



THE UNIVERSITY OF VERMONT
COLLEGE OF ENGINEERING &
MATHEMATICAL SCIENCES

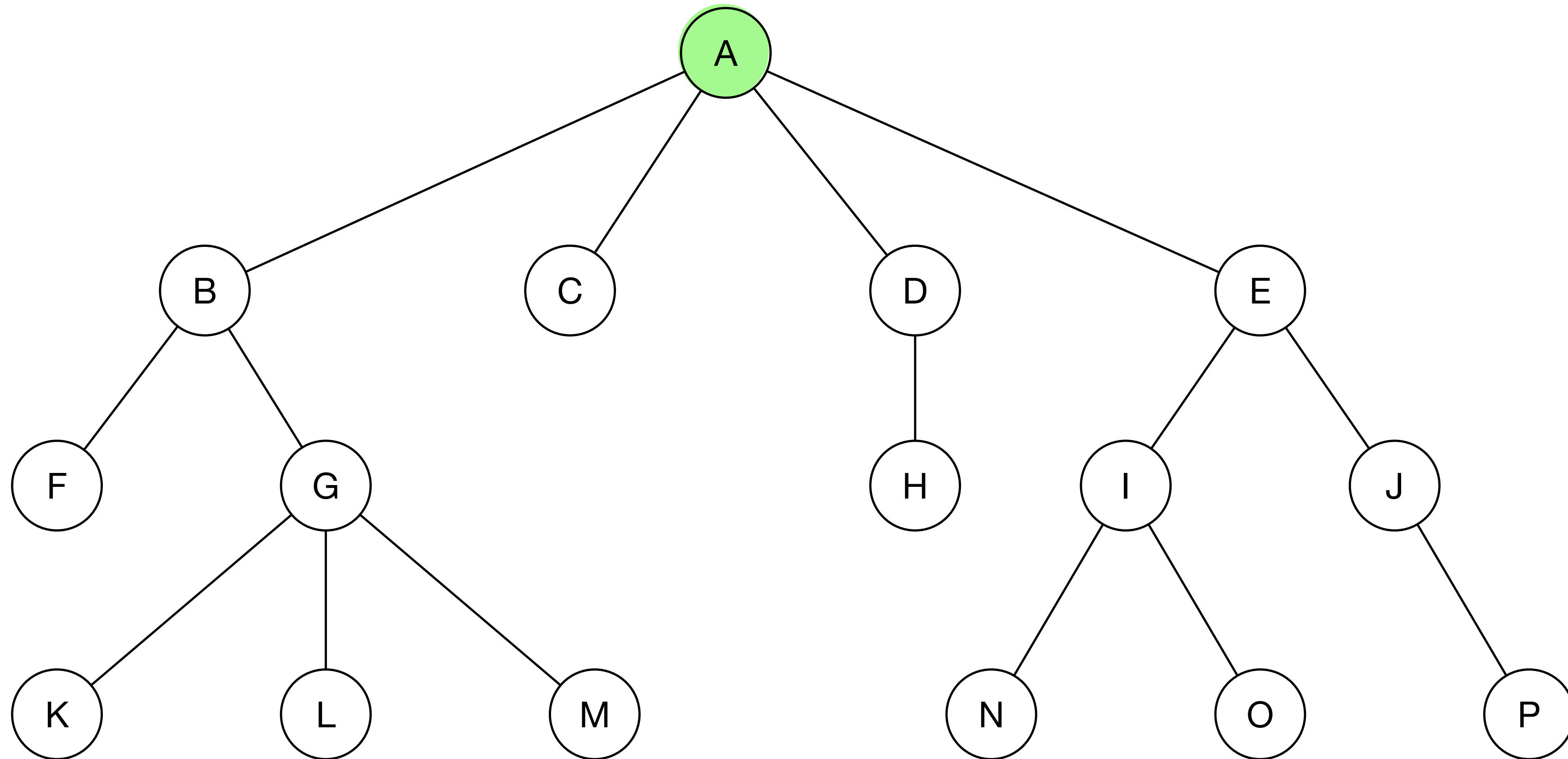
Trees, Part 2

More about trees; binary trees



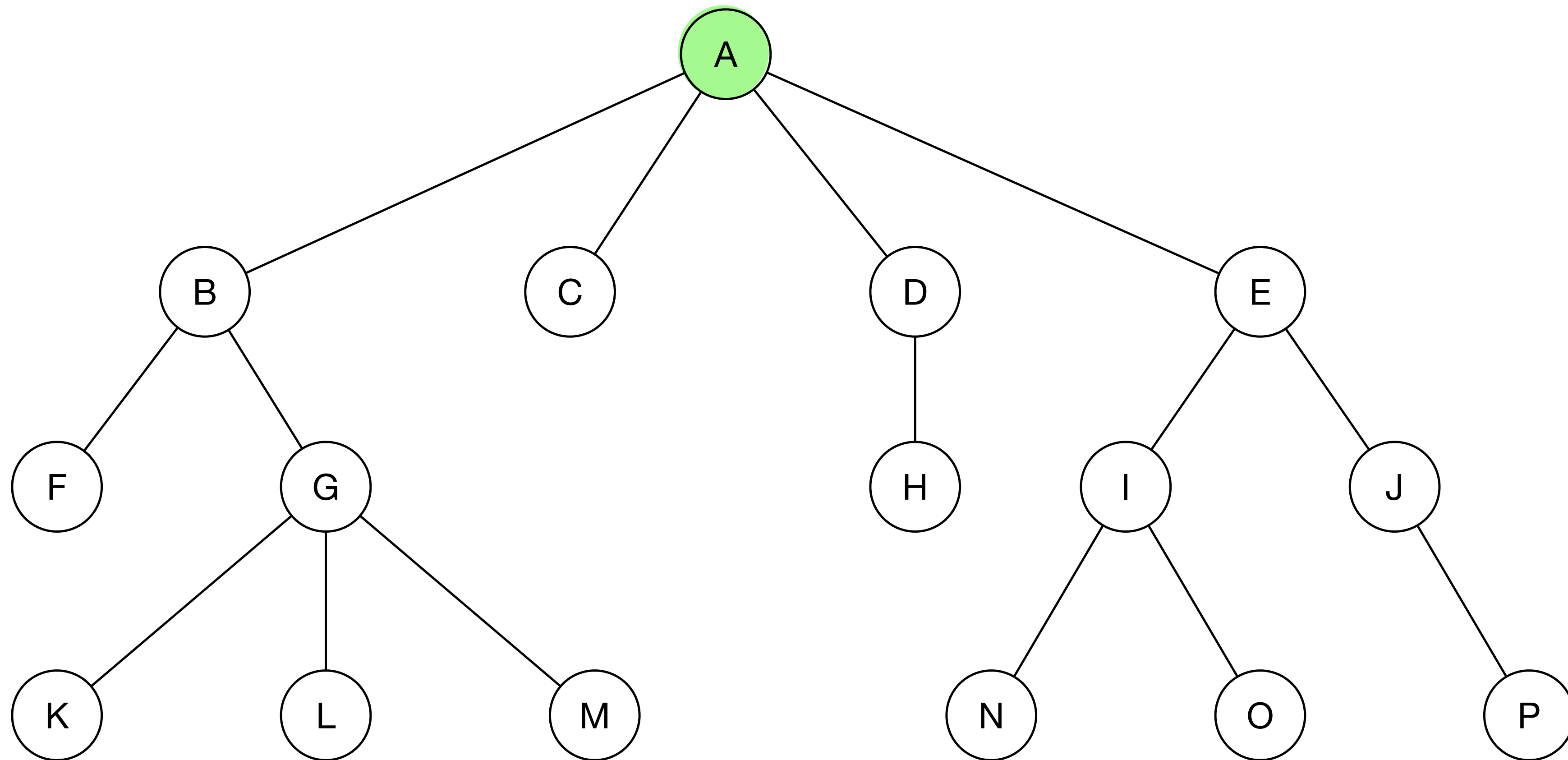
Rooted trees

One node may have special status. We call that the *root node*. If the tree has a root node we call it a *rooted tree*.



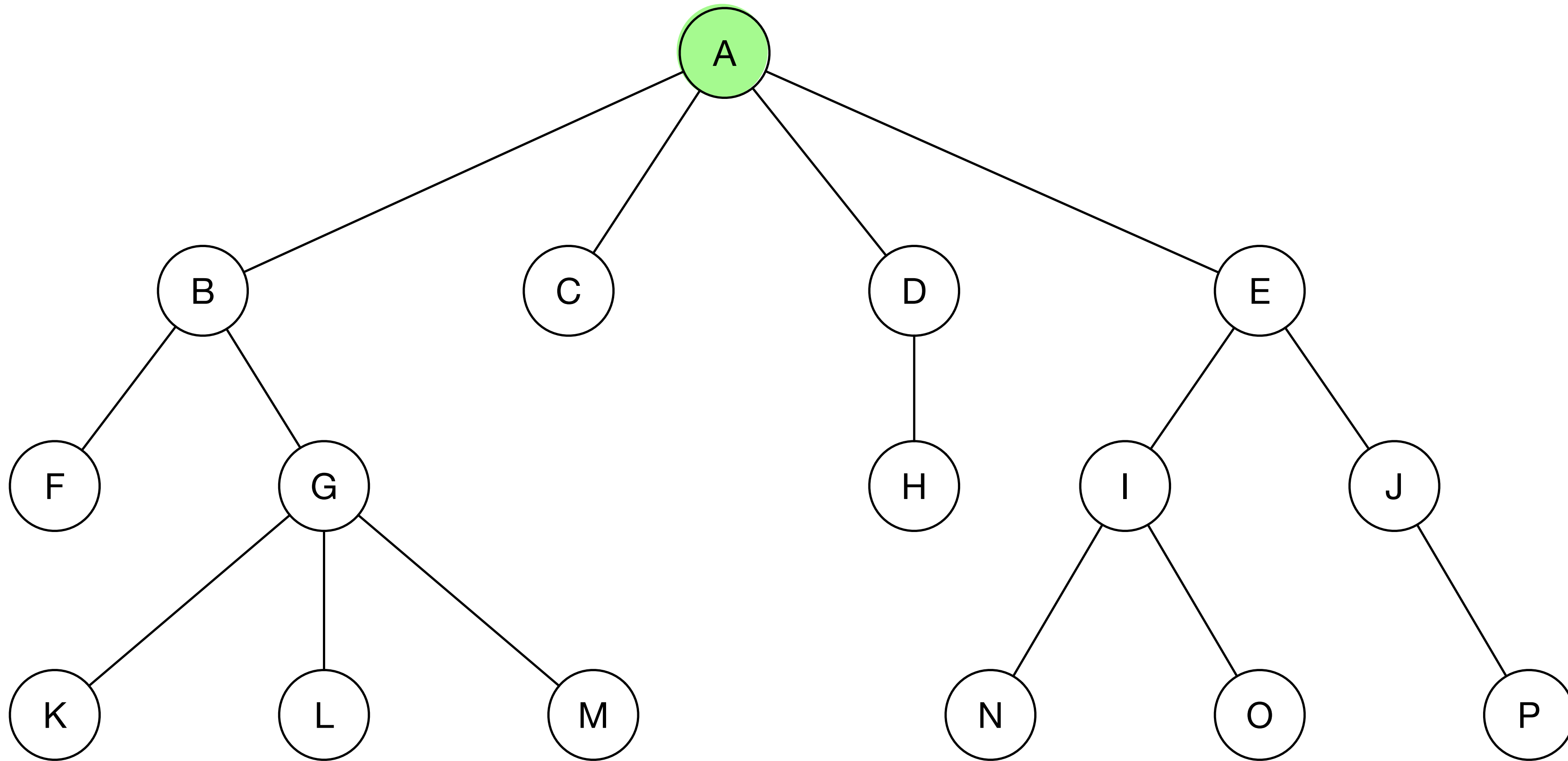
Rooted trees

Once we have a root, we can speak of nodes being *parents* and *children*.



