

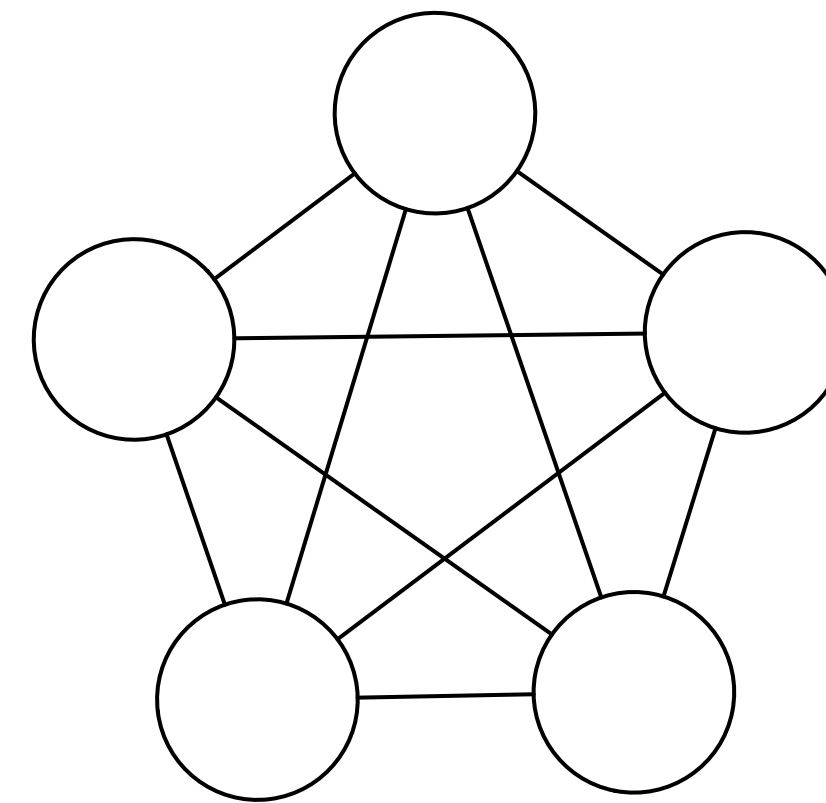
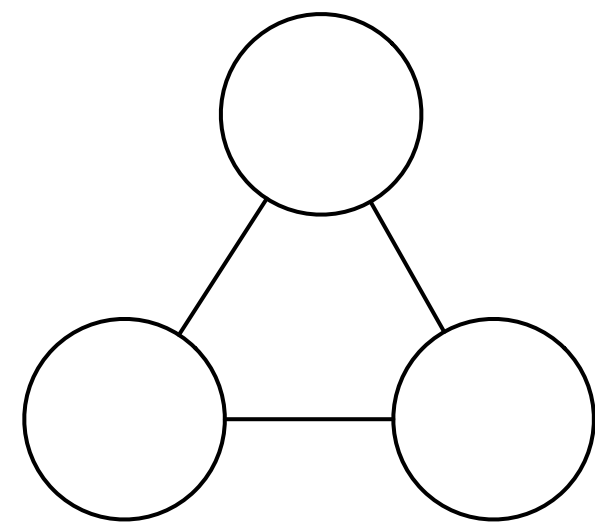
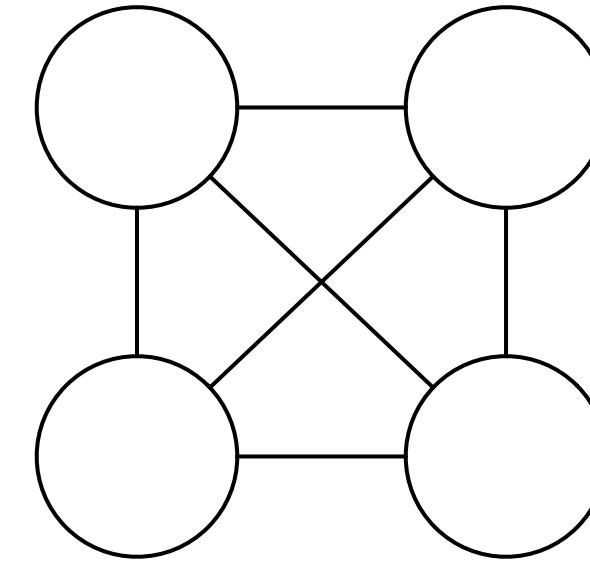
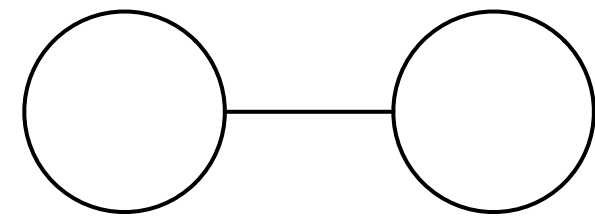


