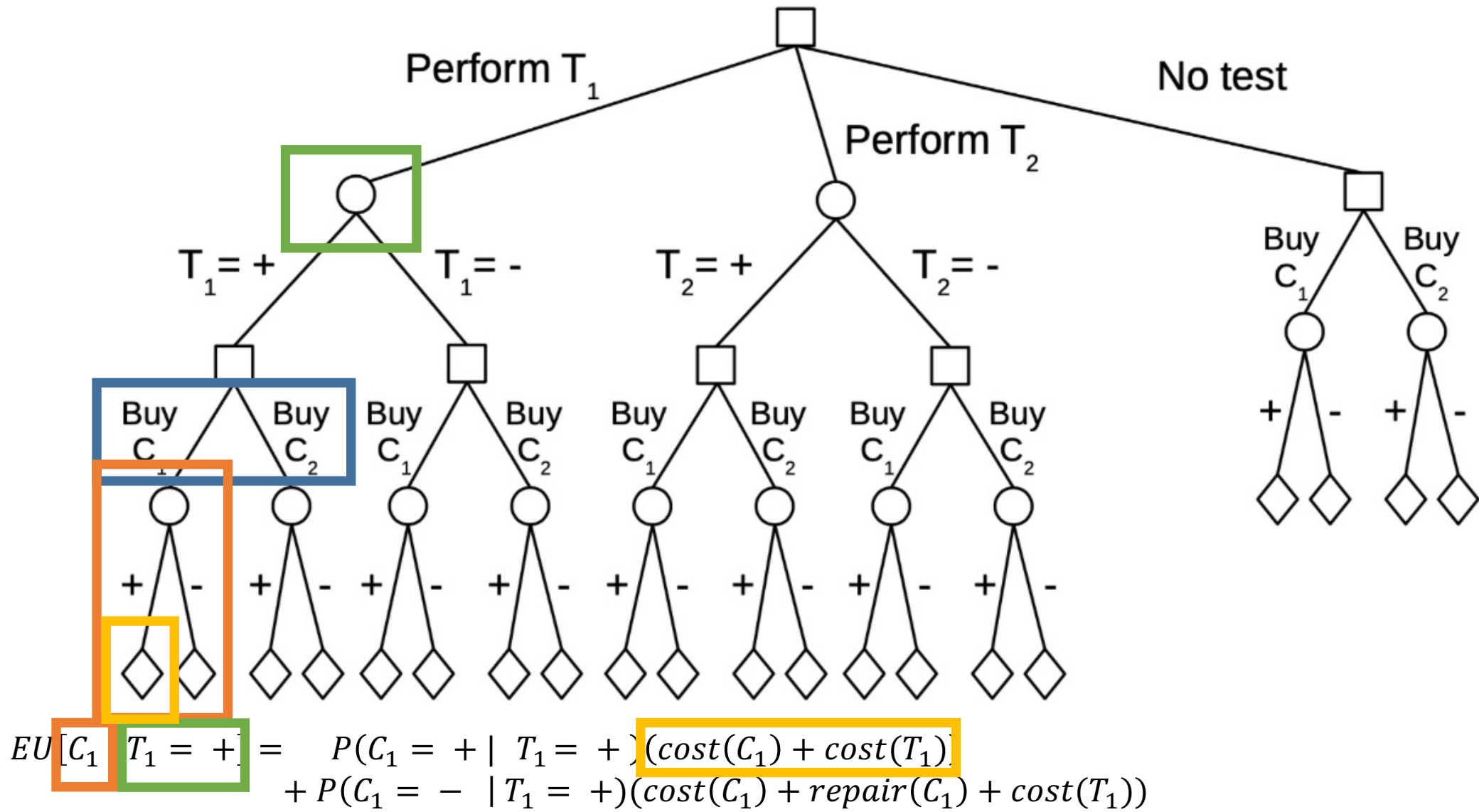


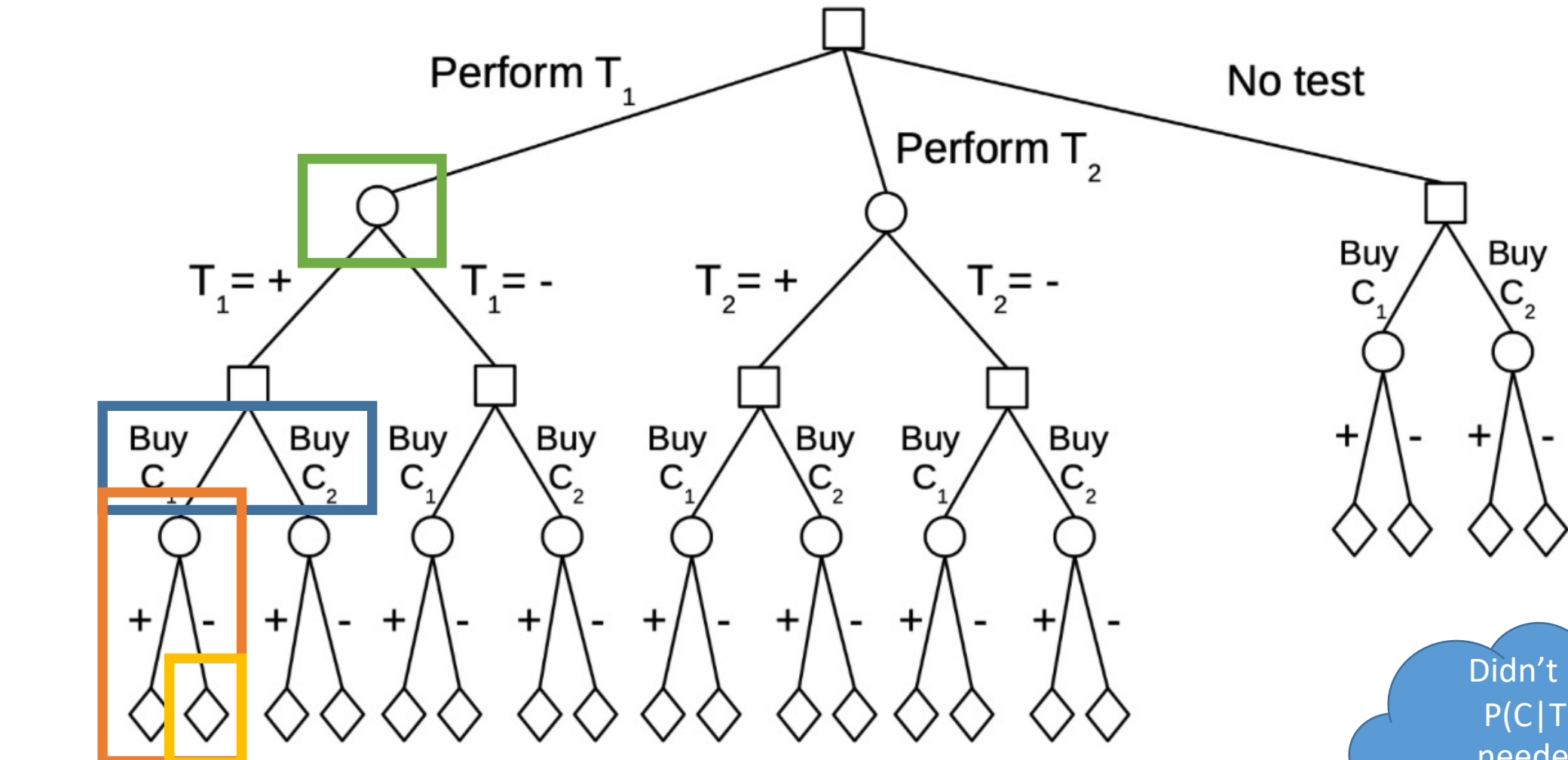
$$EU[C_1] = P(C_1 = +)(cost(C_1)) + P(C_1 = -)(cost(C_1) + repair(C_1))$$