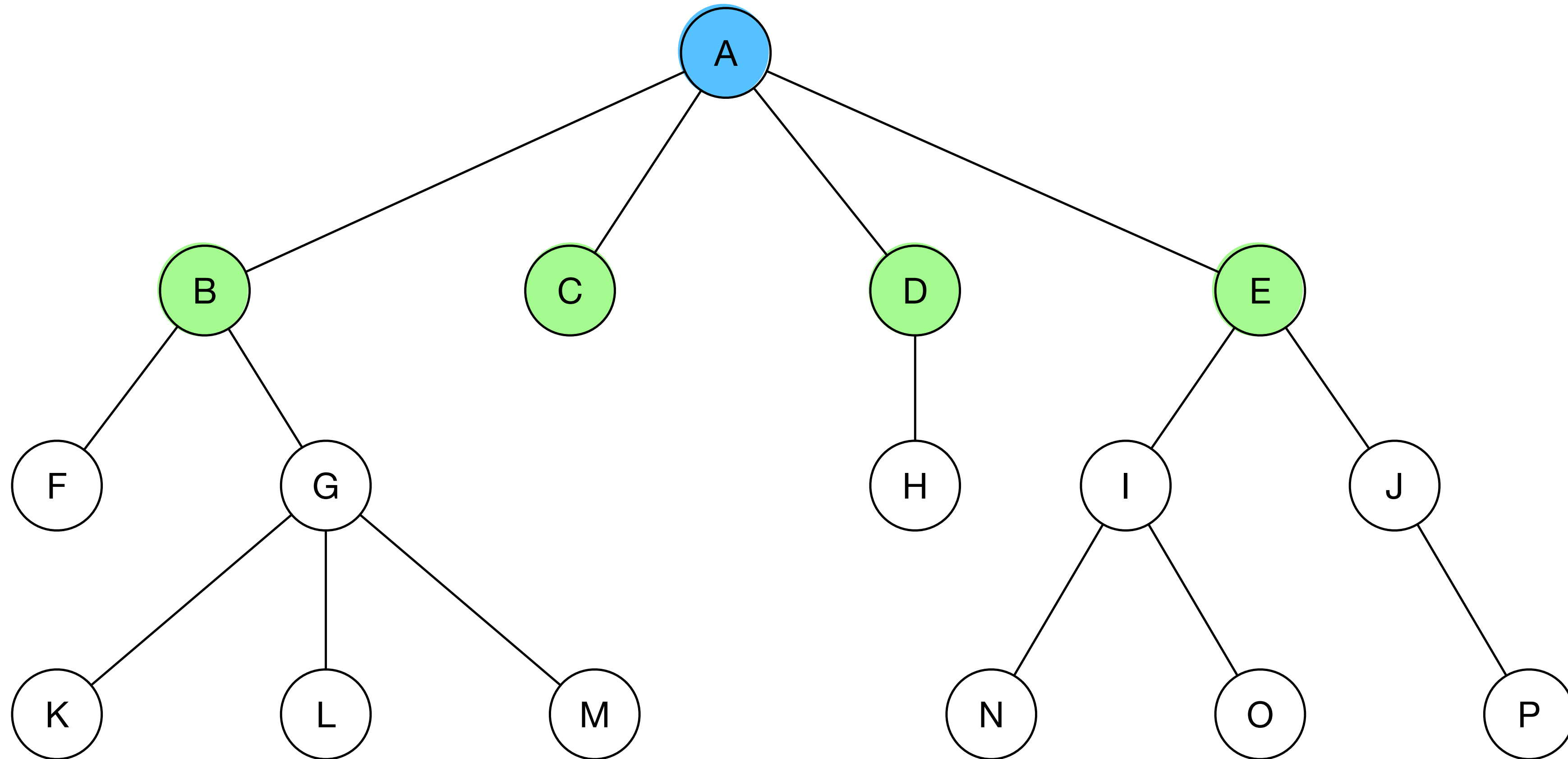
Rooted trees

The root node has a special property: it has no parents.



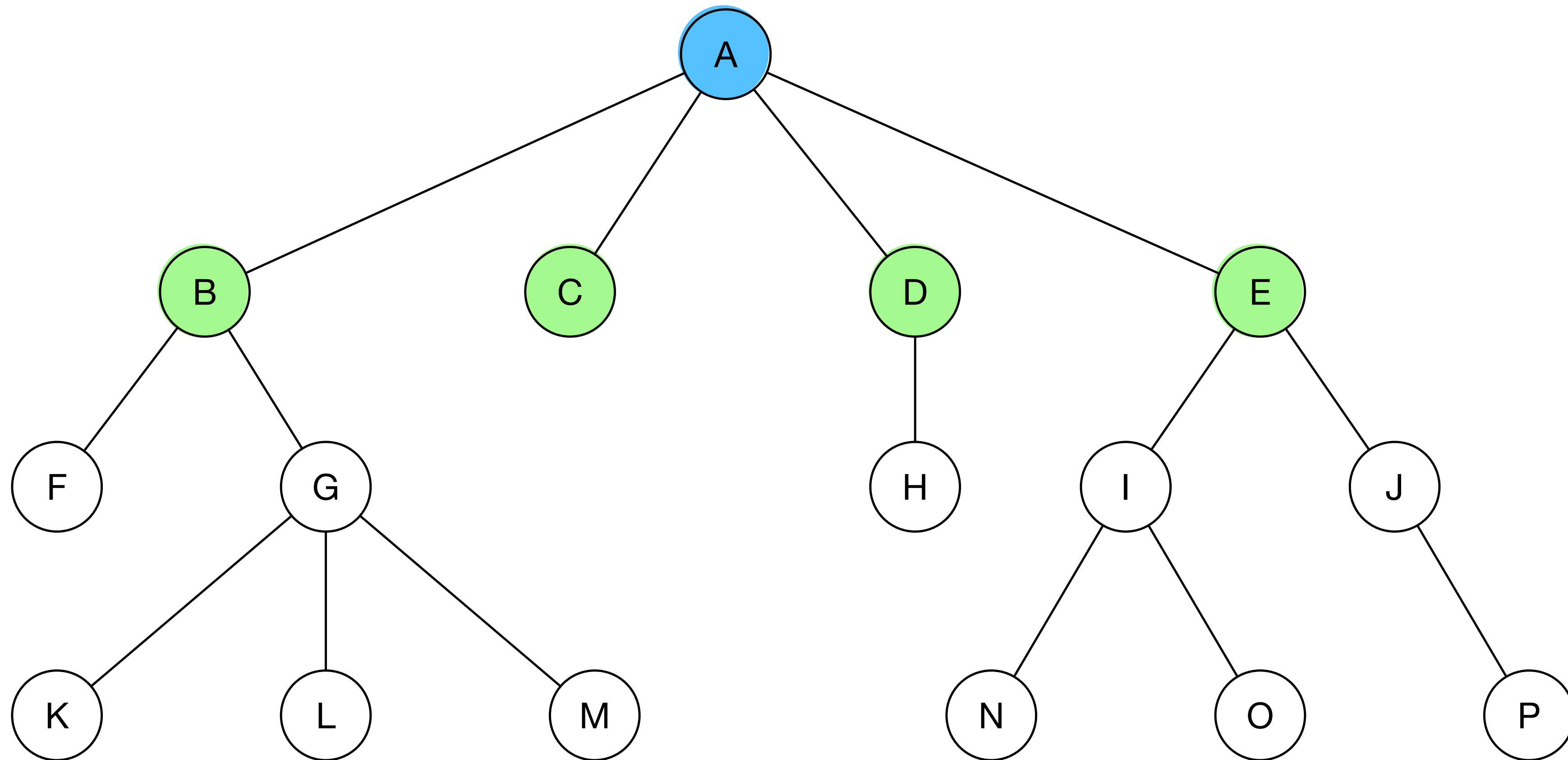
Rooted trees

Nodes one level below the root node are called the *children* of the root. For each of these nodes, B, C, D and E, the node A is called the *parent*.



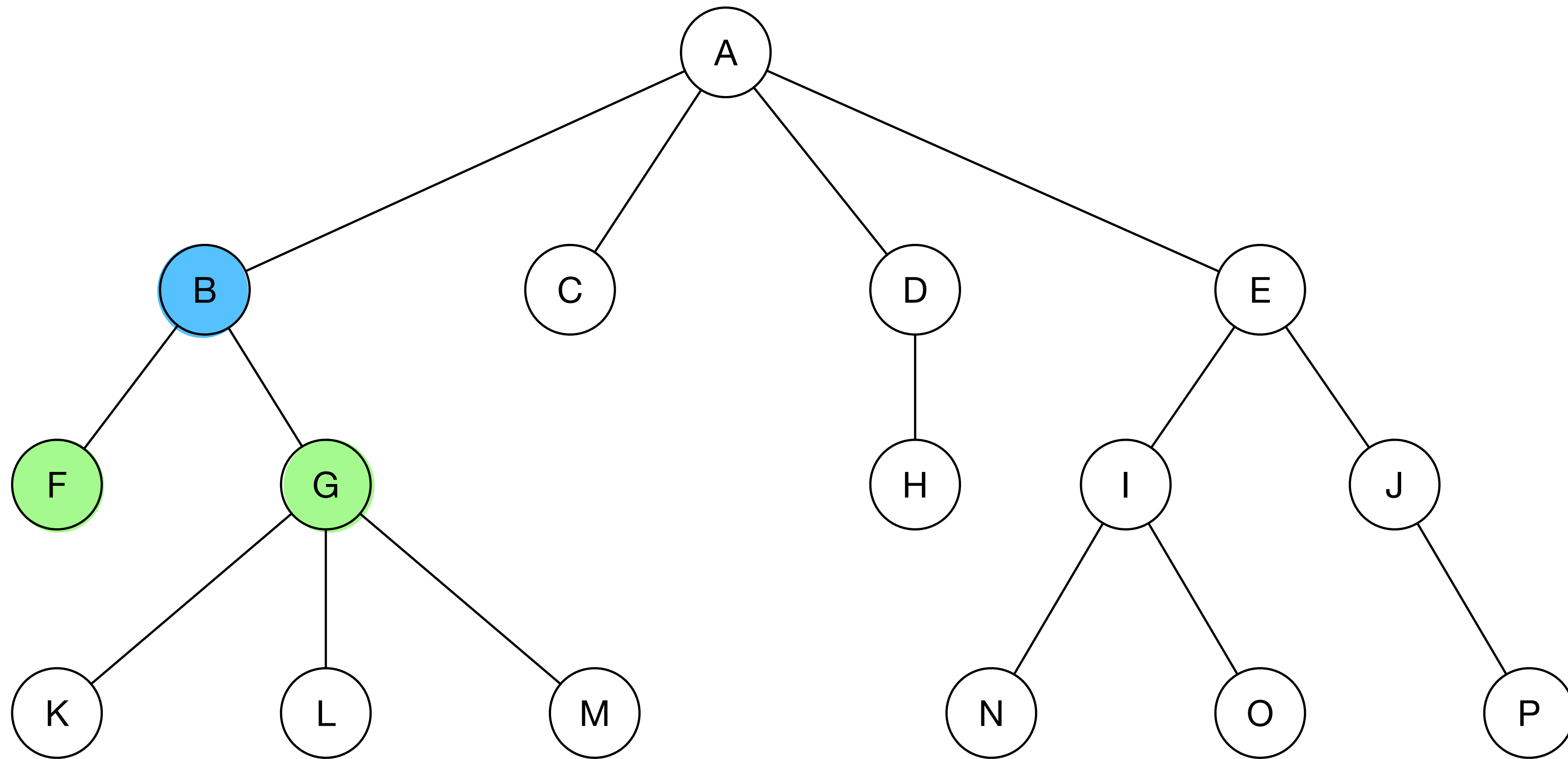
Rooted trees

A node may have more than one child, but each node (apart from the root) has only one parent.