THE UNIVERSITY OF VERMONT
COLLEGE OF ENGINEERING &
MATHEMATICAL SCIENCES

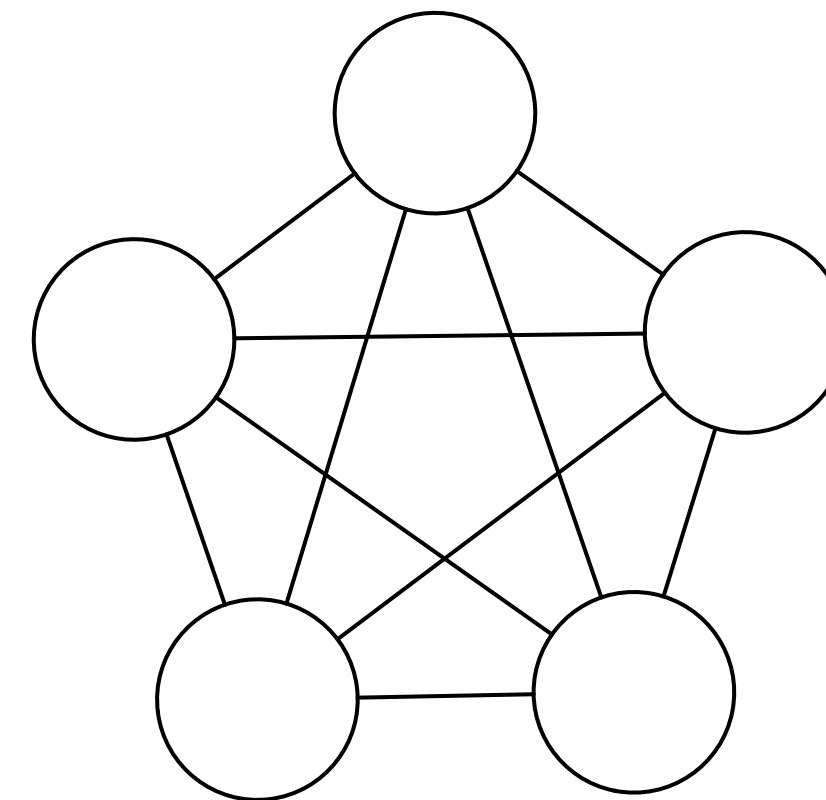
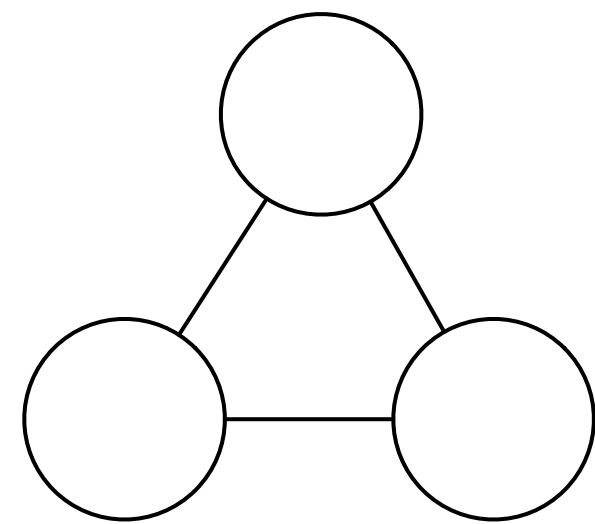
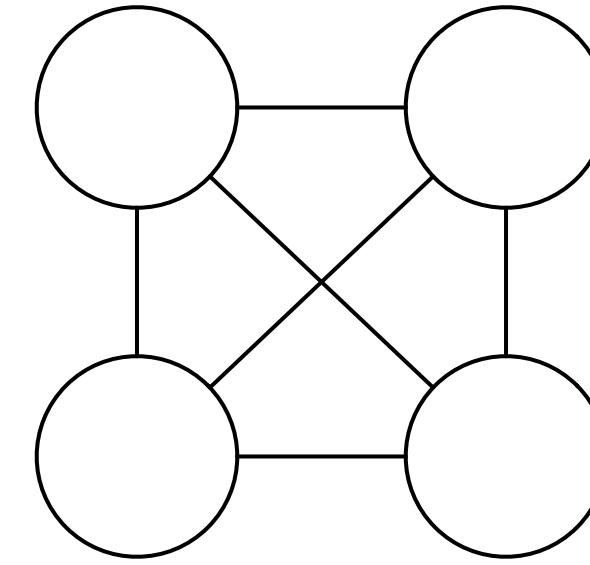