$$EU[C_2] = P(C_2 = +)(cost(C_2)) + P(C_2 = -)(cost(C_2) + repair(C_2))$$

Pick the car with the higher EU

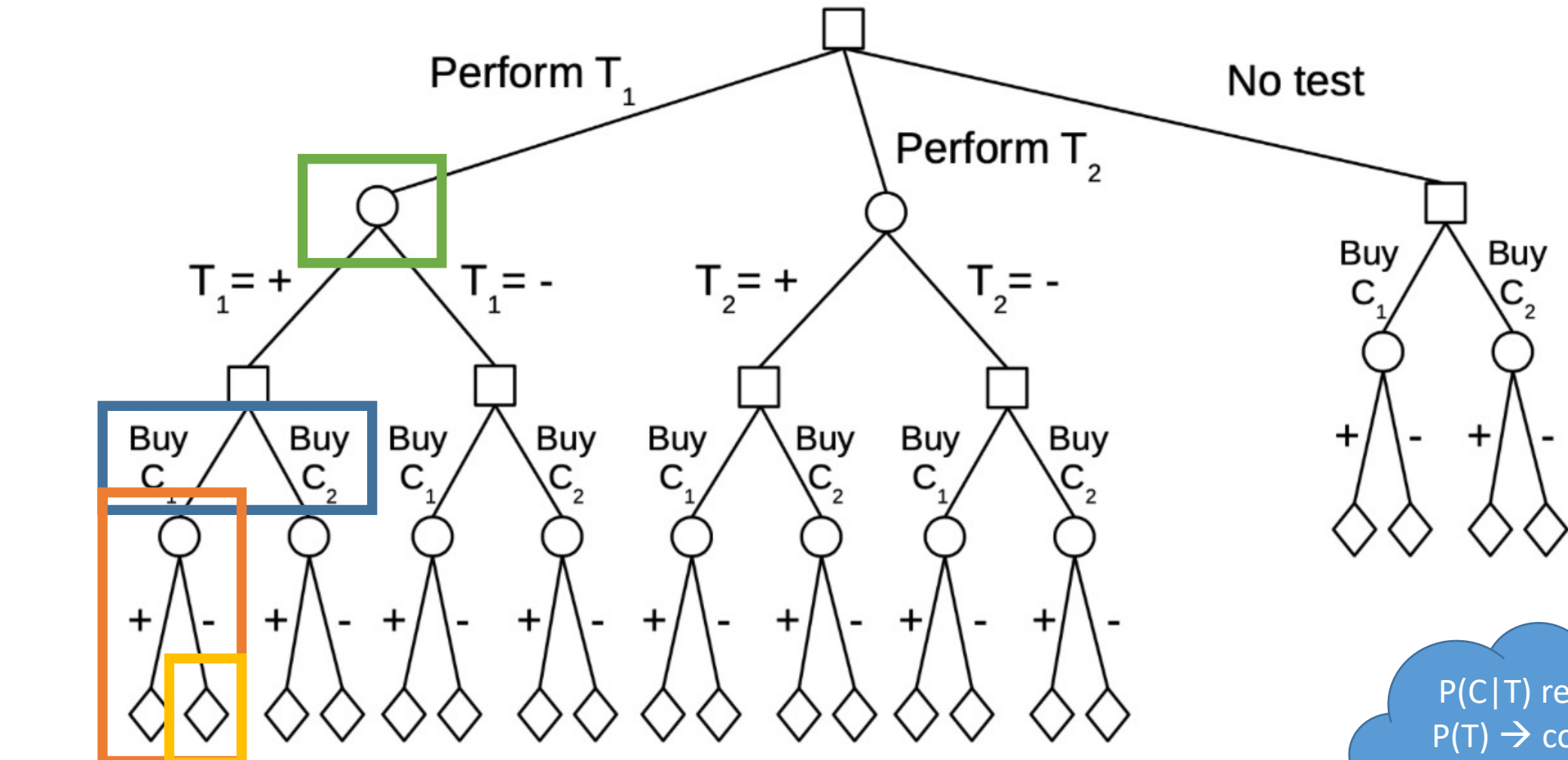






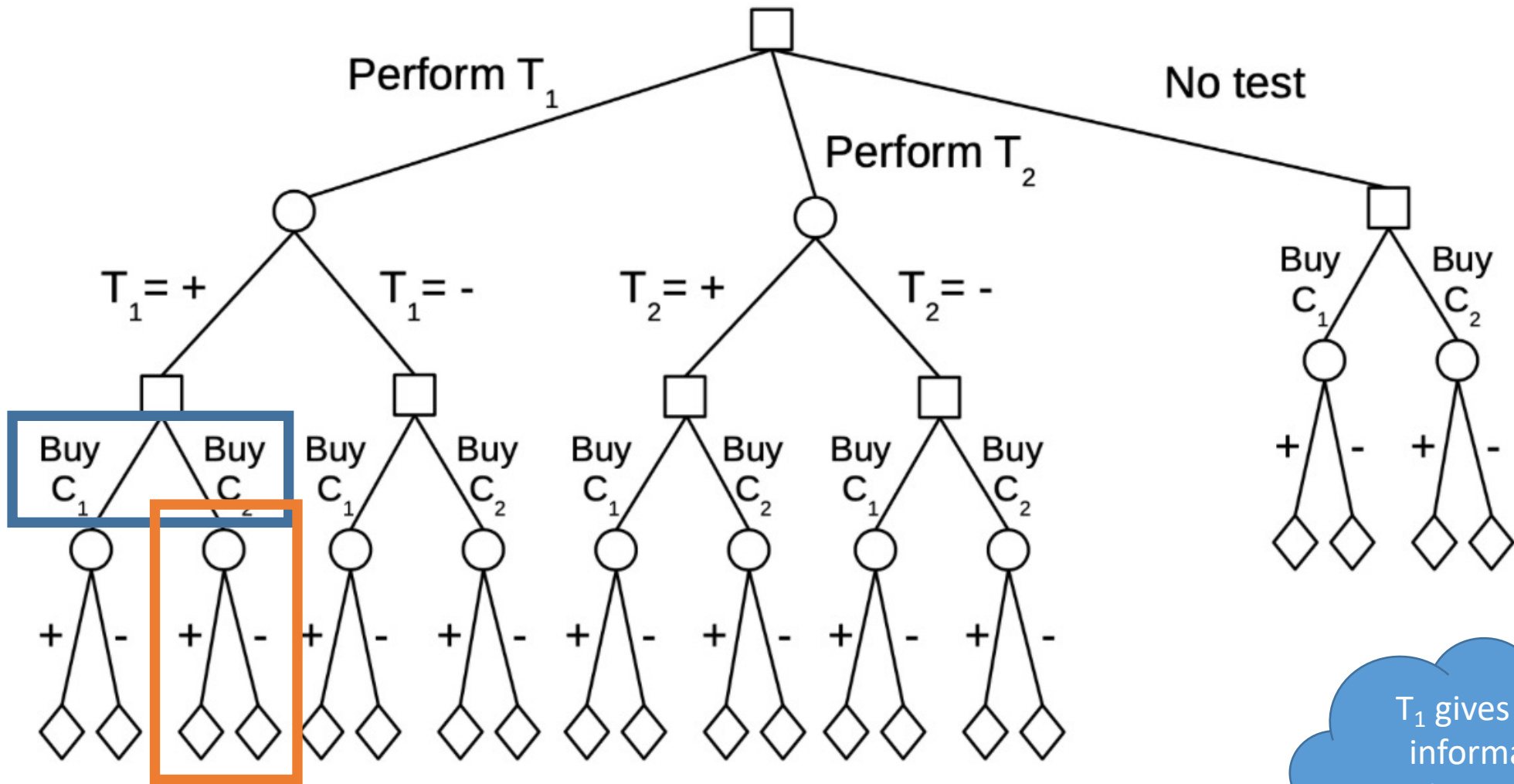
$$EU[C_1 | T_1 = +] = P(C_1 = + | T_1 = +)(cost(C_1) + cost(T_1)) + P(C_1 = - | T_1 = +)(cost(C_1) + repair(C_1) + cost(T_1))$$

Didn't have $P(C|T) \rightarrow$ needed to compute via Bayes Rule



$$EU[C_1 | T_1 = +] = P(C_1 = + | T_1 = +)(cost(C_1) + cost(T_1)) + P(C_1 = - | T_1 = +)(cost(C_1) + repair(C_1) + cost(T_1))$$