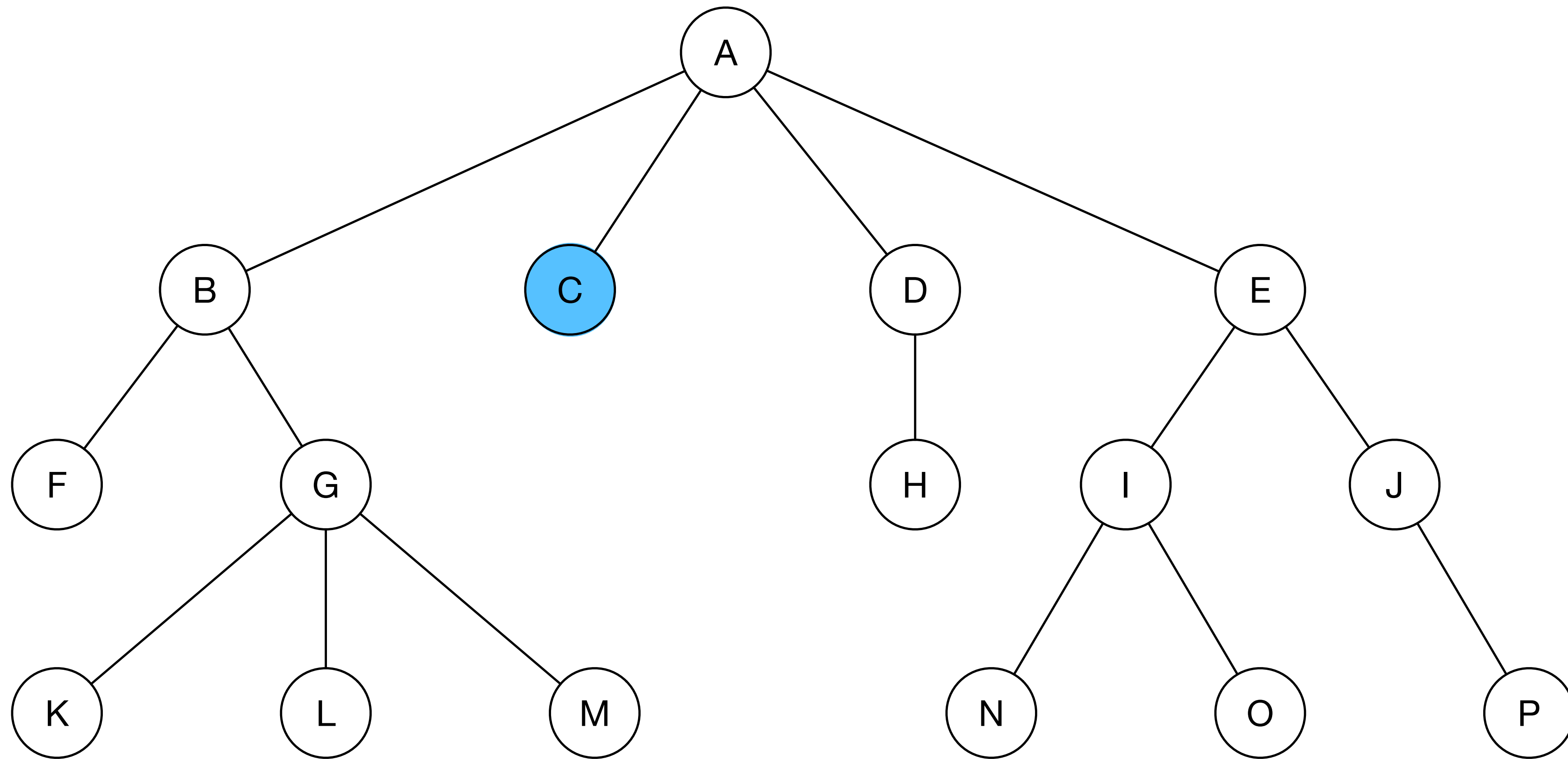
Rooted trees

B has two children, F and G.



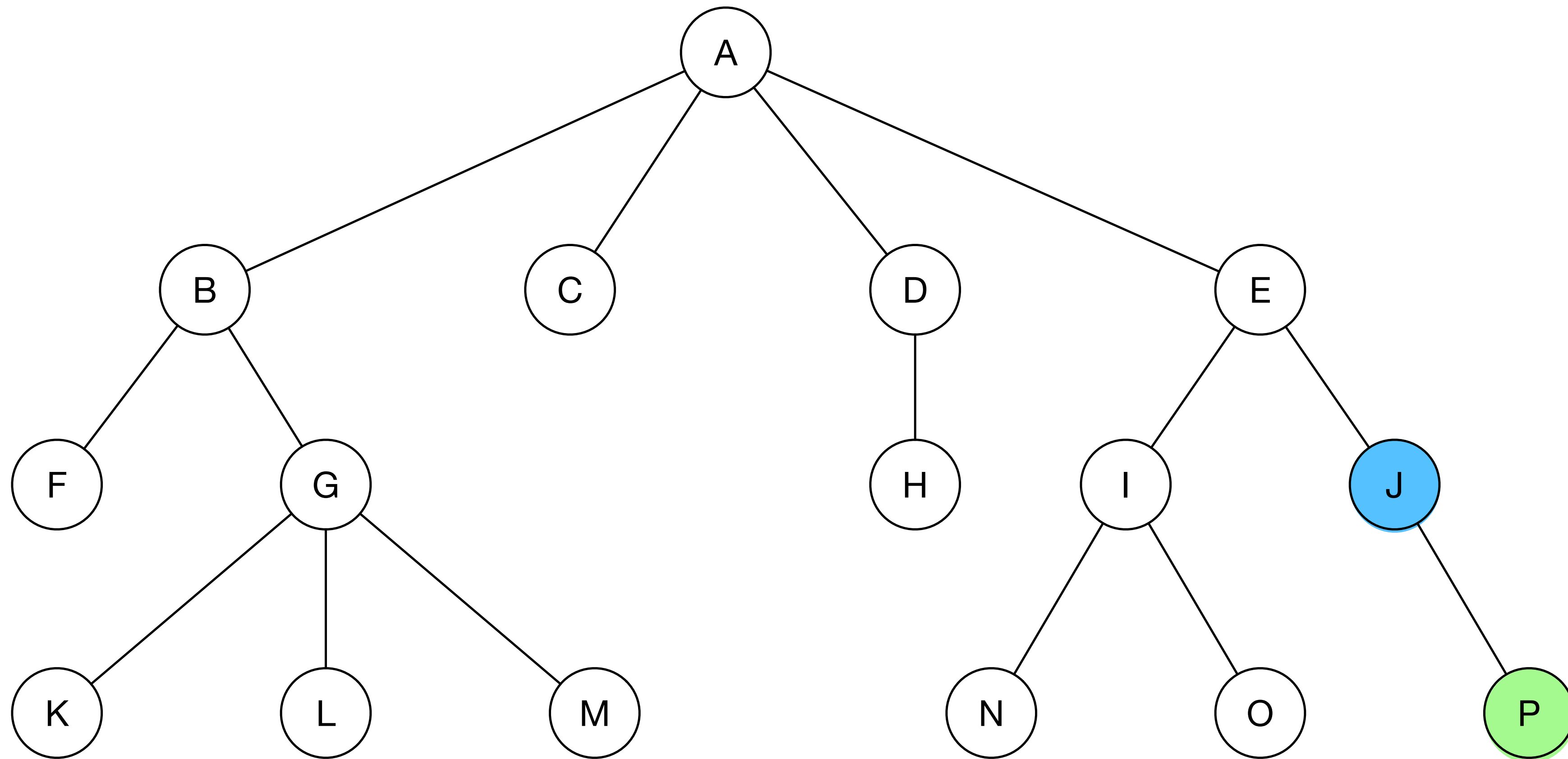
Rooted trees

C has no children.



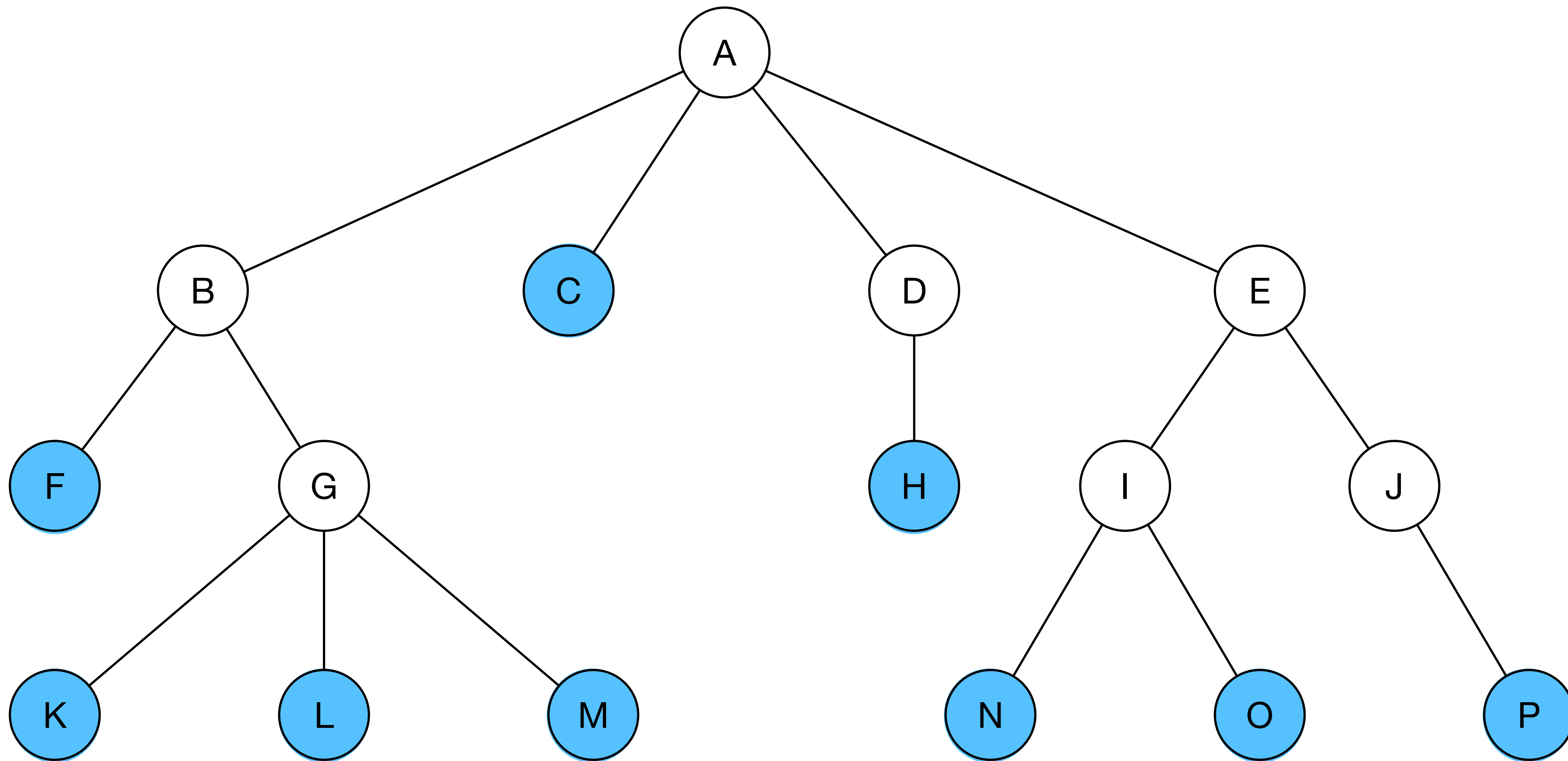
Rooted trees

The parent of P is J.