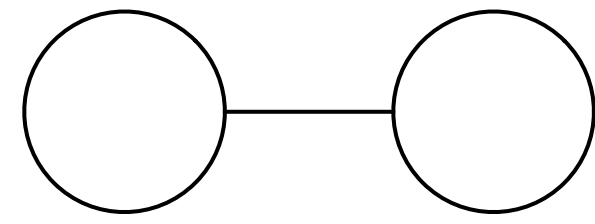
Introduction to Graphs

A sneak preview into some graph theory you'll see in CS 224

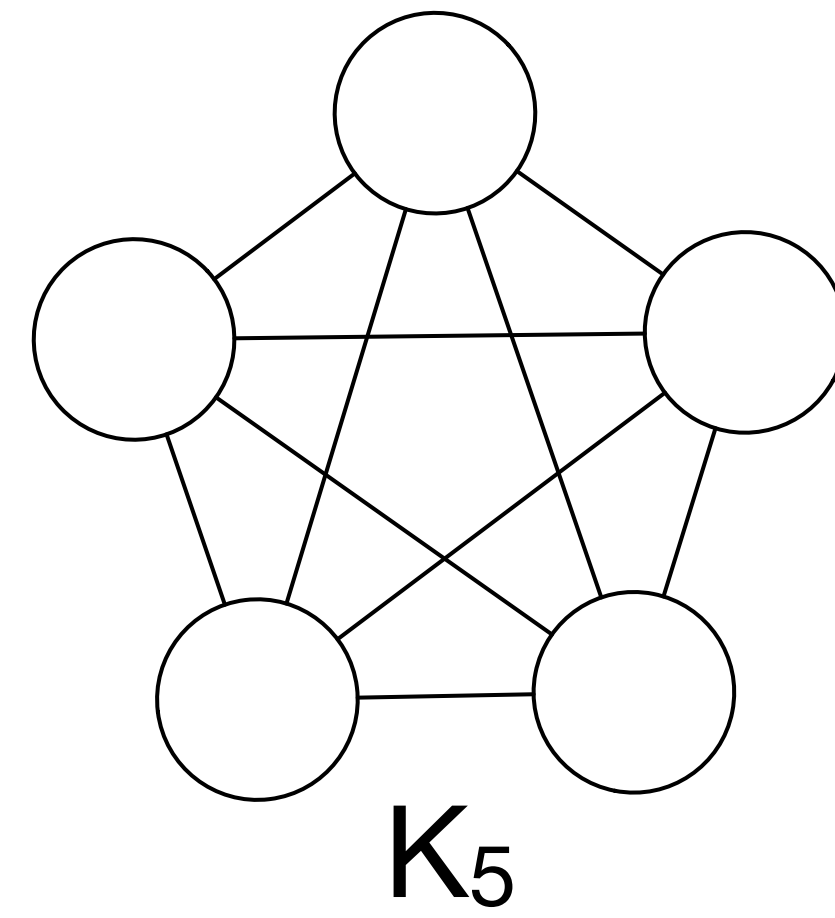
