

Exam II

Math 021 Section Z1

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- The exam is out of 60 points. There are a total of 6 questions, 10 points each.
 - **Do not discuss the contents of this exam with anyone. (including Google!)**
 - *You must show all work to receive full credit.*
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Name _____

Problem 1: Implicit Differentiation and Related Rates

(part a:) Suppose y is a function of x (i.e. $y = y(x)$). Use implicit differentiation to solve the following equation for $\frac{dy}{dx}$. Express your answer with positive exponents for all variables.

$$e^{\sin(y^3)} = \ln(x^2 + y^3)$$

(part b:) A spherical snowball is placed in the sun. The sun melts the snowball so that its radius *decreases* at a rate of $\frac{1}{4}$ inches per hour. Find the rate of change of the volume of the snowball with respect to time at the instant the radius is 4 inches.

Problem 2: (Optimization) Given the function: $f(x) = \frac{e^{x^2}(x^2 - 1)^{10}}{x^2 + 1}$

a: Find the derivative using logarithmic differentiation. Then find all critical points

b: Find the intervals where the function is increasing and decreasing.
Use this to classify these critical points.

Problem 3: Find all relative extrema, intervals of concavity and inflection points of the following equation.

$$f(x) = x^{7/3} + x^{4/3}$$

Problem 4: Find all absolute extrema of $f(x) = e^{-\frac{x^3}{3}+4x+3}$ on the interval $[0, \infty)$

Problem 5: You are trying to build a ski rack for your car. An open rectangular box is to be made by cutting a square corner from a 8ft by 3ft piece of metal and then folding up the sides. Let x represent the side-length of the square being cut from the corners of the metal. Find the value of x that maximizes the volume of the box.

Problem 6: Evaluate the following limits using L'hospital's Rule when necessary.

part a:

$$\lim_{x \rightarrow 0} \cot(2x) \sin(6x)$$

part b:

$$\lim_{x \rightarrow 0} \frac{1 - e^{2x}}{\sec(x)}$$

part c:

$$\lim_{x \rightarrow 0} (1 - 2x)^{1/x}$$