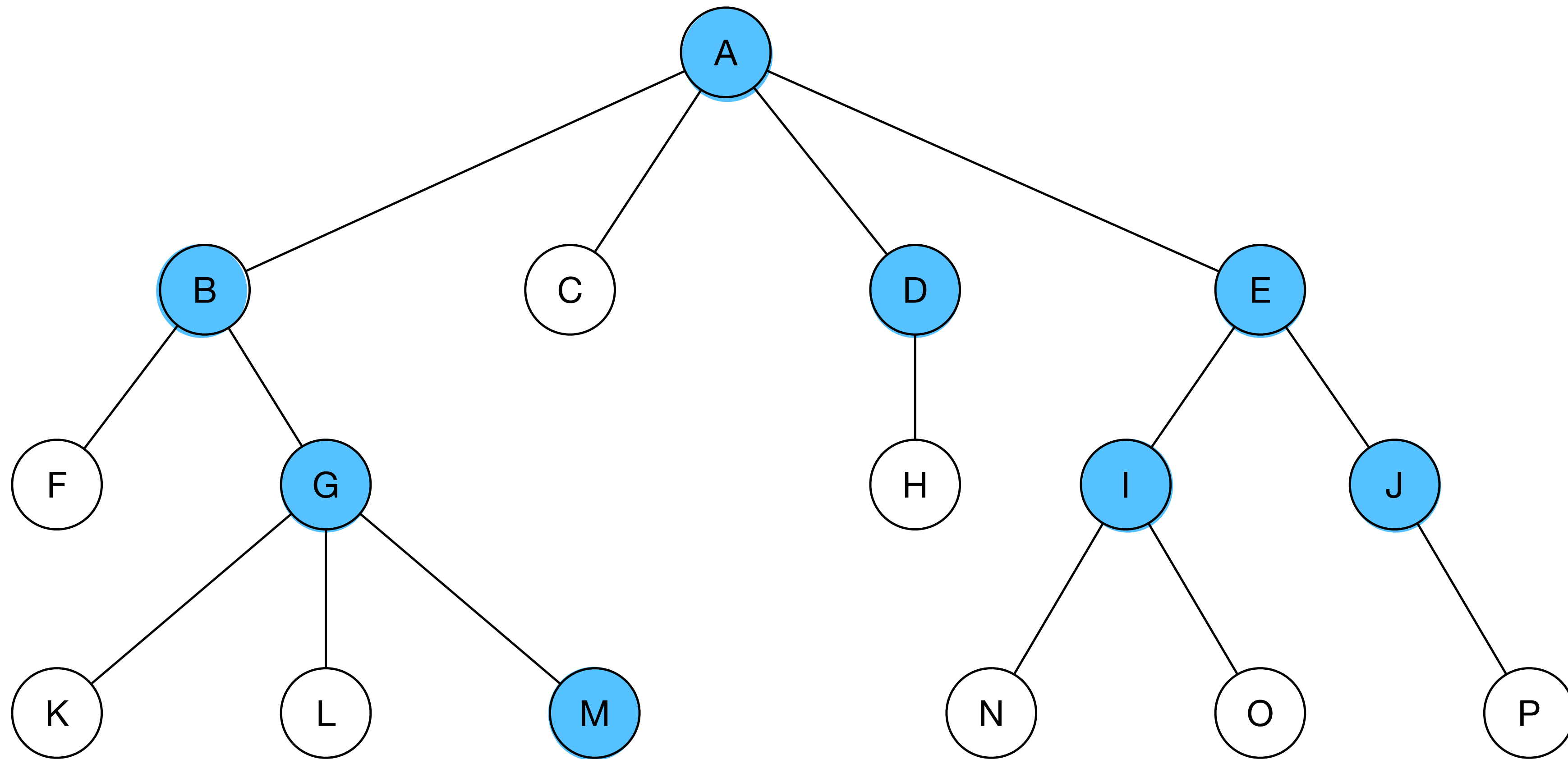
Rooted trees

Notice that the nodes with no children are precisely the leaves!



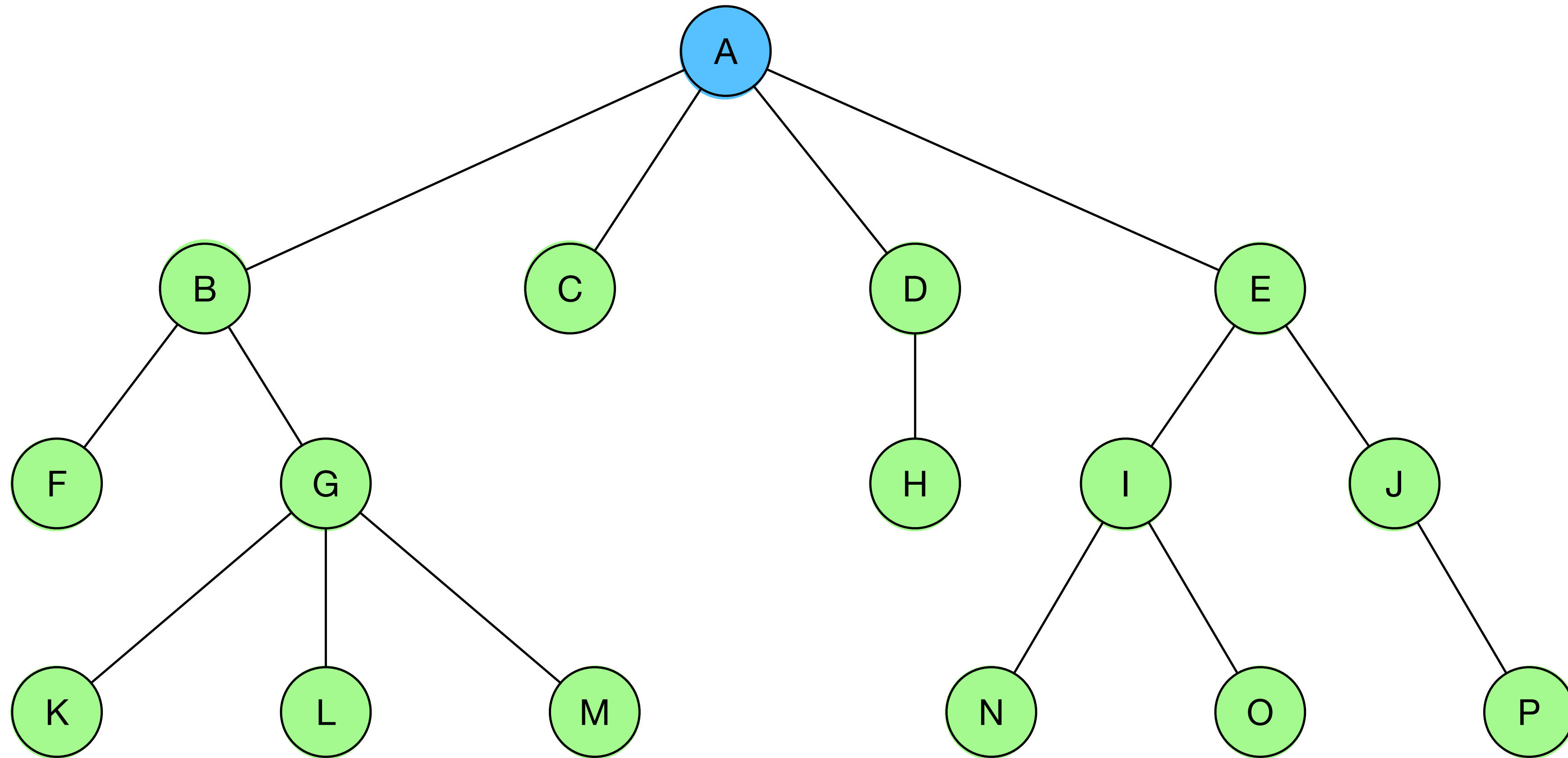
Rooted trees

The interior nodes are precisely those with at least one child.



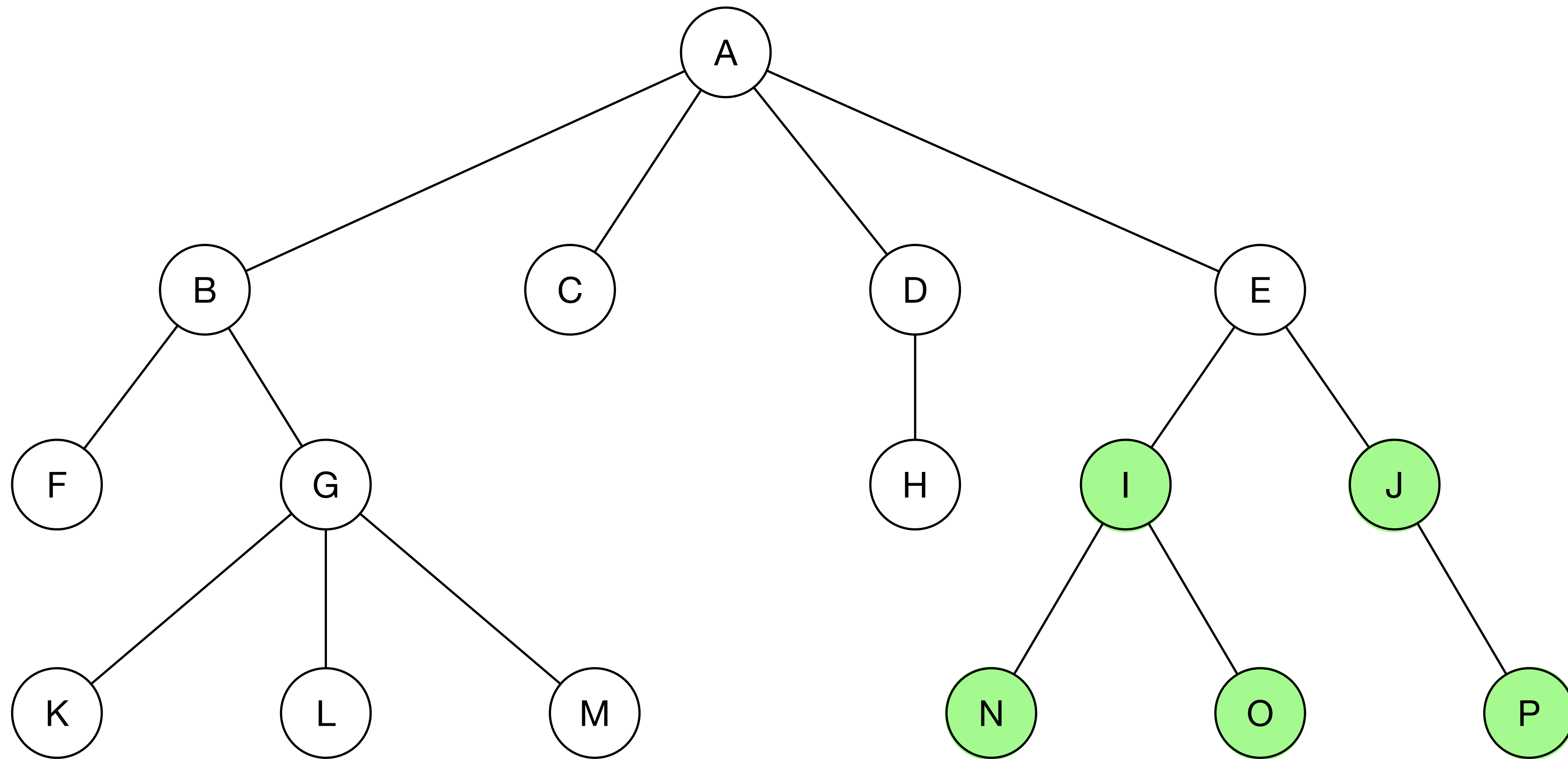
Rooted trees

We may speak of *ancestors* and *descendants*. Apart from the root itself all nodes in the tree are descended from the root. The root is the ancestor of all.



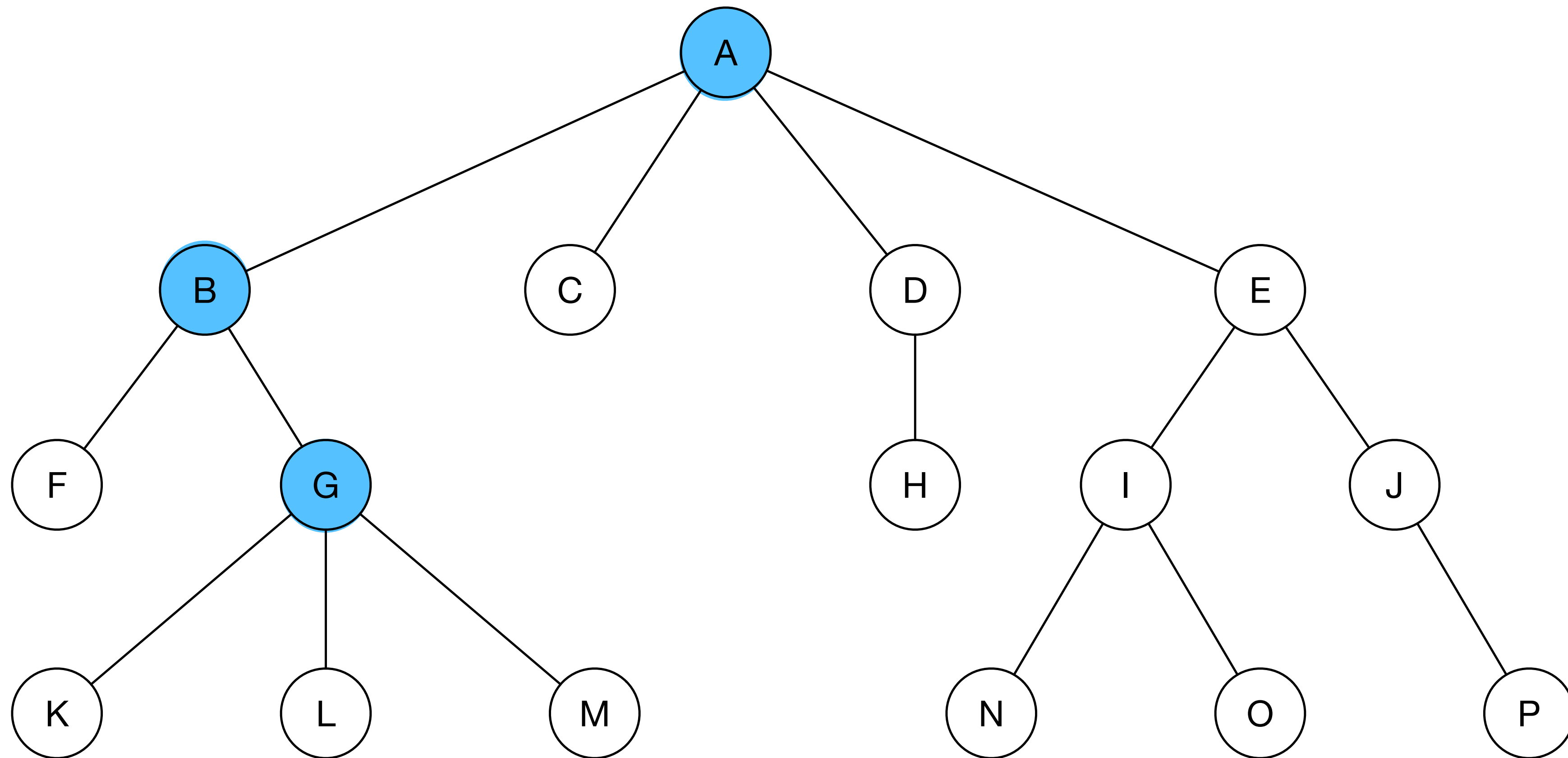
Rooted trees

These are the descendants of E.