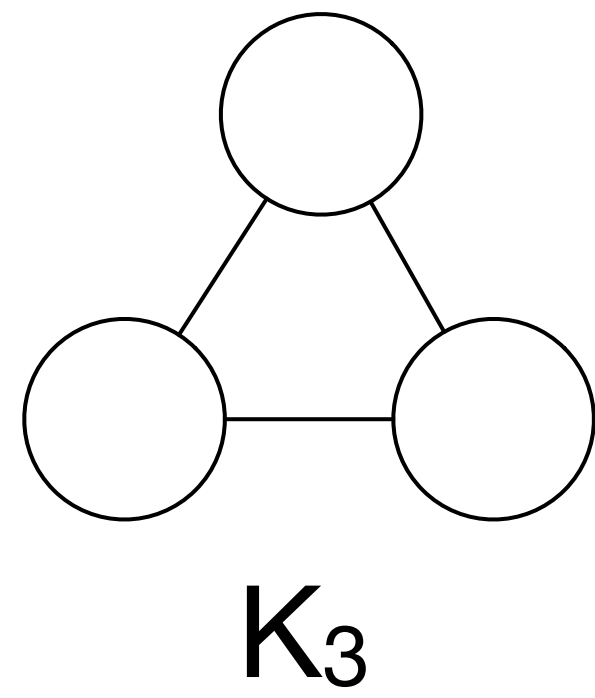
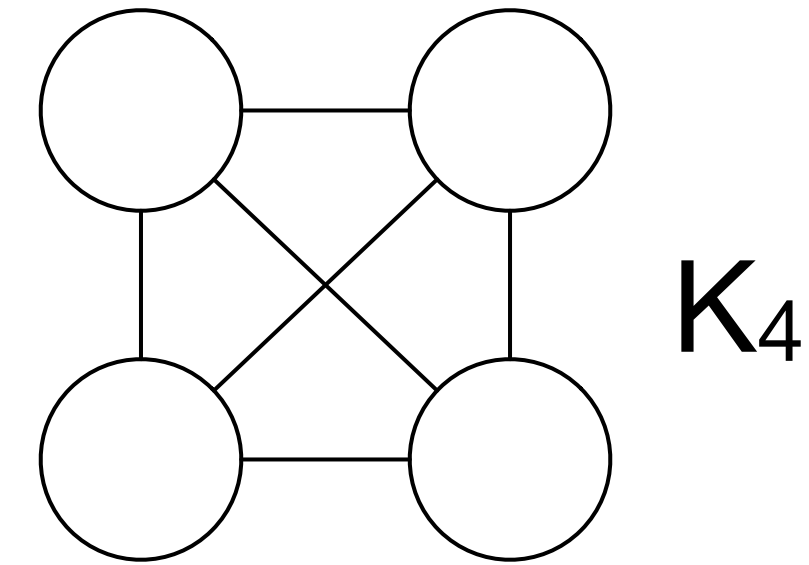
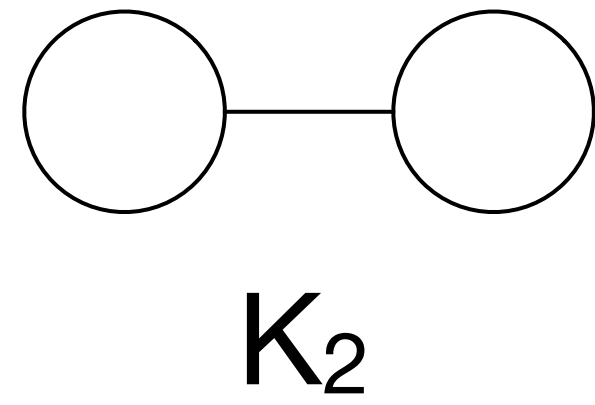
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