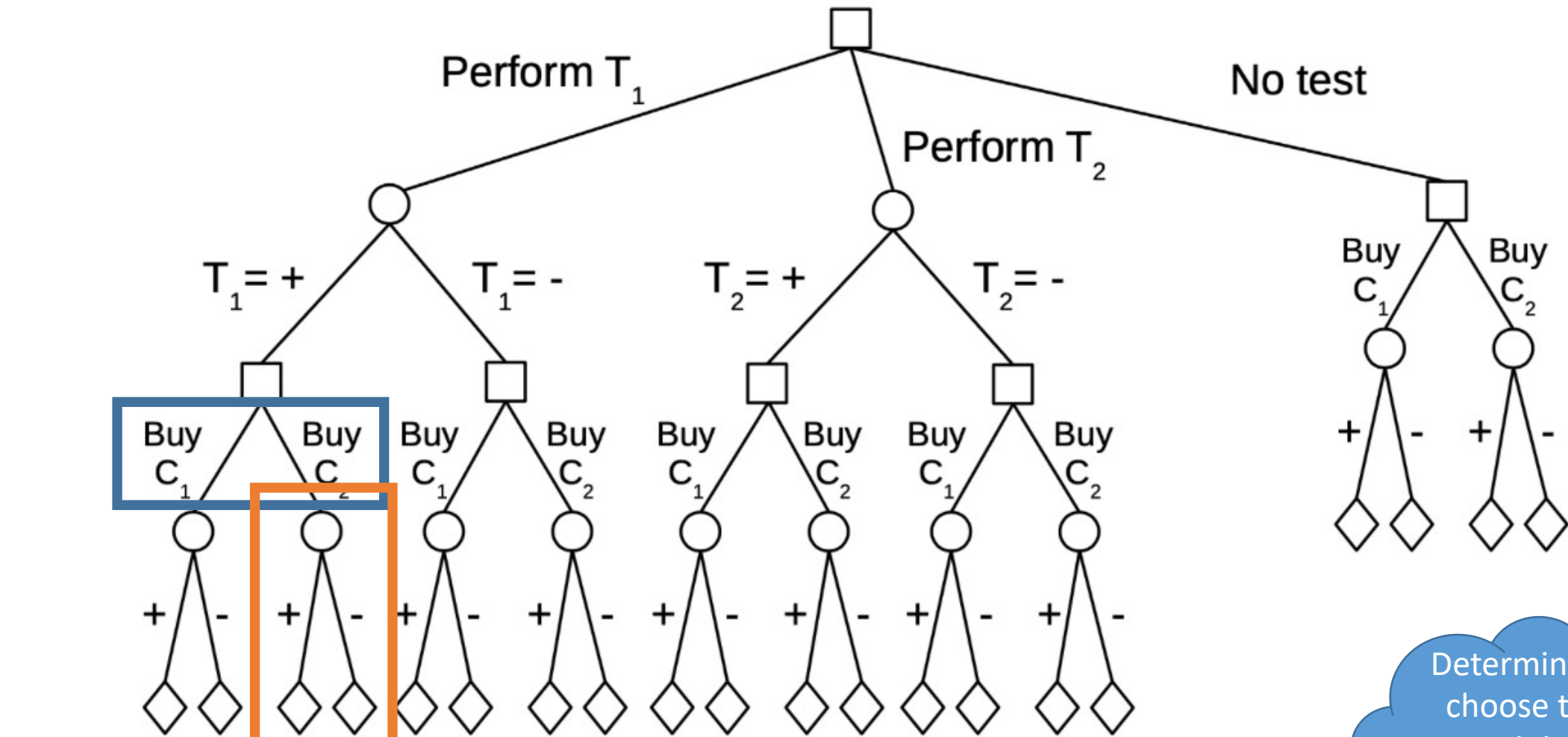
P(C|T) required
P(T) → compute
via law of total
probability



T_1 gives us no information about $C_2 \rightarrow$ independent

$$EU[C_1 | T_1 = +] = P(C_1 = + | T_1 = +)(cost(C_1) + cost(T_1)) + P(C_1 = - | T_1 = +)(cost(C_1) + repair(C_1) + cost(T_1))$$

$$EU[C_2 | T_1 = +]$$

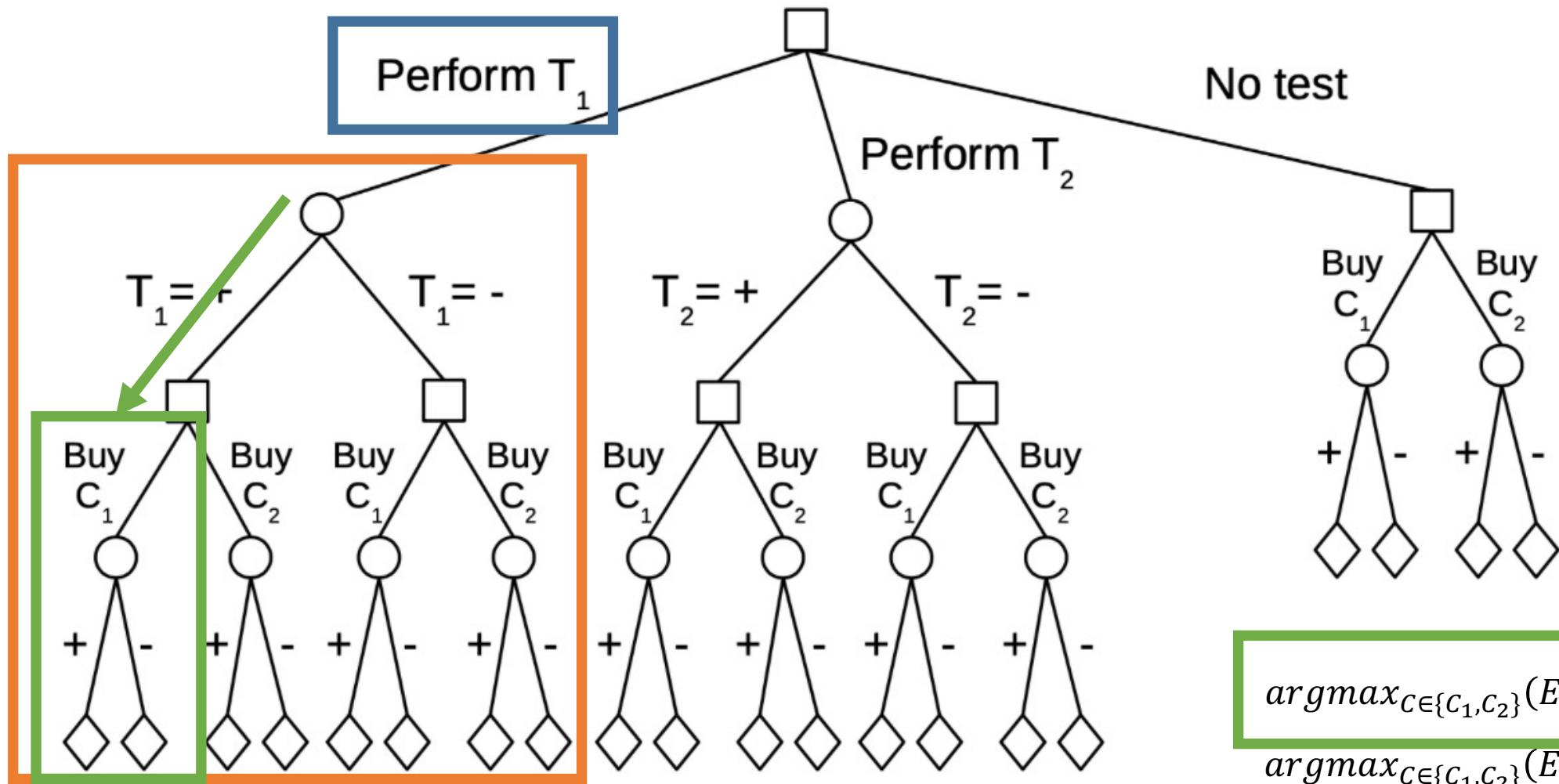


Deterministically choose the car with higher expected utility given $T_1 = +$

$$EU[C_1 | T_1 = +] = P(C_1 = + | T_1 = +)(cost(C_1) + cost(T_1)) + P(C_1 = - | T_1 = +)(cost(C_1) + repair(C_1) + cost(T_1))$$