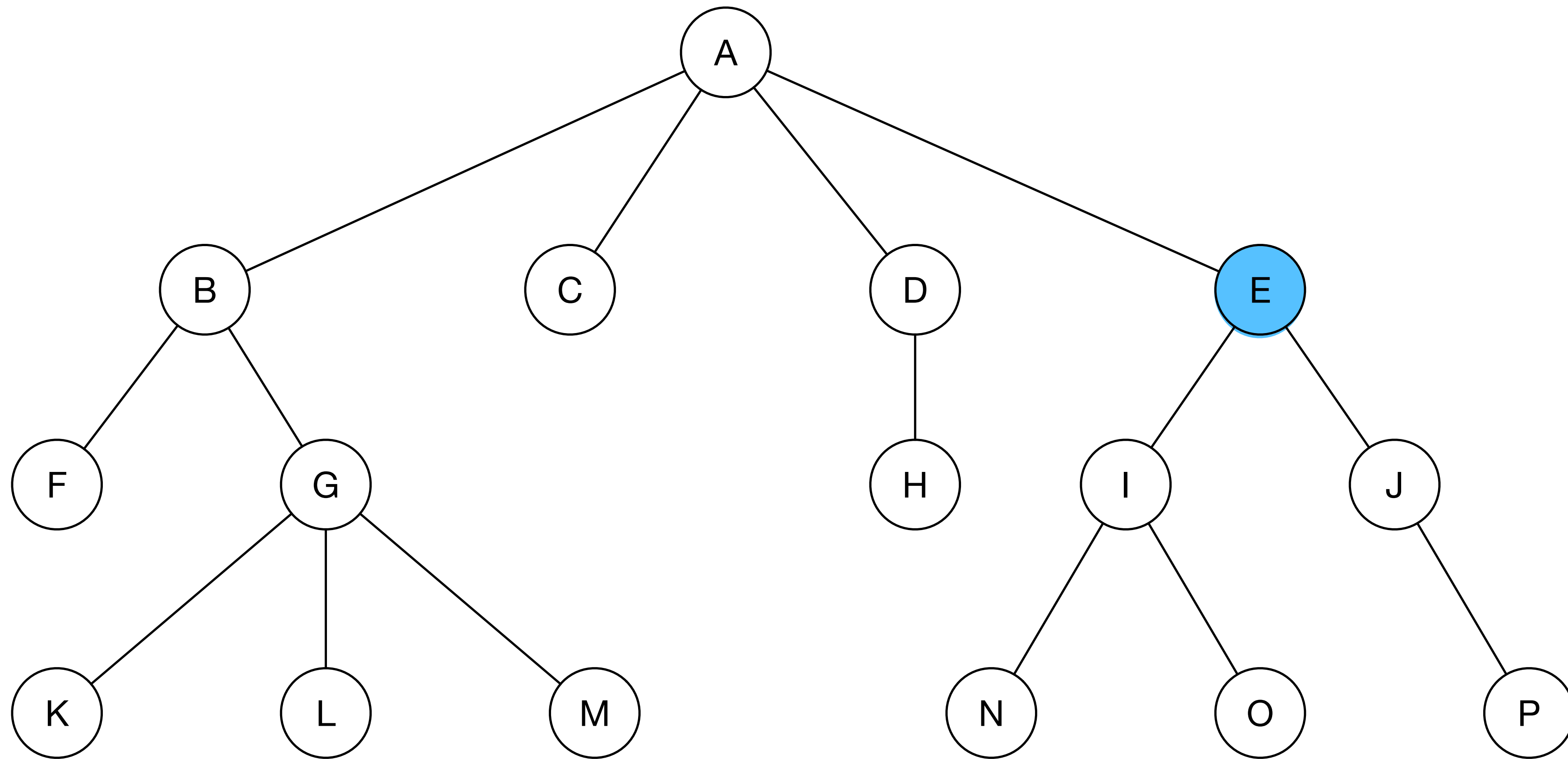
Rooted trees

These are the ancestors of K.



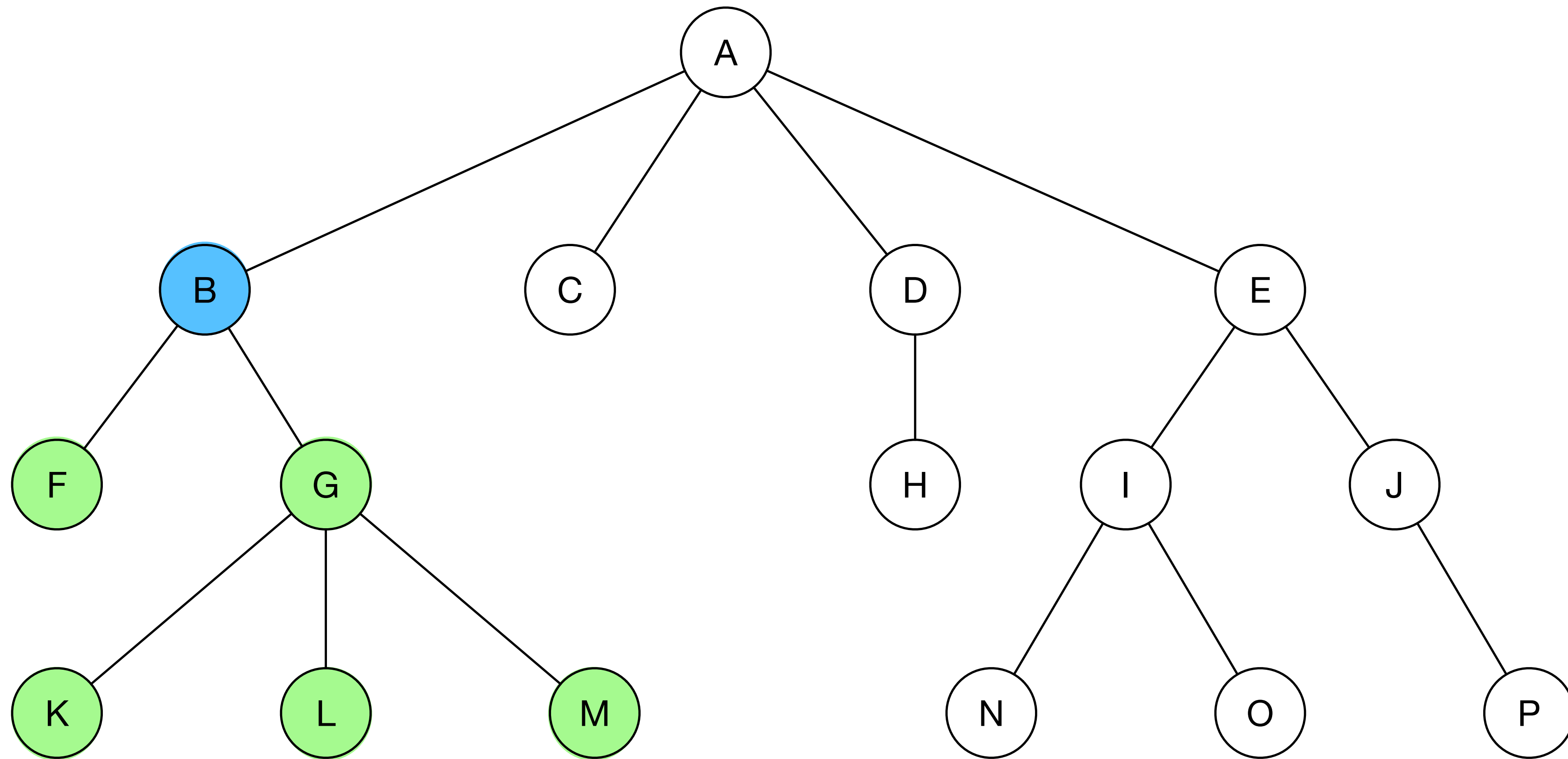
Rooted trees

E is the least (or first) common ancestor of N and P.



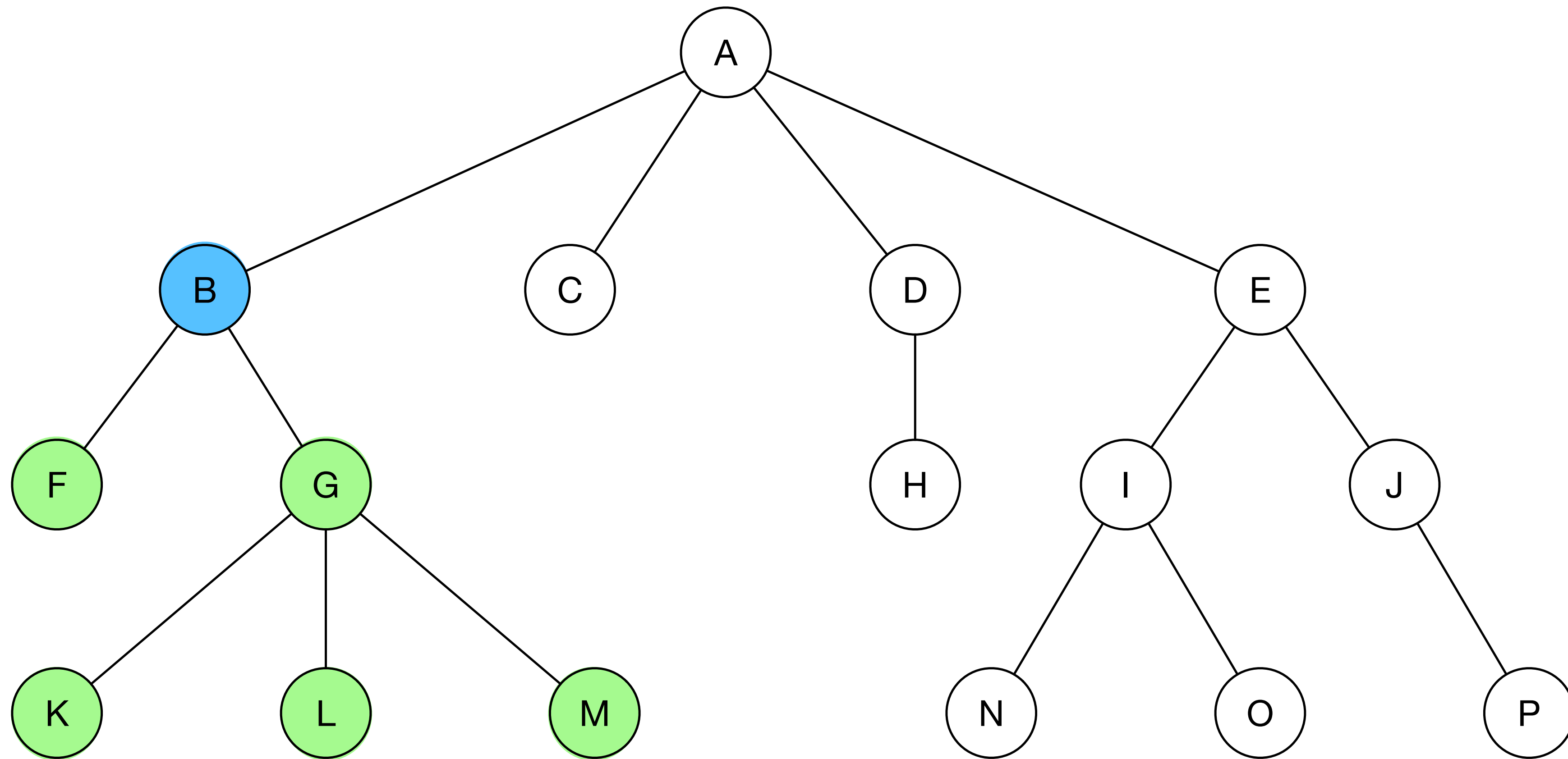
Subtrees

We may refer to a node and its descendants as a *subtree*.