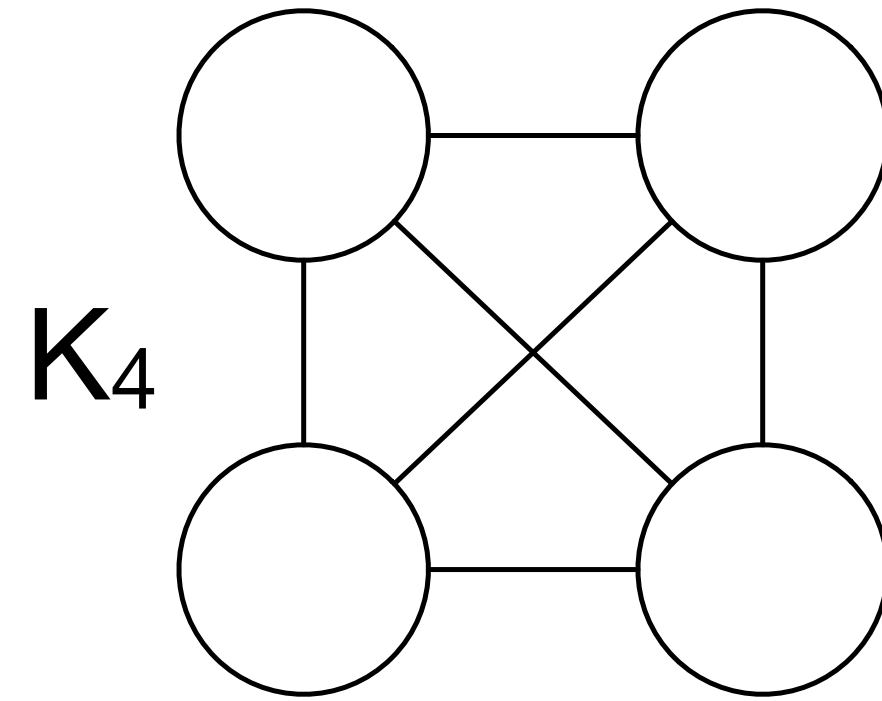
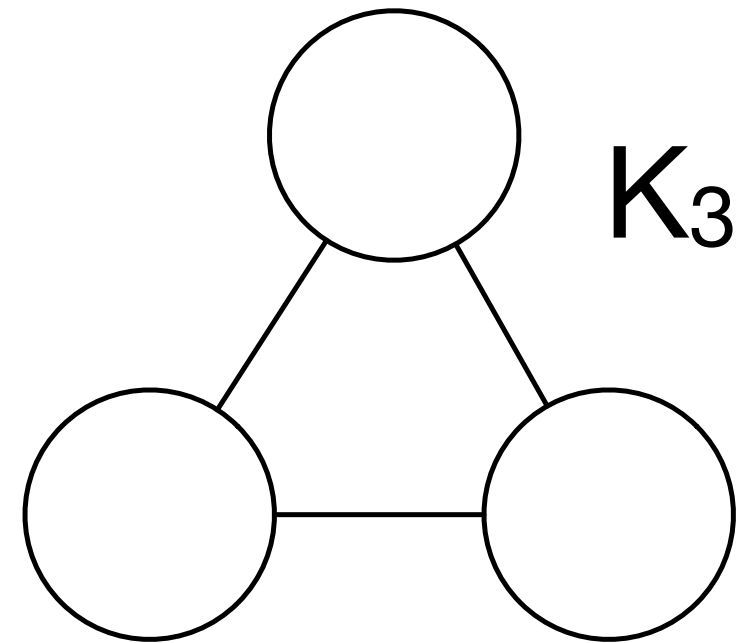
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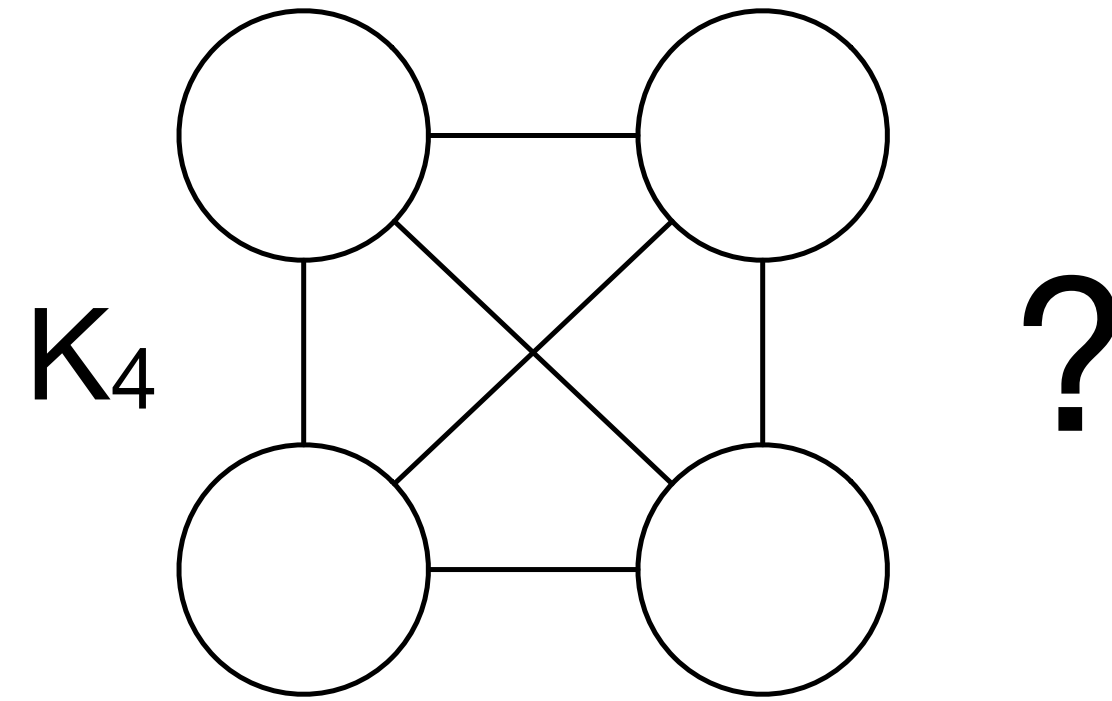