$$EU[C_2 | T_1 = +] = P(C_2 = +)(cost(C_2) + cost(T_1)) + P(C_2 = -)(cost(C_2) + repair(C_2) + cost(T_1))$$

$$argmax_{C \in \{C_1, C_2\}}(EU[C | T_1 = +])$$

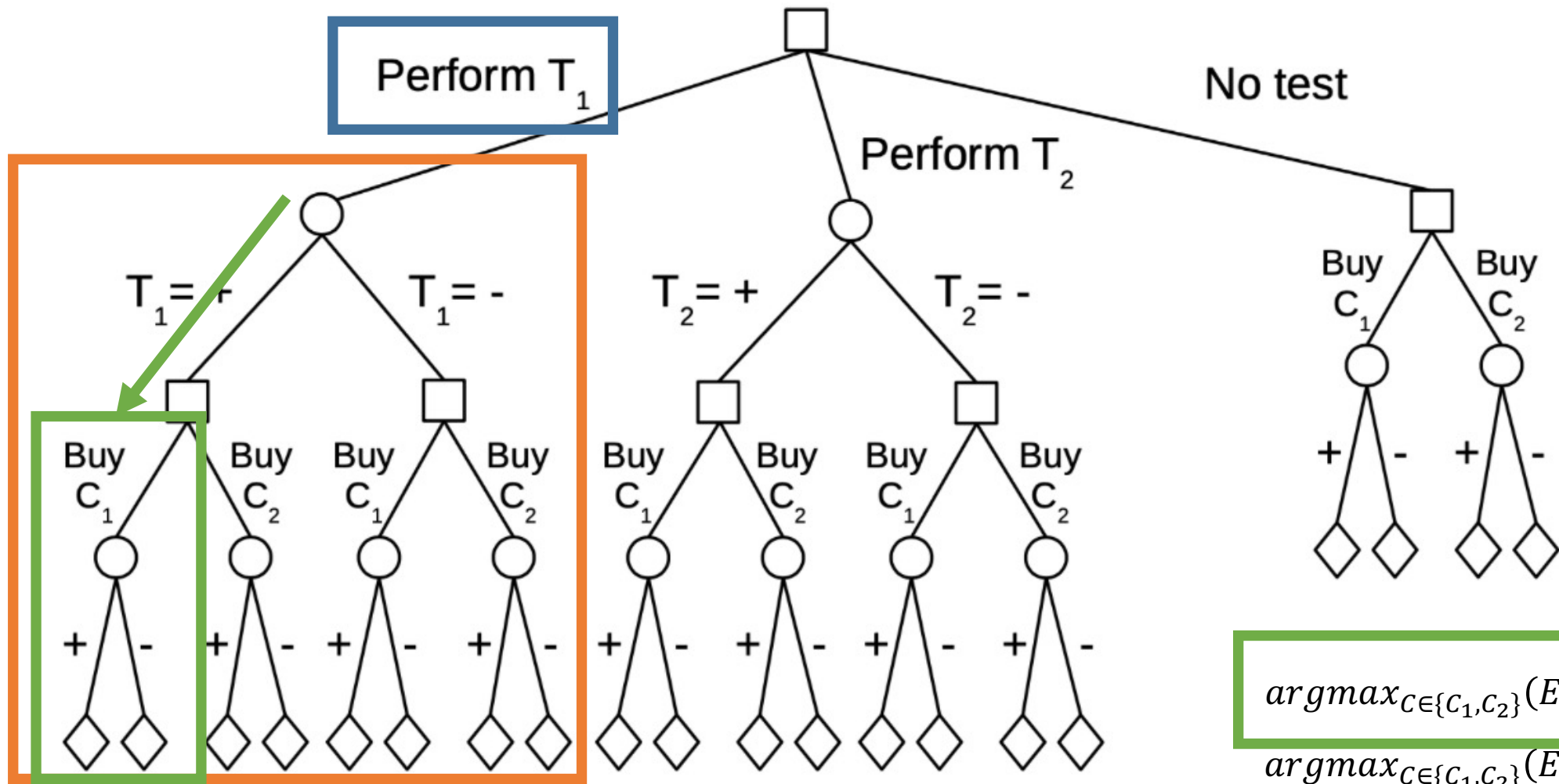


$$EU[T_1] = P(T_1 = +) U(T_1 = +) + P(T_1 = -) U(T_1 = -)$$

????

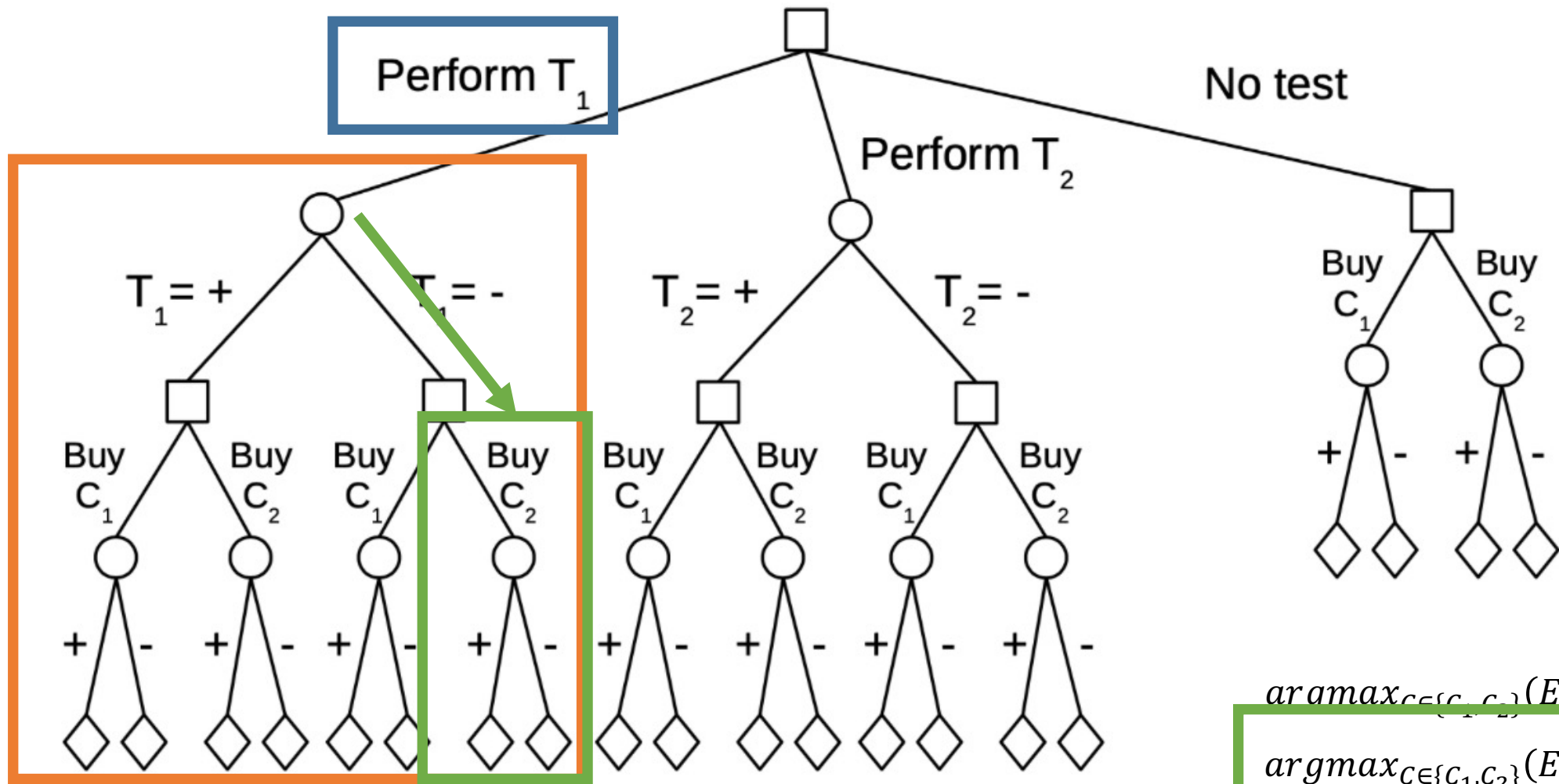
If we deterministically choose C_1 when $T_1 = +$, ...

$$\begin{aligned} & \operatorname{argmax}_{C \in \{C_1, C_2\}} (EU[C | T_1 = +]) \\ & \operatorname{argmax}_{C \in \{C_1, C_2\}} (EU[C | T_1 = -]) \end{aligned}$$



$$EU[T_1] = P(T_1 = +)EU(C_1 | T_1 = +) + P(T_1 = -)U(T_1 = -)$$

If we deterministically choose C_1 when $T_1 = +$, ...



$$\operatorname{argmax}_{C \in \{C_1, C_2\}} (EU[C|T_1 = +])$$

$$\operatorname{argmax}_{C \in \{C_1, C_2\}} (EU[C|T_1 = -])$$

$$EU[T_1] = P(T_1 = +)EU(C_1|T_1 = +) + P(T_1 = -)U(T_1 = -)$$

If we deterministically choose C_2 when $T_1 = -$, ...