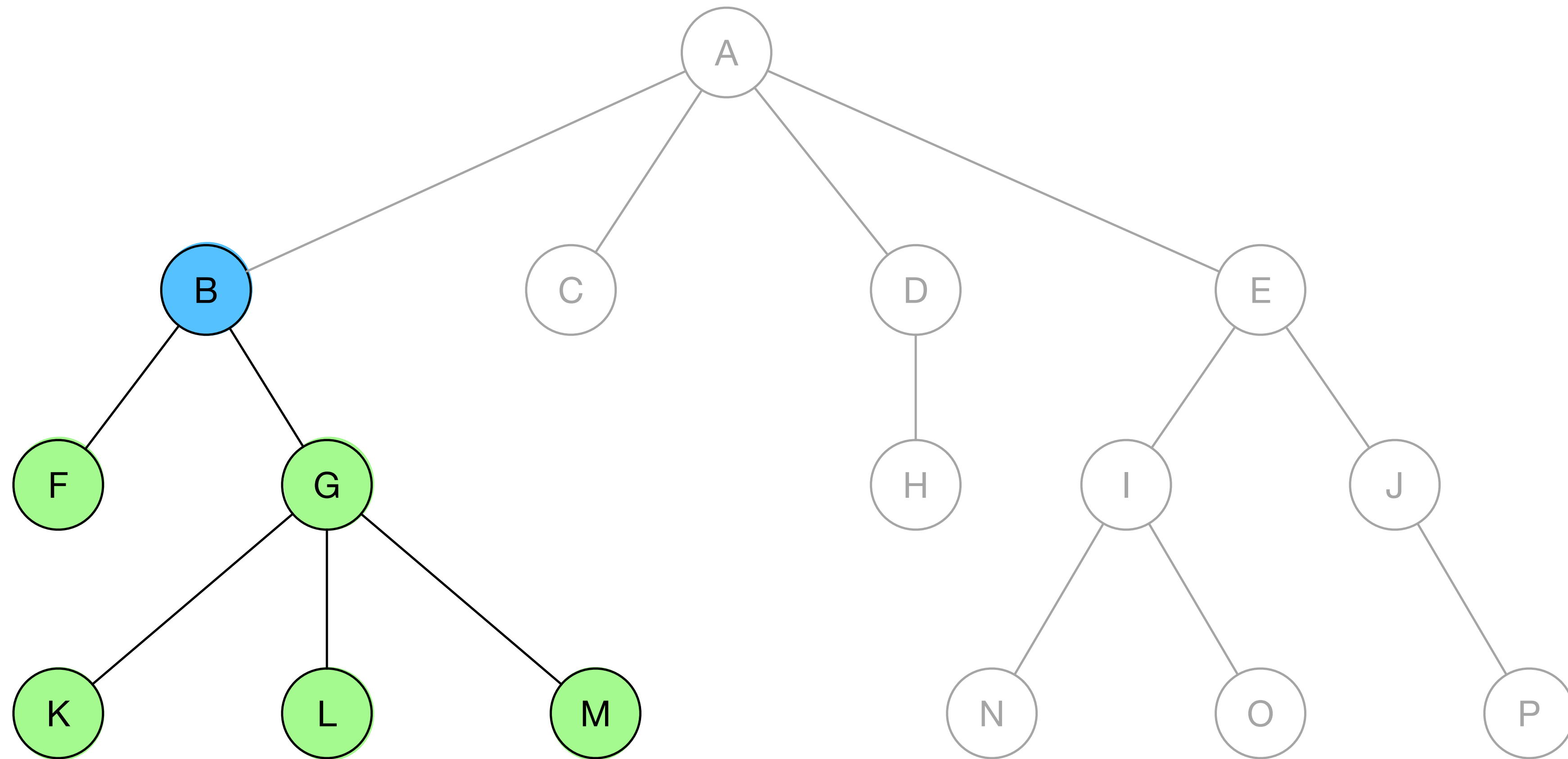
Subtrees

Here is the subtree rooted at B.



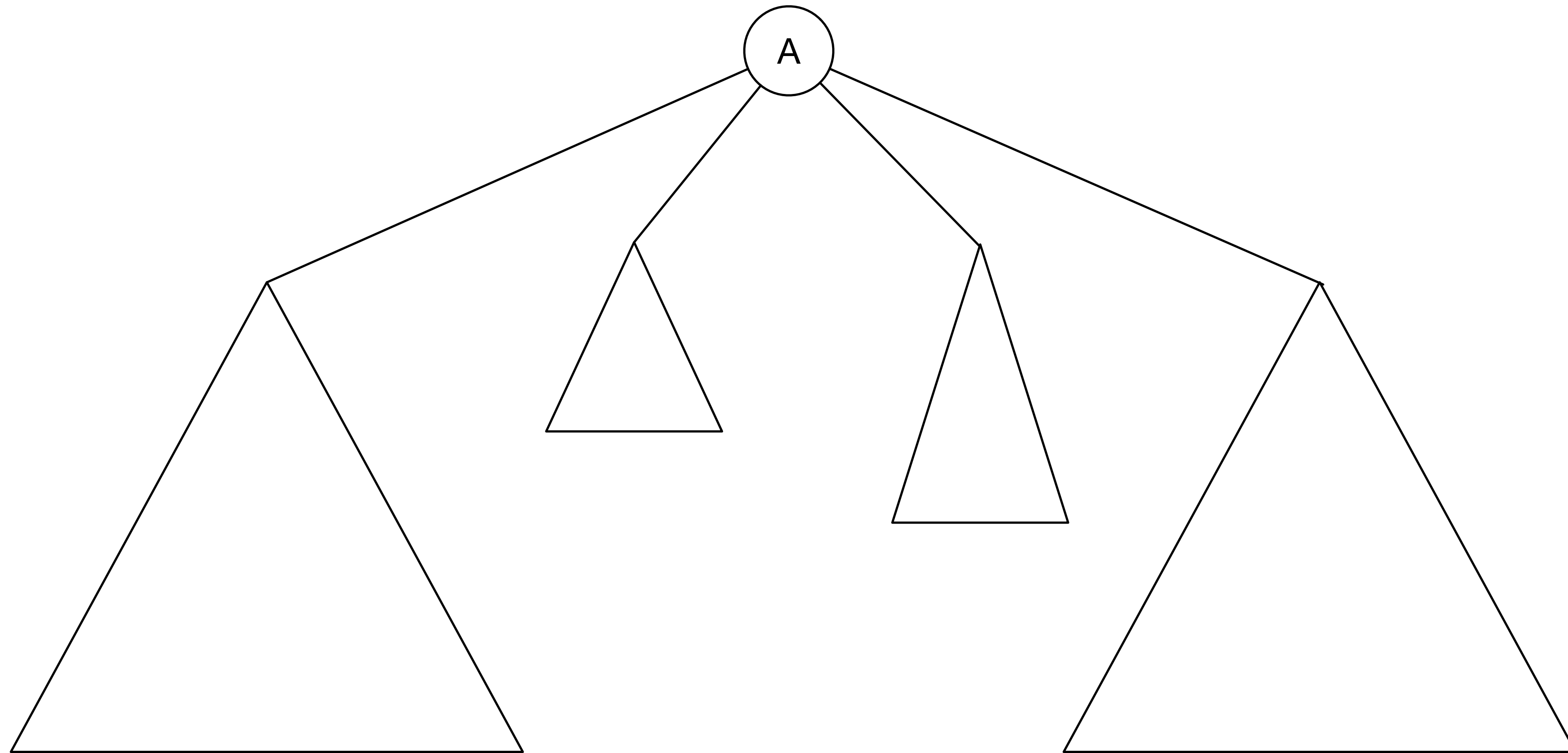
Subtrees

Here is the subtree rooted at B.



Subtrees

Sometimes, when we don't care about detail we just show subtrees as triangles. Here is a tree with four subtrees.



Binary trees

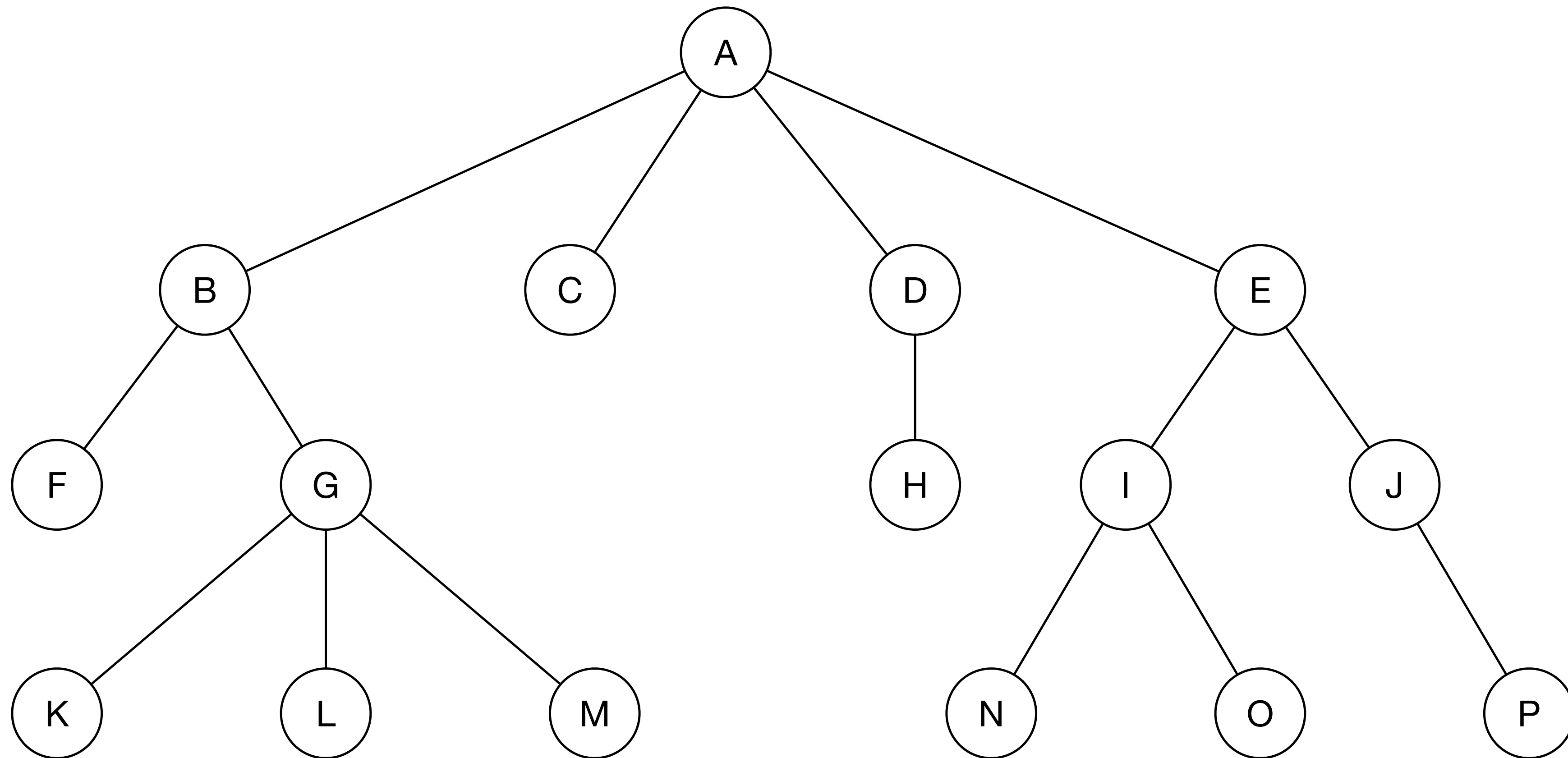
Many of the trees we'll work with are *binary trees*.

What is a binary tree?

Definition: A binary tree is a tree in which each parent can have at most two children.

