

Math 395 - Fall 2021  
Beginner Reading 5

This reading is “due” on Monday October 4 at 11:59pm.

This week you are invited to read Section 4.5 of Dummit and Foote. As you go along, you can answer the following questions to test your understanding and bring your attention to the most important concepts.

**Section 4.5**

1. If  $p$  is a prime, what is a  $p$ -group?
2. Still with  $p$  a prime, what is a Sylow  $p$ -subgroup of a group  $G$ ?
3. What is  $\text{Syl}_p(G)$ ? What is  $n_p(G)$ ?
4. True or false: Any two Sylow  $p$ -subgroups of a group  $G$  are isomorphic.
5. What are the Sylow 2-subgroups of  $S_3$ ? What are its Sylow 3-subgroups?
6. What are the possibilities for the structure of a group of order  $pq$ , for  $p$  and  $q$  distinct primes?
7. There are a lot more examples, and it can get a bit overwhelming on a first read. If your eyes are glazing over, please focus on the example of groups of order 30, because the technique of assuming that no Sylow  $p$ -subgroup is normal and then showing that this implies there are too many elements in  $G$  is a technique we use all the time. After that you can take a break and come back to this section later.