When uncertainty comes from another agent's actions

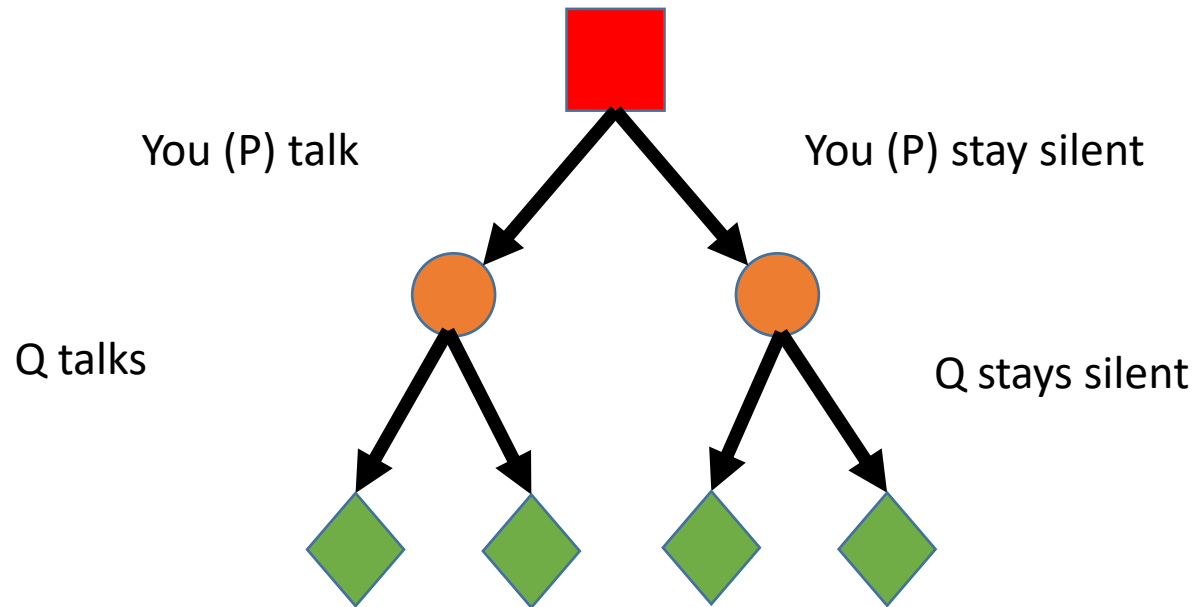
Car example: taking an action in one branch closes off possibilities in another

- Randomness comes from
 - Epistemic uncertainty about effects of past actions (e.g., accuracy of test results)
 - Epistemic uncertainty about future state (e.g., quality of car)

Consider the case when randomness comes from another agent's actions...

Example: Prisoner's Dilemma

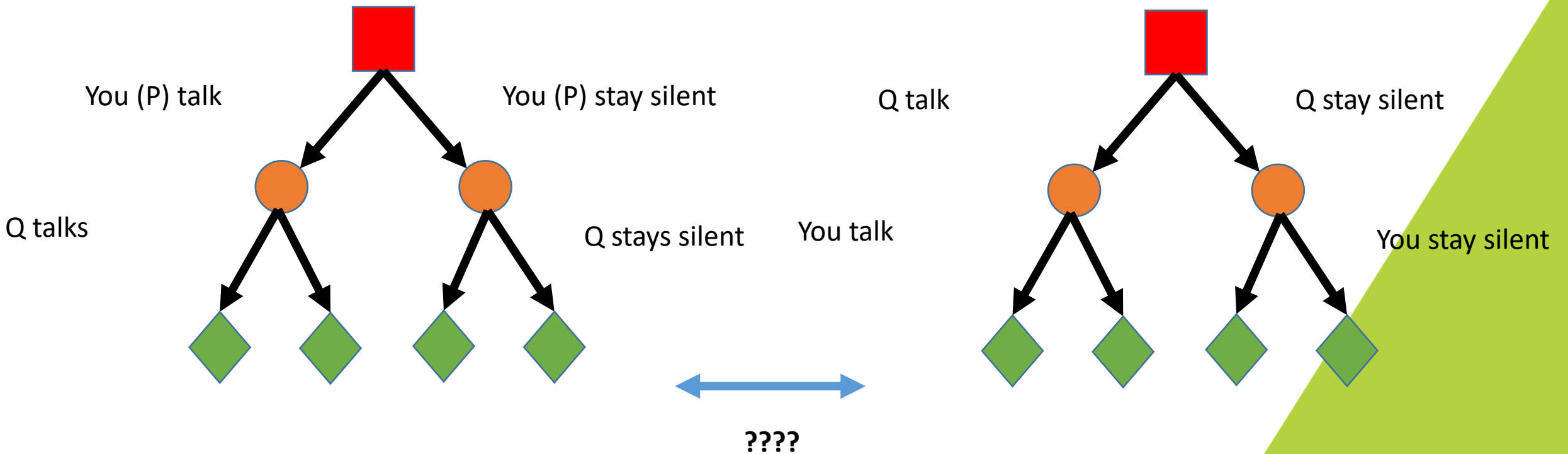
You (agent P) and an accomplice (agent Q) have been arrested for a crime...



But Q also knows all this and must make the same choices...

Example: Prisoner's Dilemma

You (agent P) and an accomplice (agent Q) have been arrested for a crime...



Both parties know this

Example: Prisoner's Dilemma

P and Q have been arrested for a crime and separated for interrogations. Each has the choice of whether or not to confess and each action is assumed to be rational. You don't know how your accomplice will act. What do you do?

Utility function:
Collective cost?

	P silent	P talks
Q silent	(0, 0)	(-2, -5)
Q talks	(-5, -2)	(-10, -10)

Both parties know this matrix

Example: Prisoner's Dilemma

P and Q have been arrested for a crime and separated for interrogations. Each is given the choice of whether or not to confess and each action is assumed to be rational. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	0	-7
Q talks	-7	-20

Utility function:
Collective cost?

Both choose silent if
both are using the
same utility function

Both parties know this matrix

Example: Prisoner's Dilemma

P and Q have been arrested for a crime and separated for interrogation. Each has the choice of whether or not to confess and each action is assumed to be rational. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	0	-7
Q talks	-7	-20

Utility function:
Collective cost?

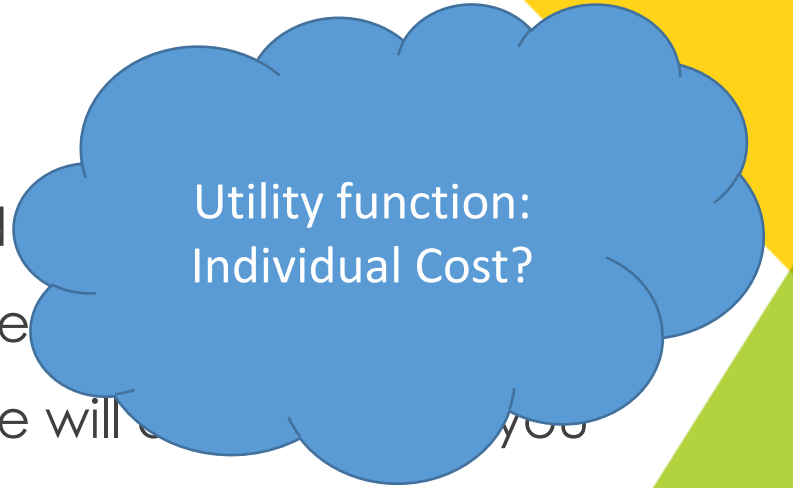
What if one uses a
different utility
function?