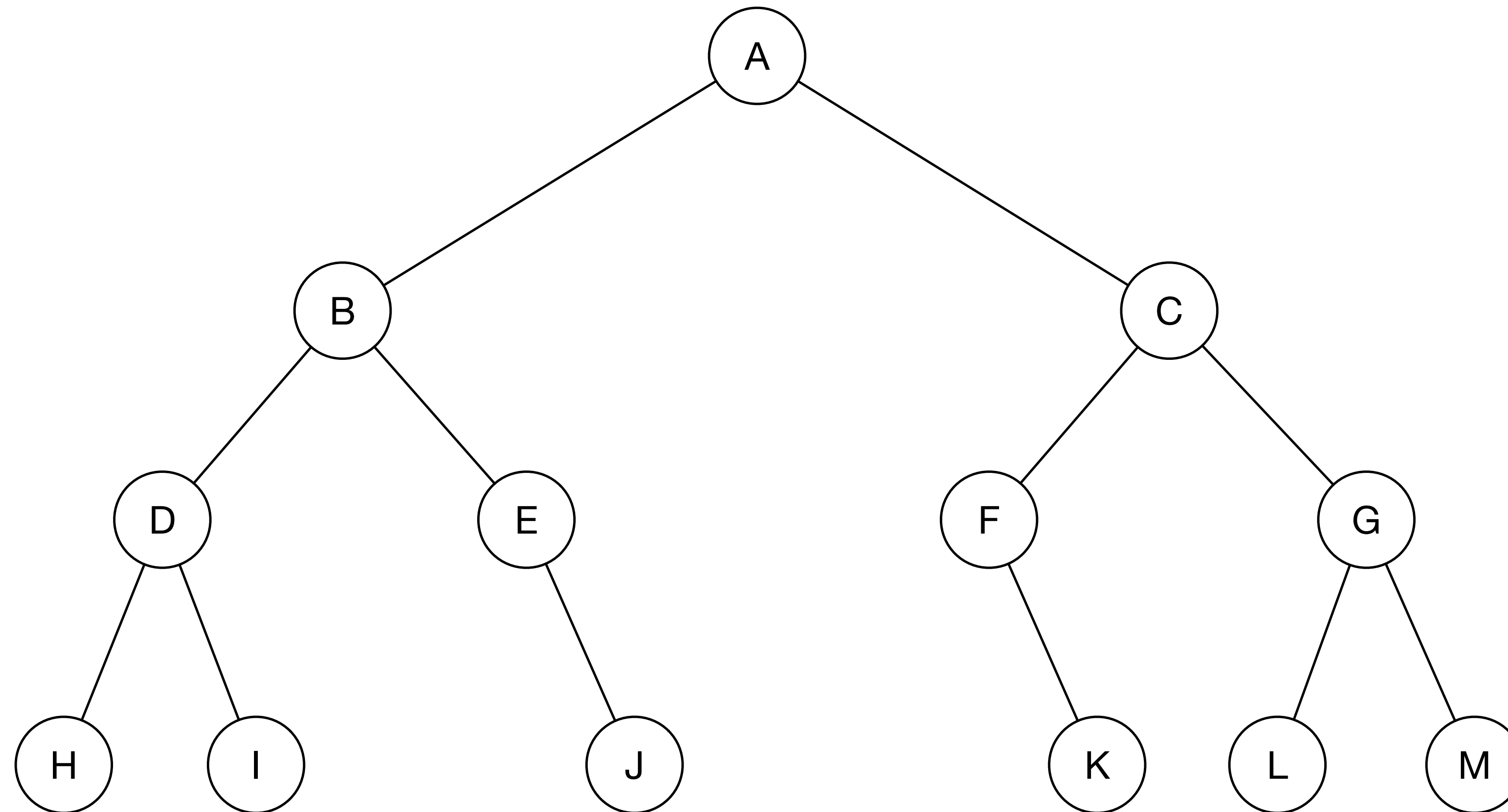
Binary trees

The example we've been using so far is *not* a binary tree.



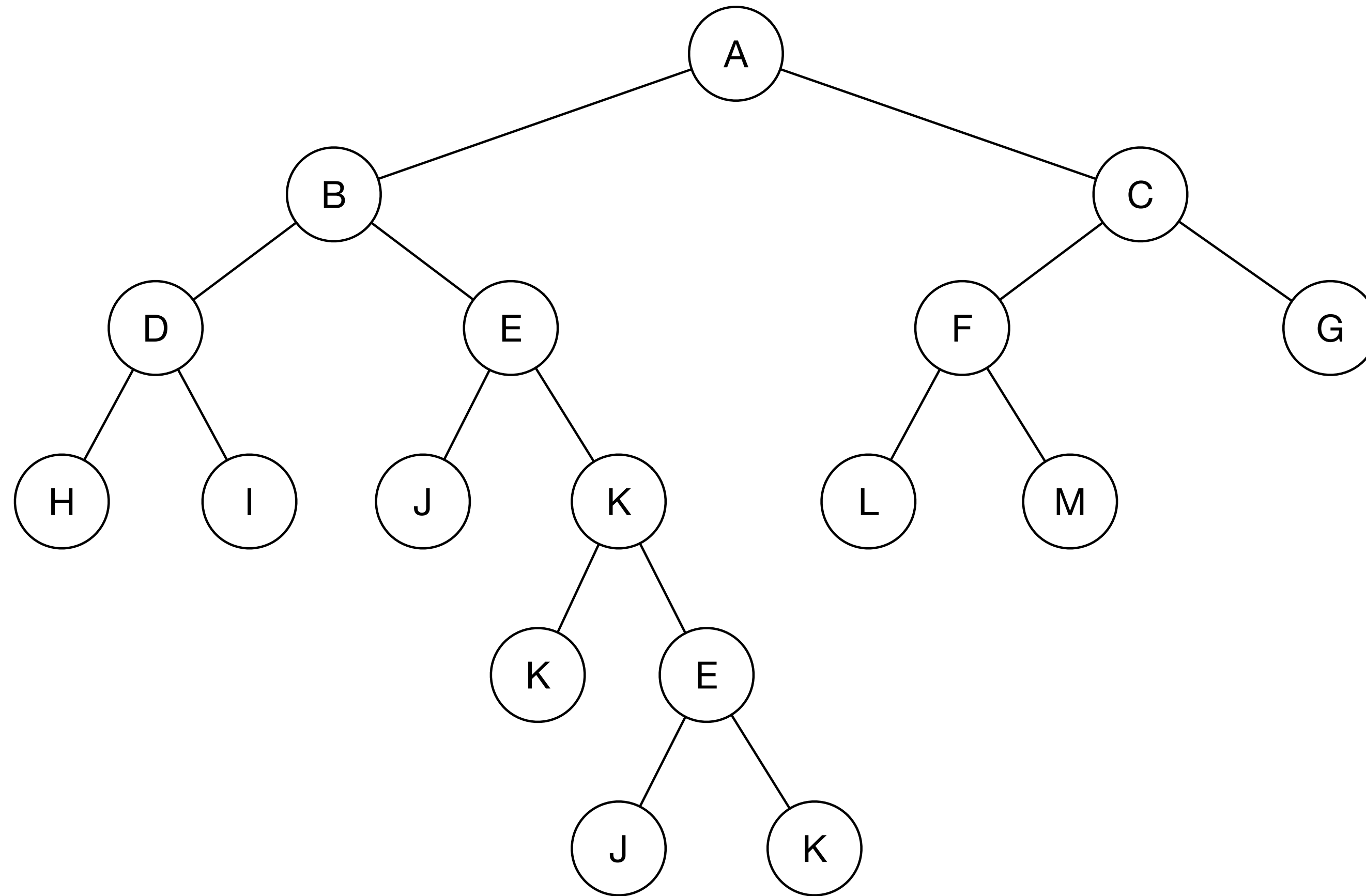
Binary trees

Here's an example of a binary tree.



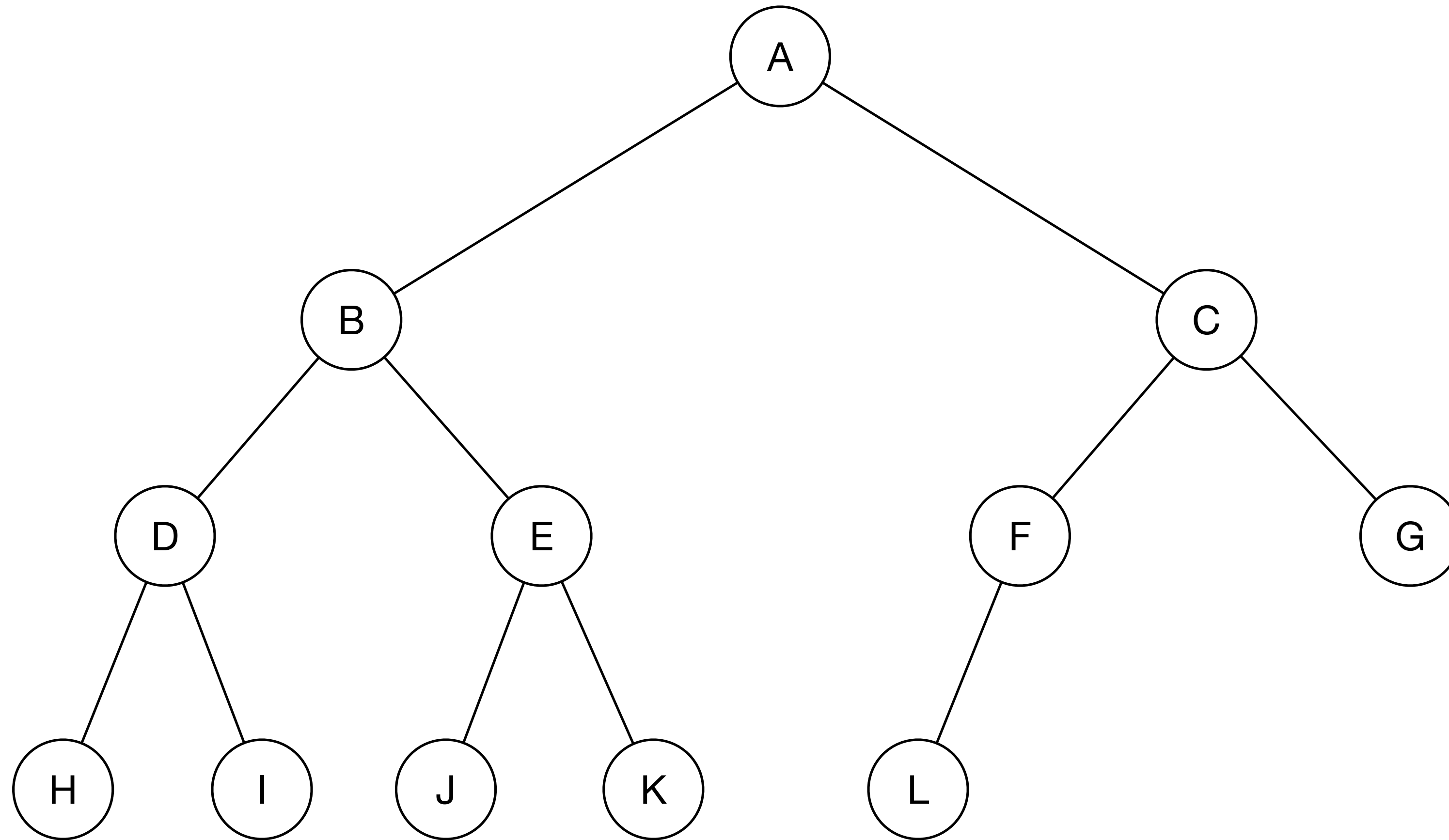
Binary trees

A *full binary tree* is a binary tree in which each node has either two children or none at all.



