Math 395 - Fall 2021
Beginner Reading 1

This reading is "due" on Monday September 6 at 11:59pm.

This week you are invited to read Chapter 1 of Dummit and Foote. As you go along, you can answer the following questions to test your understanding and bring your attention to the most important concepts.

Section 1.1

1. Give an example of a concrete set G with a binary operation.
2. The set of integers $\mathbb{Z}$ is an abelian group under the binary operation $+$ (usual addition). What is the identity of this group? What is the inverse of the number 2 under this operation?
3. Read Proposition 1 carefully. What is $(a \star b)^{-1}$?
4. Read Proposition 2 and its proof carefully. **Why** do the right and left cancellation laws hold in a group?
5. Consider again the group $\mathbb{Z}$ with the binary operation $+$ (usual addition). What is the order of the element 2?

Section 1.2

6. Suppose that you see the group D_6. What are the two possibilities for the number of elements it can have? Which one is the case if you are reading Dummit and Foote?
7. Write a presentation for D_8, where D_8 is the dihedral group with 16 elements.

Section 1.3

8. Write the cycle decomposition of the element of S_6 that sends 1 to 5, 2 to 6, 3 to 1, 4 to 4, 5 to 3, and 6 to 2.
9. Products in S_6 are given by composition. What is $(143) \circ (15)(23)$?
10. True or false: $(12)(34) = (34)(12)$.
11. True or false: $(123)(34) = (34)(123)$.
12. Recall the permutation described in problem 8. What is the order of this permutation?

For a first reading you may skip Sections 1.4 and 1.5, but do note that they cover matrix groups and the quaternion group Q_8 so you can read them if these groups come up later.

Section 1.6

13. Is every group homomorphism a group isomorphism?

14. How many isomorphism classes of groups of order 6 are there in total?

Section 1.7

15. After you have read Example 5, consider the group $G = S_3$ acting on itself by left multiplication. Explicitly write down the permutation representation of this group action. In other words, this group action gives a group homomorphism $\varphi\colon S_3 \to S_6$, since S_3 has six elements. Give explicitly the image of each element of S_3 under the homomorphism φ.