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess, you will receive a sentence associated with a cost. You don't know how your accomplice will confess or not confess. What do you do?



	P silent	P talks
Q silent	(0, 0)	(-2, -5)
Q talks	(-5, -2)	(-10, -10)

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice does not, you will receive a 10-year sentence and your accomplice will receive a 1-year sentence. If you do not confess and your accomplice does, you will receive a 1-year sentence and your accomplice will receive a 10-year sentence. If both of you confess, you will each receive a 5-year sentence. If neither of you confesses, you will each receive a 0-year sentence. Confessing is associated with a cost. You don't know how your accomplice will choose to confess or not. What do you do?

Utility function:
Individual cost?

	P silent	P talks
Q silent	0	-2
Q talks	-5	-10

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are being interrogated. You have the choice of whether or not to confess. If you confess, you will receive a sentence associated with a cost. You don't know how your accomplice will confess. What should you do?

Utility function:
Individual cost?

	P silent	P talks
Q silent	0	-5
Q talks	-2	-10

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice does not, you will receive a 5-year sentence and your accomplice will receive a 2-year sentence. If you confess and your accomplice also confesses, you will receive a 10-year sentence and your accomplice will receive a 10-year sentence. If you do not confess and your accomplice does not confess, you will receive a 0-year sentence and your accomplice will receive a 0-year sentence. If you do not confess and your accomplice confesses, you will receive a 5-year sentence and your accomplice will receive a 2-year sentence. You do not know how your accomplice will choose to confess or not confess. What do you do?

Utility function:
Individual Cost?

	 P silent	P talks
 Q silent	<div style="border: 2px solid red; padding: 5px;">$(0, 0)$</div>	$(-2, -5)$
Q talks	$(-5, -2)$	$(-10, -10)$

Both parties know this matrix

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice does not, you will receive a 5-year sentence, while your accomplice will receive a 2-year sentence. If you confess and your accomplice also confesses, you will both receive a 10-year sentence. If neither of you confesses, you will both receive a 0-year sentence. Confessing is associated with a cost. You don't know how your accomplice will choose to act. What do you do?

Local reasoning, rather than global

		Decision/action	
		P silent	P talks
Uncertainty	Q silent	(0, 0)	(-2, -5)
	Q talks	(-5, -2)	(-10, -10)
Weak case			

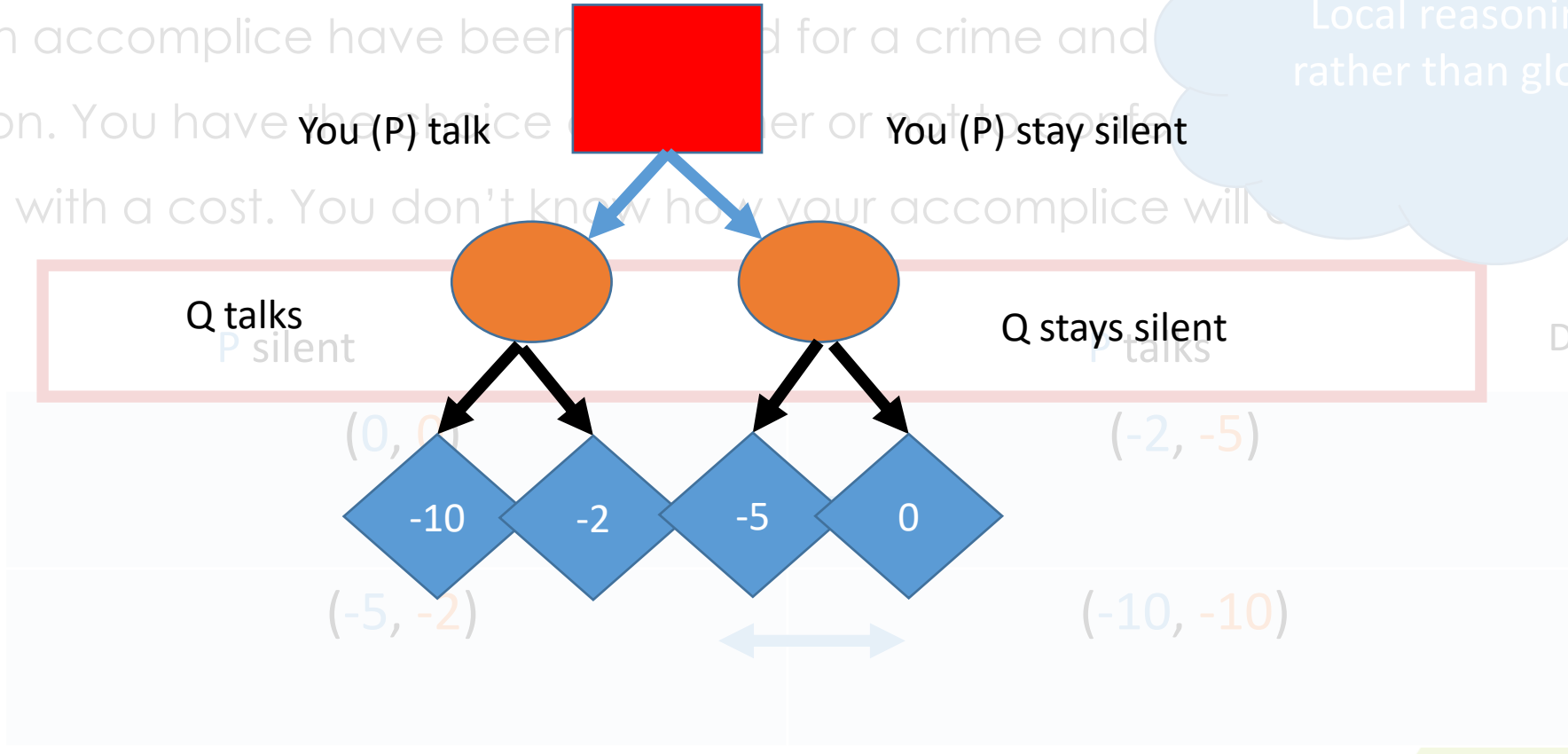
Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice to talk or stay silent. Talking is associated with a cost. You don't know how your accomplice will respond. What do you do?

Local reasoning, rather than global

Uncertainty

Q talks
Q silent



Decision/action

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice does not, you will receive a 5-year sentence and your accomplice will receive a 2-year sentence. If you confess and your accomplice also confesses, you will receive a 10-year sentence and your accomplice will receive a 10-year sentence. If you do not confess and your accomplice does not confess, you will receive a 0-year sentence and your accomplice will receive a 0-year sentence. If you do not confess and your accomplice confesses, you will receive a 5-year sentence and your accomplice will receive a 2-year sentence. You don't know how your accomplice will choose to confess or not confess. What do you do?