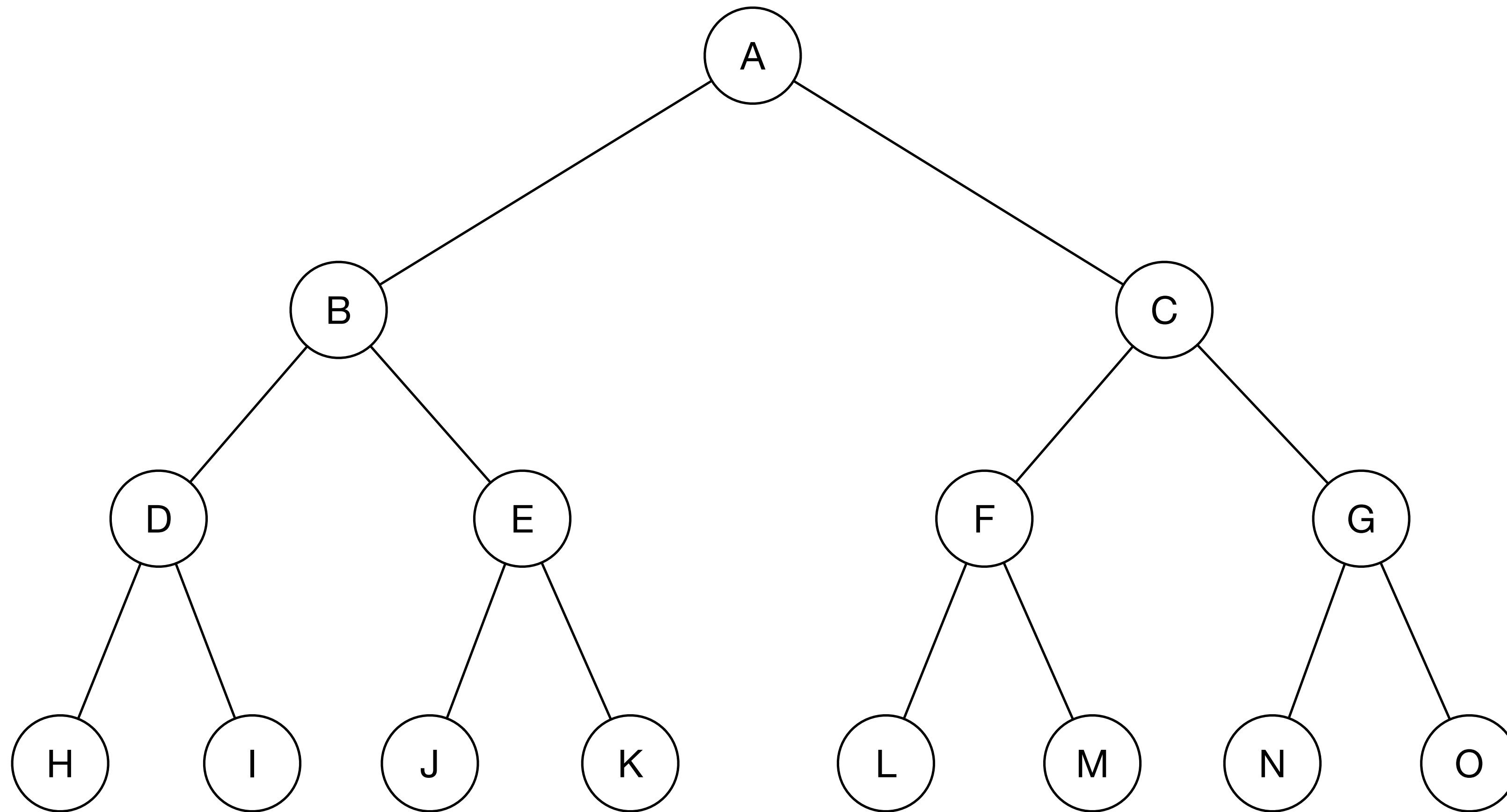
Binary trees

A complete binary tree is a tree in which each level is full, with the possible exception of the last.



Binary trees

A perfect binary tree is a tree in which each level is full.



Binary tree

Are all trees binary?

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children.

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children.

Are all binary trees full?

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete?

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Are all complete trees perfect?

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Are all complete trees perfect? **NO**. The lowest level may not be filled.

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Are all complete trees perfect? **NO**. The lowest level may not be filled.

Are all perfect trees full?

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Are all complete trees perfect? **NO**. The lowest level may not be filled.

Are all perfect trees full? **YES**. Every node in a perfect tree has 0 or 2 children.

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Are all complete trees perfect? **NO**. The lowest level may not be filled.

Are all perfect trees full? **YES**. Every node in a perfect tree has 0 or 2 children.

Can a binary tree have an odd number of nodes?

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Are all complete trees perfect? **NO**. The lowest level may not be filled.

Are all perfect trees full? **YES**. Every node in a perfect tree has 0 or 2 children.

Can a binary tree have an odd number of nodes? **YES**. Sure, why not?

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Are all complete trees perfect? **NO**. The lowest level may not be filled.

Are all perfect trees full? **YES**. Every node in a perfect tree has 0 or 2 children.

Can a binary tree have an odd number of nodes? **YES**. Sure, why not?

Can a perfect tree have an odd number of nodes?

Binary tree

Are all trees binary? **NO**. Some trees have nodes with more than two children?

Are all binary trees full? **NO**. Some binary trees have nodes with only one child.

Are all perfect trees complete? **YES**.

Are all complete trees perfect? **NO**. The lowest level may not be filled.

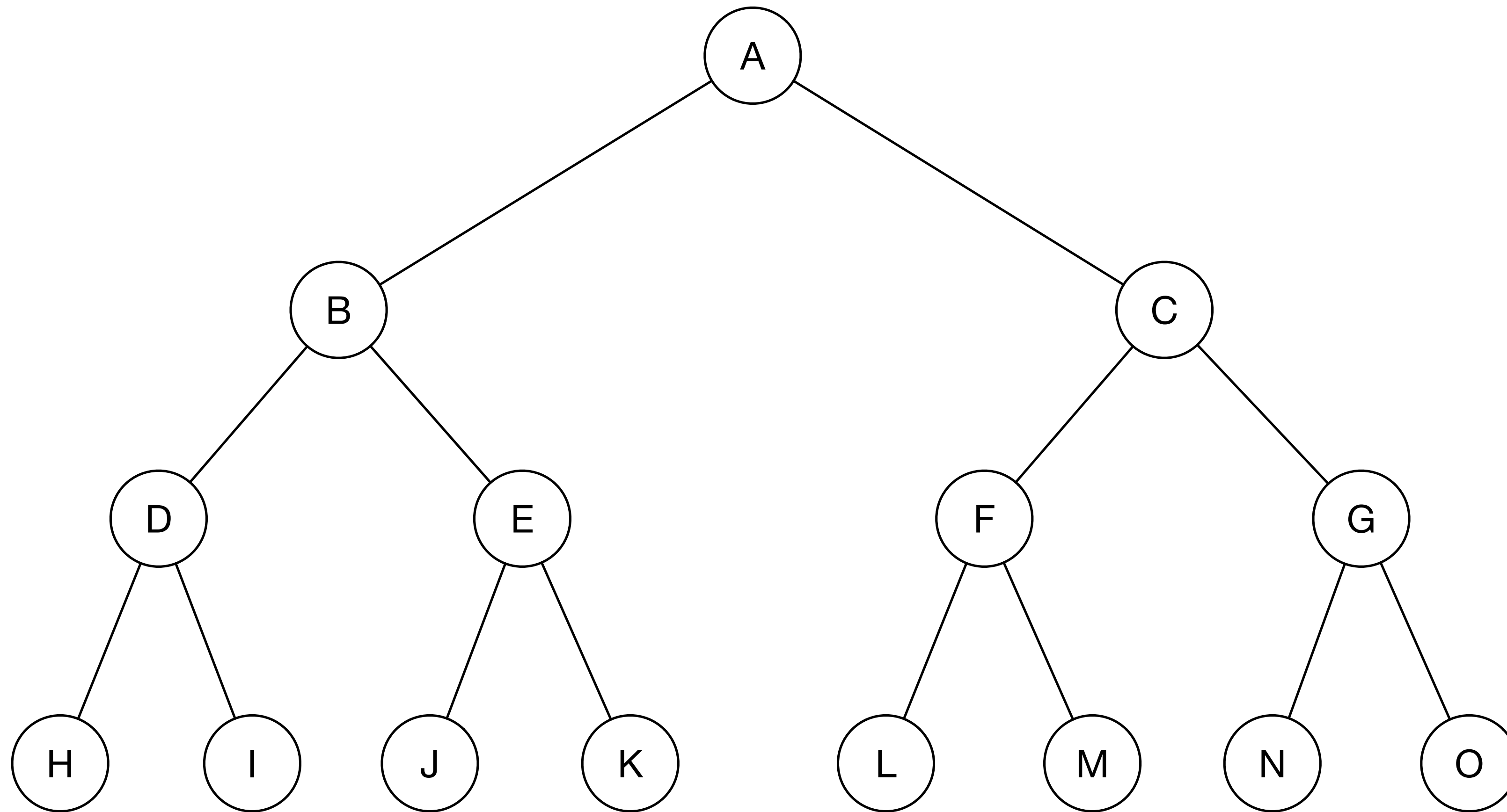
Are all perfect trees full? **YES**. Every node in a perfect tree has 0 or 2 children.

Can a binary tree have an odd number of nodes? **YES**. Sure, why not?

Can a perfect tree have an odd number of nodes? **YES**. In fact, it must!

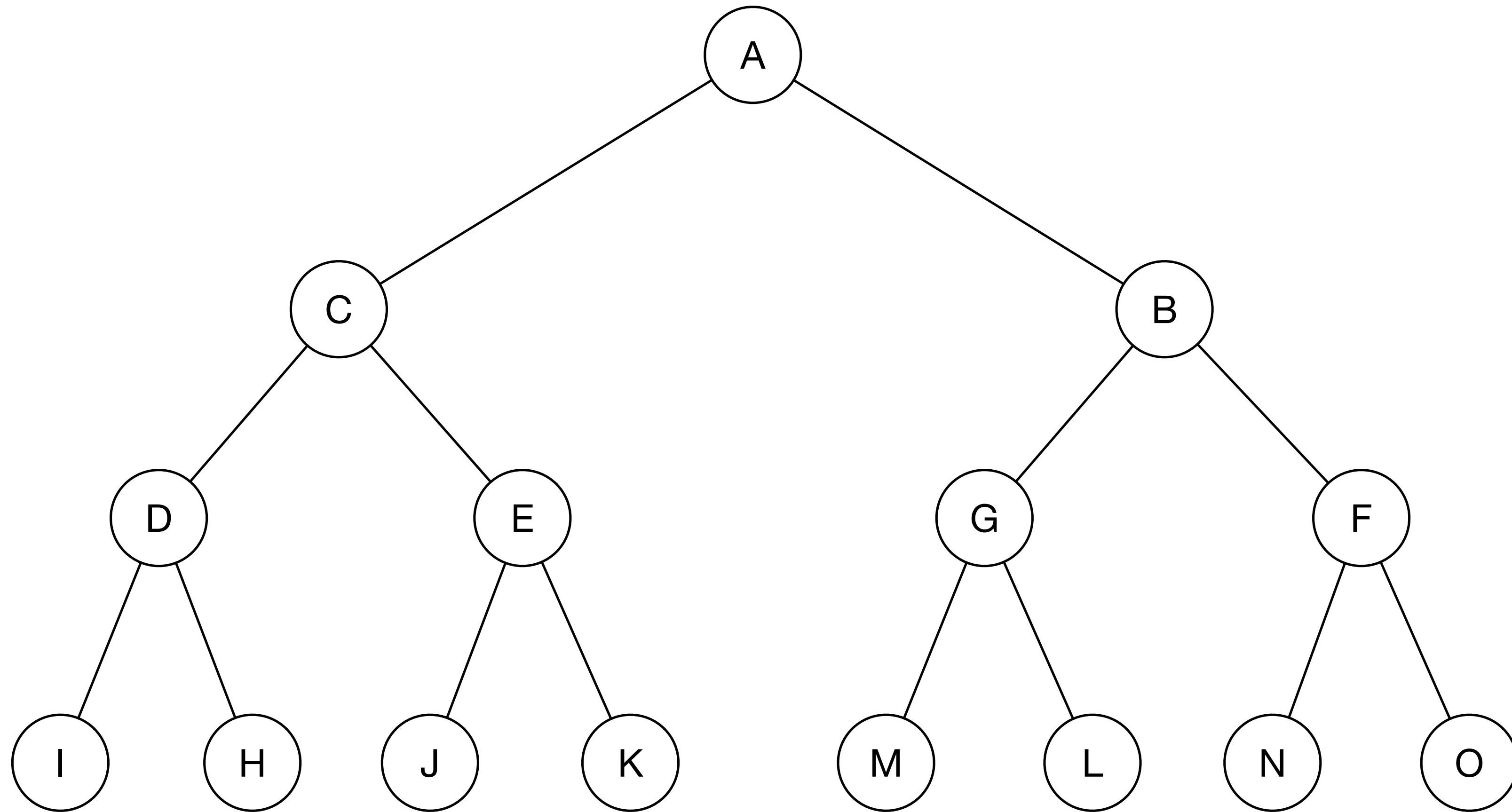
Ordered and unordered trees

Some trees are ordered and some are unordered. This tree is ordered.



Ordered and unordered trees

This tree is unordered.



More to follow...