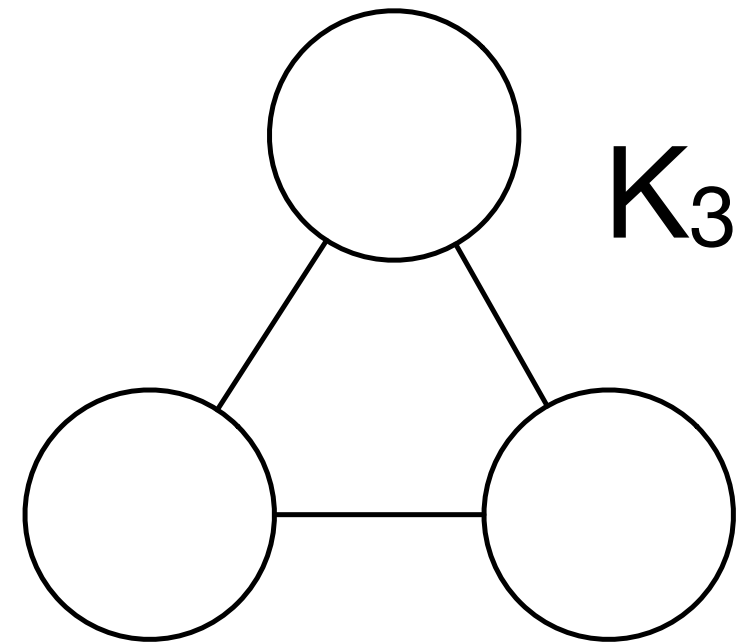


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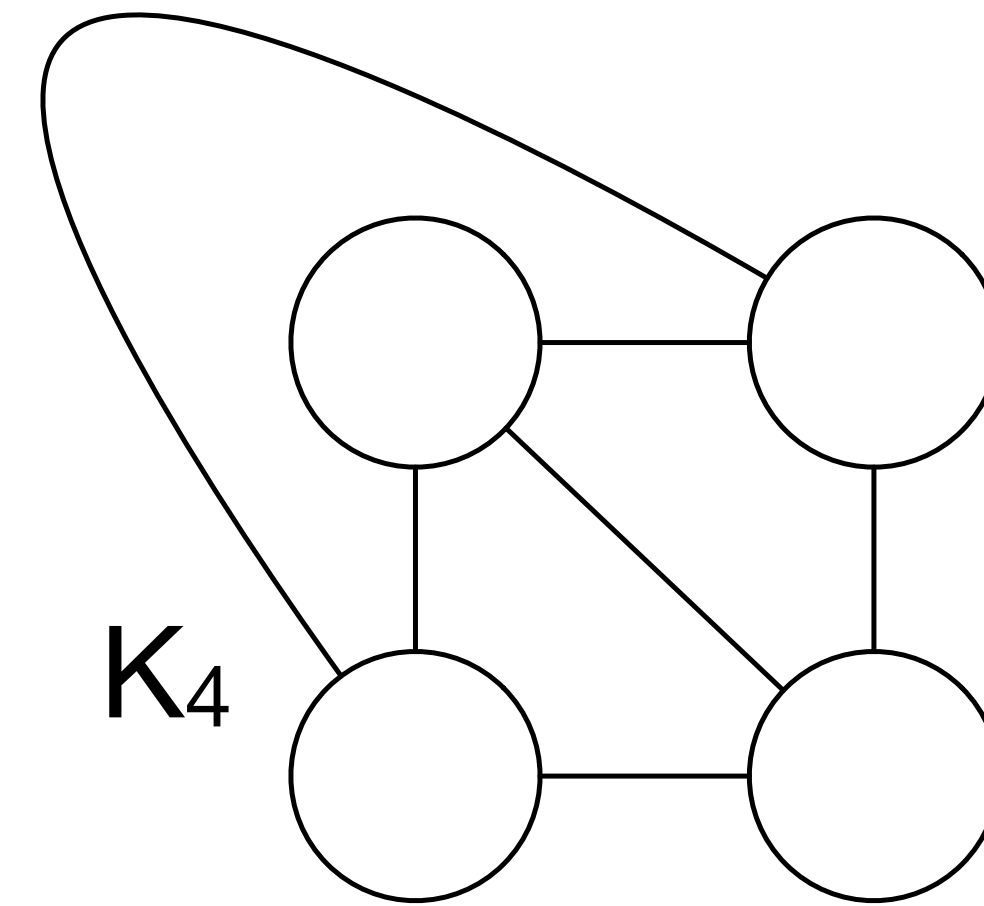