Local reasoning, rather than global

		Decision/action	
		P silent	P talks
Uncertainty	Q silent	(0, 0)	(-2, -5)
	Q talks	(-5, -2)	(-10, -10)

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

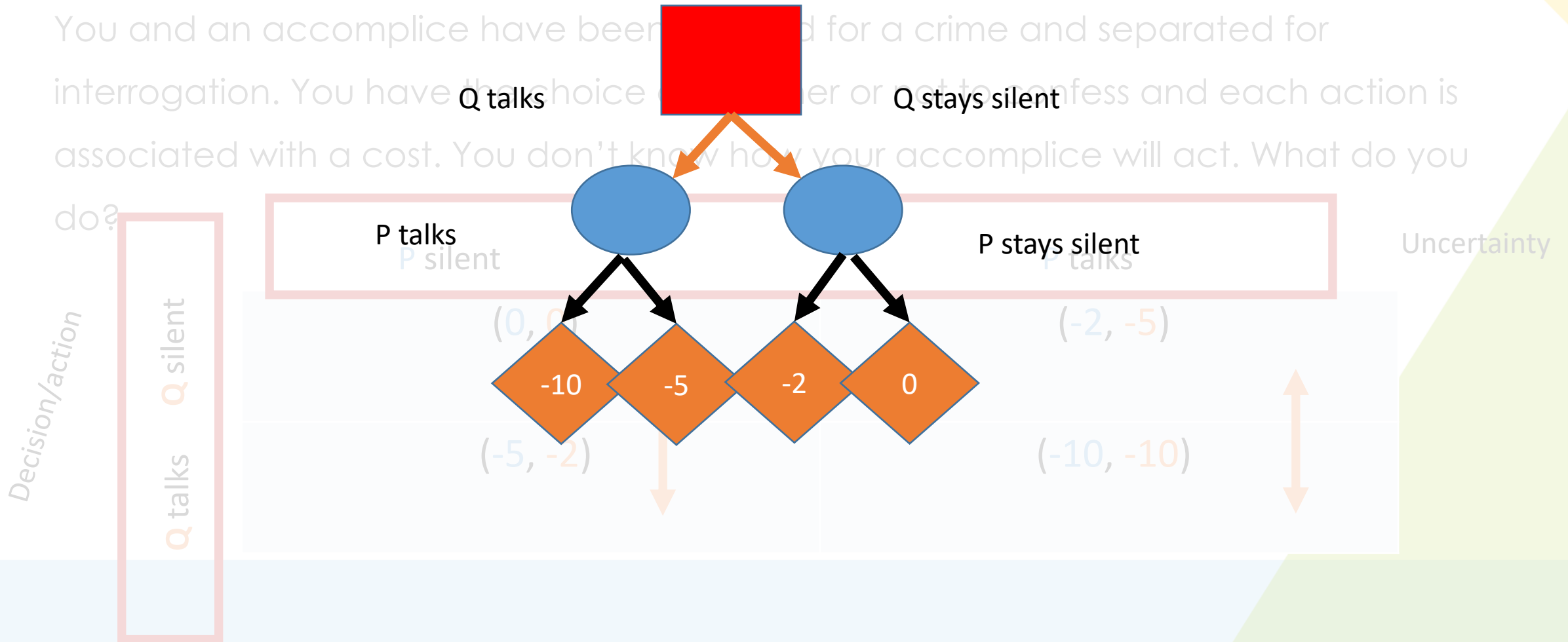
		P		Uncertainty
		silent	talks	
Q	silent	(0, 0)	(-2, -5)	
	talks	(-5, -2)	(-10, -10)	

Decision/action

Weak case

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice to confess or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?



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	P silent	P talks
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Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	$(0, 0)$	$(-2, -5)$
Q talks	$(-5, -2)$	$(-10, -10)$

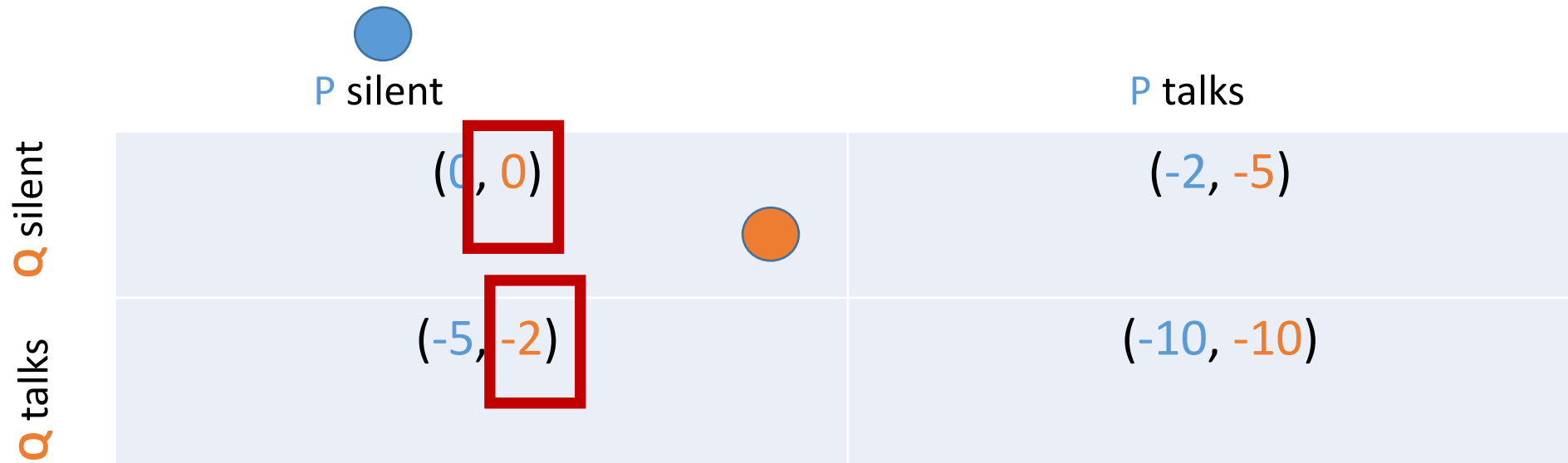
Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	(0, 0)	(-2, -5)
Q talks	(-5, -2)	(-10, -10)

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?






A 2x2 payoff matrix for a Prisoner's Dilemma. The vertical axis represents Player Q's choices: 'Q silent' (top) and 'Q talks' (bottom). The horizontal axis represents Player P's choices: 'P silent' (left) and 'P talks' (right). The payoffs are shown as (P's payoff, Q's payoff). The cells for (P silent, Q silent) with payoff (0, 0) and (P silent, Q talks) with payoff (-5, -2) are highlighted with red boxes. A blue circle is positioned above the 'P silent' column, and an orange circle is positioned to the right of the 'Q silent' row.

	P silent	P talks
Q silent	(0, 0)	(-2, -5)
Q talks	(-5, -2)	(-10, -10)






Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	 P silent	P talks
 Q silent	(0, 0)	(-2, -5)
 Q talks	(-5, -2)	(-10, -10)

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	 P silent	P talks
 Q silent	(0, 0) 	(-2, -5) 
 Q talks	(-5, -2)	(-10, -10) 

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	 P silent	P talks
 Q silent	<div style="border: 2px solid red; padding: 5px;">$(0, 0)$</div>	$(-2, -5)$
Q talks	$(-5, -2)$	$(-10, -10)$

Weak case

Same as globally optimal case for both utility functions

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to prison for interrogation. You have the choice of whether or not to confess to the crime, which is associated with a cost. You don't know how your accomplice will choose to do or not to do you do?

Consider a different payoff matrix



	P silent	P talks
Q silent	$(-2, -2)$	$(0, -15)$
Q talks	$(-15, 0)$	$(-10, -10)$

Incentivize talking

Still globally optimal for collective utility?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	 P silent	P talks
 Q silent	-4	-15
Q talks	-15	-20

Incentivize talking

Still globally optimal for collective utility?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to prison for interrogation. You have the choice of whether or not to confess to the crime, which is associated with a cost. You don't know how your accomplice will confess or not to you do?

Assume Q uses collective utility, but P uses individual utility...

	P silent	P talks
Q silent	-2	0
Q talks	-15	-10

Incentivize talking

Still globally optimal for individual utility for P?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to jail for interrogation. You have the choice of whether or not to confess to the crime. Confessing is associated with a cost. You don't know how your accomplice will act. What do you do?

What if Q knows P's utility function and decides to mimic?

	P silent	P talks
Q silent	-2	-15
Q talks <td>0</td> <td>-10</td>	0	-10

Incentivize talking

Still globally optimal for individual utility?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to jail for interrogation. You have the choice of whether or not to confess to the crime. Confessing is associated with a cost. You don't know how your accomplice will act. What do you do?

What if Q knows P's utility function and decides to mimic?

	P silent	P talks
Q silent	-2	-15
Q talks	0	-10

Incentivize talking

Still globally optimal for individual utility?

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	$(-2, -2)$	$(0, -15)$
Q talks	$(-15, 0)$	$(-10, -10)$

Incentivize talking

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	-4	-15
Q talks	-15	-20

Worst possible global outcome!

