Math 395 - Fall 2021
Beginner Reading 4

This reading is "due" on Monday September 27 at 11:59pm.

This week you are invited to read Sections 4.1, 4.2, 4.3, 4.4 and 4.6 of Dummit and Foote (all of Chapter 4 except for Section 4.5 which we will cover next week). As you go along, you can answer the following questions to test your understanding and bring your attention to the most important concepts.

Section 4.1

1. Explain in your own words why every action of a group G on a set A can be thought of as a faithful action of $G/\ker\varphi$ on A, where $\ker\varphi$ is the kernel of the group action (or of the permutation representation associated to this action).

2. True or false: Given a group homomorphism $G \to S_A$, where S_A is the set of permutations of A with binary operation given by composition, we can give a unique group action of G on A.

3. True or false: The index of the stabilizer of a in G is equal to the number of elements in the orbit of a under the action of G.

Section 4.2

4. True or false: The action of a group on itself by left multiplication is transitive and faithful.

5. Let G be a group and H a subgroup of G. Let G act by left multiplication on the set of left cosets of H. What is the stabilizer of the coset H?

6. What does Cayley's Theorem say?

7. What is the left regular representation of G?

8. True or false: Let G be a group of order $45 = 3^2 \cdot 5$, and H be a subgroup of G of order 15. Then H is normal.

Section 4.3

9. What is a conjugacy class?

10. Let G be a group, and $h \in G$. How many distinct conjugates does h have in G?

11. What does the Class Equation say?

12. Let p be a prime and G be a group of order p^2. What are the possibilities for the isomorphism class of G?

13. How can you tell if two elements are conjugate in S_n?

Section 4.4

14. In the statement of Proposition 13, the authors state that $G/C_G(H)$ is isomorphic to a subgroup of $\mathrm{Aut}(H)$. In class we said that $N_G(H)/C_G(H)$ was isomorphic to a subgroup of $\mathrm{Aut}(H)$. Is there a contradiction here or is everything okay?

15. Can an abelian group have nontrivial inner automorphisms? What about nontrivial outer automorphisms?

16. True or false: If H is the unique subgroup of G of a given order, then H is normal in G.

17. What is the automorphism group of the cyclic group of order n?

18. True or false: There exists a nonabelian group of order $15 = 3 \cdot 5$.

Section 4.6

19. No questions, but please read to get a sense of how the proof goes!