Math 395 - Fall 2021
Beginner Reading 7

This reading is “due” on Monday October 18 at 11:59pm.

This week you are invited to read Chapter 6 of Dummit and Foote. As you go along, you can answer the following questions to test your understanding and bring your attention to the most important concepts.

Section 6.1

1. True or false: Let G be any group. Then it is possible for $Z(G) = 1$, or in other words for the center of G to be trivial.
2. The topic of nilpotent groups is interesting, but historically hasn't been a big focus of the algebra qualifying exam at UVM. For this reason I will not cover the upper central series in this course.
3. True or false: Let G be a finite group such that for all positive integers n dividing $\#G$, G has at most n elements such that $g^n = 1$. Then G is cyclic.
4. True or false: Let H and K be subgroups of a group G. Then $[H, K]$ is the set containing all of the elements of G of the form $h^{-1}k^{-1}hk$ for $h \in H$ and $k \in K$.
5. In your own words, what is the connection between the derived series of a group G and G being solvable?
6. True or false: Subgroups of solvable groups are solvable.
7. True or false: Homomorphic images of solvable groups are solvable.
8. True or false: Quotients of solvable groups are solvable.
9. It can be helpful to know the statements of Burnsides's Theorem and the Feit-Thompson theorem, but please note that in many instances you are explicitly not allowed to use them on the qualifying exam to show that a group is solvable.
10. Though you may read the proof of the Fundamental Theorem of Finite Abelian Groups, you can also hold off since this will be proved in much more generality next semester in Algebra IV.

Section 6.2

11. This section is very intense! I suggest you begin with reading methods 1 and 5 (“counting elements” and “studying normalizers of intersections of Sylow p-subgroups”) and come back to this section when you have more experience with qualifying exam problems. Until then, summarize for yourself briefly what each of these two methods entail.

Section 6.3

12. What is the universal property of the free group $F(S)$ on a set S?

13. What is a word on a set S?

14. Explain in your own words the difference between a free group $F(S)$ of rank r, and the free abelian group of rank r $\mathbb{Z}^r$.

15. True or false: If a group G has a presentation (S, R), then the kernel of the map $F(S) \to G$ is just the set R.