Incentivize talking

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and are in separate cells during interrogation. You have the choice of whether or not to confess. If you confess and your accomplice does not, you will receive a 15-year sentence and your accomplice will go free. If you confess and your accomplice also confesses, you will both receive a 10-year sentence. If neither of you confesses, you will both receive a 2-year sentence. You don't know how your accomplice will choose to confess or not. What do you do?

Local reasoning, rather than global

	P silent	P talks
Q silent	(-2, -2)	(0, -15)
Q talks	(-15, 0)	(-10, -10)

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

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	P silent	P talks
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Q silent	$(-2, -2)$	$(0, 15)$
Q talks	$(-15, 0)$	$(-10, -10)$

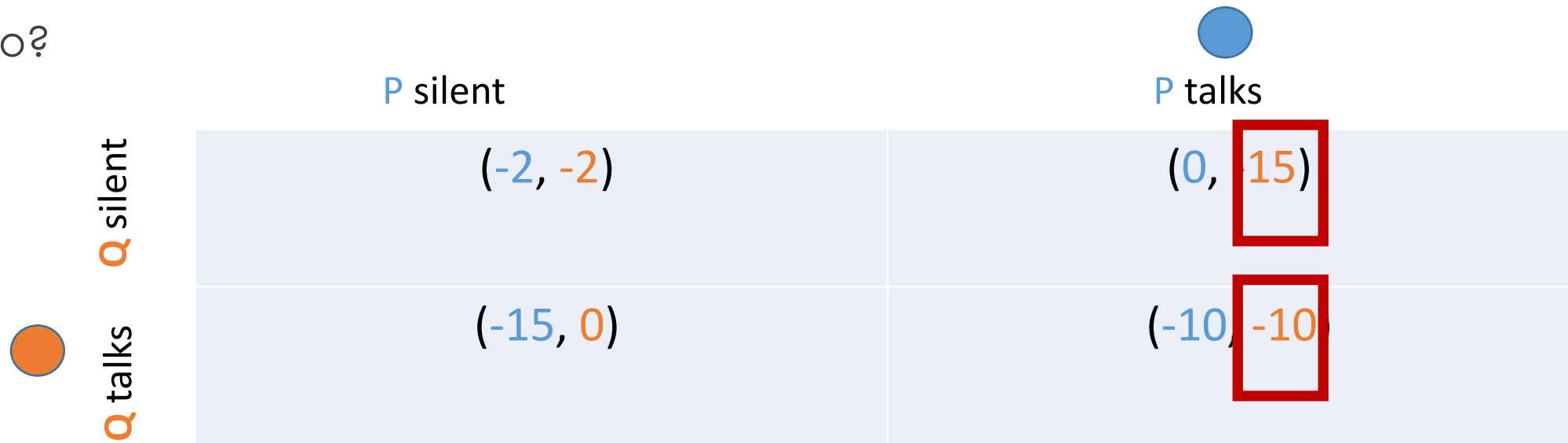
Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

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A 2x2 payoff matrix for a Prisoner's Dilemma. The columns represent Player P's choices: 'P silent' and 'P talks'. The rows represent Player Q's choices: 'Q silent' and 'Q talks'. Each cell contains a pair of payoffs (P's payoff, Q's payoff). The payoffs are: (P silent, Q silent) = (-2, -2); (P talks, Q silent) = (0, 15); (P silent, Q talks) = (-15, 0); (P talks, Q talks) = (-10, -10). The payoffs (0, 15) and (-10, -10) are highlighted with red boxes. A blue circle is positioned above the 'P talks' column header, and an orange circle is positioned to the left of the 'Q talks' row header.

	P silent	P talks
Q silent	$(-2, -2)$	$(0, 15)$
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	P silent	P talks
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Q talks	$(-15, 0)$	$(-10, -10)$

Same result!

Incentivize talking

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. Loser pays the winner.

	P plays 1	P plays 2
Q plays 1	$(+2, -2)$	$(-3, +3)$
Q plays 2	$(-3, +3)$	$(+4, -4)$

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. Loser pays the winner.

	P plays 1	P plays 2
Q plays 1	0	0
Q plays 2	0	0

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. Loser pays the winner.

	P plays 1	P plays 2
Q plays 1	+2	-3
Q plays 2	-3	+4

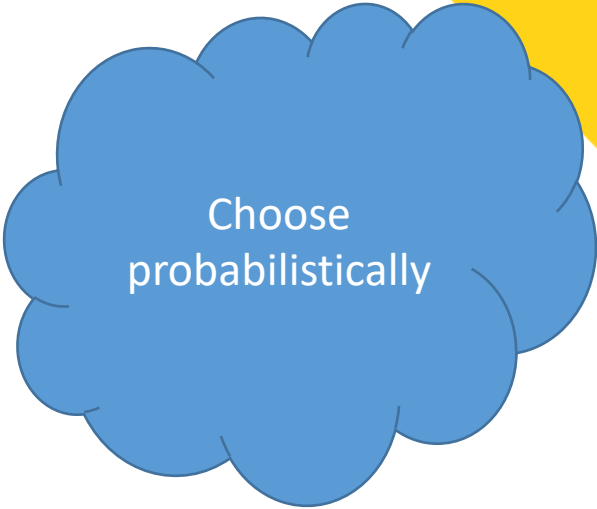
Example: 2-finger Morra





Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. Loser pays the winner.

	P plays 1	P plays 2
Q plays 1	-2	+3
Q plays 2	+3	-4

Example: 2-finger Morra

Choose between 1 and 2 fingers. P wins if sum is even. Q wins if sum is odd. P pays the winner.



	P plays 1	P plays 2
Q plays 1	$(+2, -2)$ 	$(-3, +3)$ 
Q plays 2	$(-3, +3)$ 	$(+4, -4)$ 

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	$(-4, -4)$	$(0, -6)$
Q talks	$(-5, -2)$	$(-10, -10)$

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	-8	-6
Q talks	-7	-20


Example: Prisoner's Dilemma

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	P silent	P talks
Q silent	-4	0
Q talks	-5	-10

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?



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A 2x2 payoff matrix for a Prisoner's Dilemma. The rows represent Player Q's actions (silent or talks) and the columns represent Player P's actions (silent or talks). The payoffs are shown in orange text within light blue cells. A red rectangular box highlights the cell where Q talks and P is silent, with a payoff of -2. A blue circle is positioned above the 'P talks' column header, and an orange circle is positioned to the left of the 'Q talks' row header.

	P silent	P talks
Q silent	-4	-6
Q talks	-2	-10

No free ride for Q

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?


	P silent	P talks
Q silent	$(-4, -4)$	$(0, -6)$
Q talks	$(-5, -2)$	$(-10, -10)$

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
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Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and separated for interrogation. You have the choice of whether or not to confess and each action is associated with a cost. You don't know how your accomplice will act. What do you do?

	P silent	P talks
Q silent	$(-4, -4)$ 	$(0, -6)$ 
Q talks	$(-5, -2)$ 	$(-10, -10)$ 

Example: Prisoner's Dilemma

You and an accomplice have been arrested for a crime and sent to jail for an extended period of time. You are in separate rooms during interrogation. You have the choice of whether or not to confess to the crime. If you confess and your accomplice confesses, you will both receive a 4-year sentence. If you confess and your accomplice stays silent, you will receive a 5-year sentence and your accomplice will receive a 2-year sentence. If you stay silent and your accomplice confesses, you will receive a 6-year sentence and your accomplice will receive a 0-year sentence. If you both stay silent, you will both receive a 10-year sentence. You don't know how your accomplice will choose to confess or stay silent. What do you do?

Suppose we model Q's choice probabilistically...

	P silent	P talks
Q silent	$(-4, -4)$	$(0, -6)$
Q talks	$(-5, -2)$	$(-10, -10)$

No free ride for Q

Do stuff on board

Vocabulary & Concepts

- Always assume local decision making (all players maximizing individual utility)
- **Zero sum** – every entry in global collective payoff is 0
- **Pure strategy** – always pick the same action no matter what
- **Mixed strategy** – pick an action probabilistically
- **Dominant strategy** – one action is strictly better no matter what the other plays does