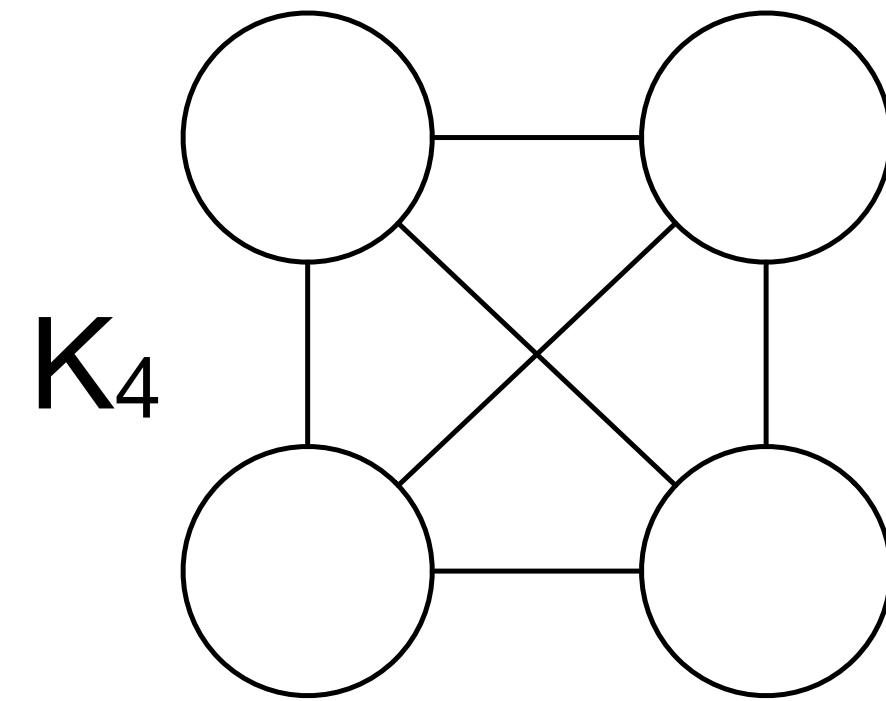
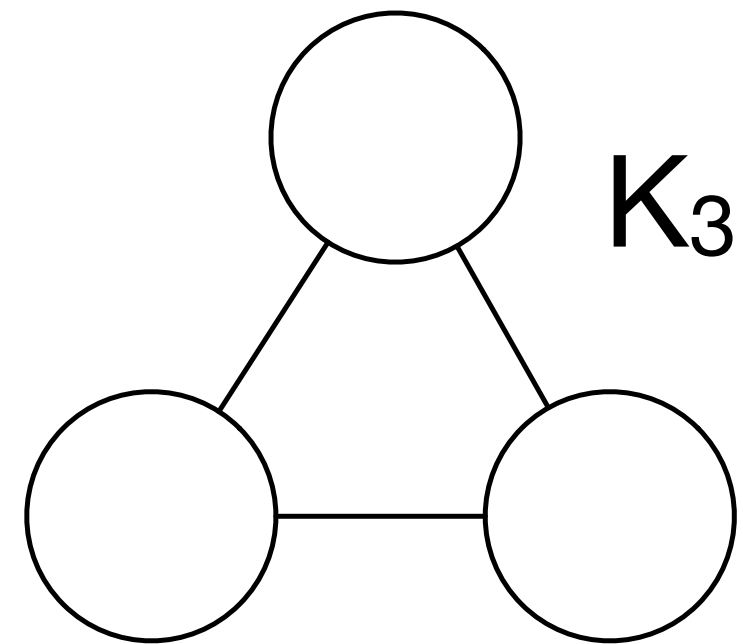


If we can draw a graph on a 2D surface without having any edges cross we call such a graph *planar*.

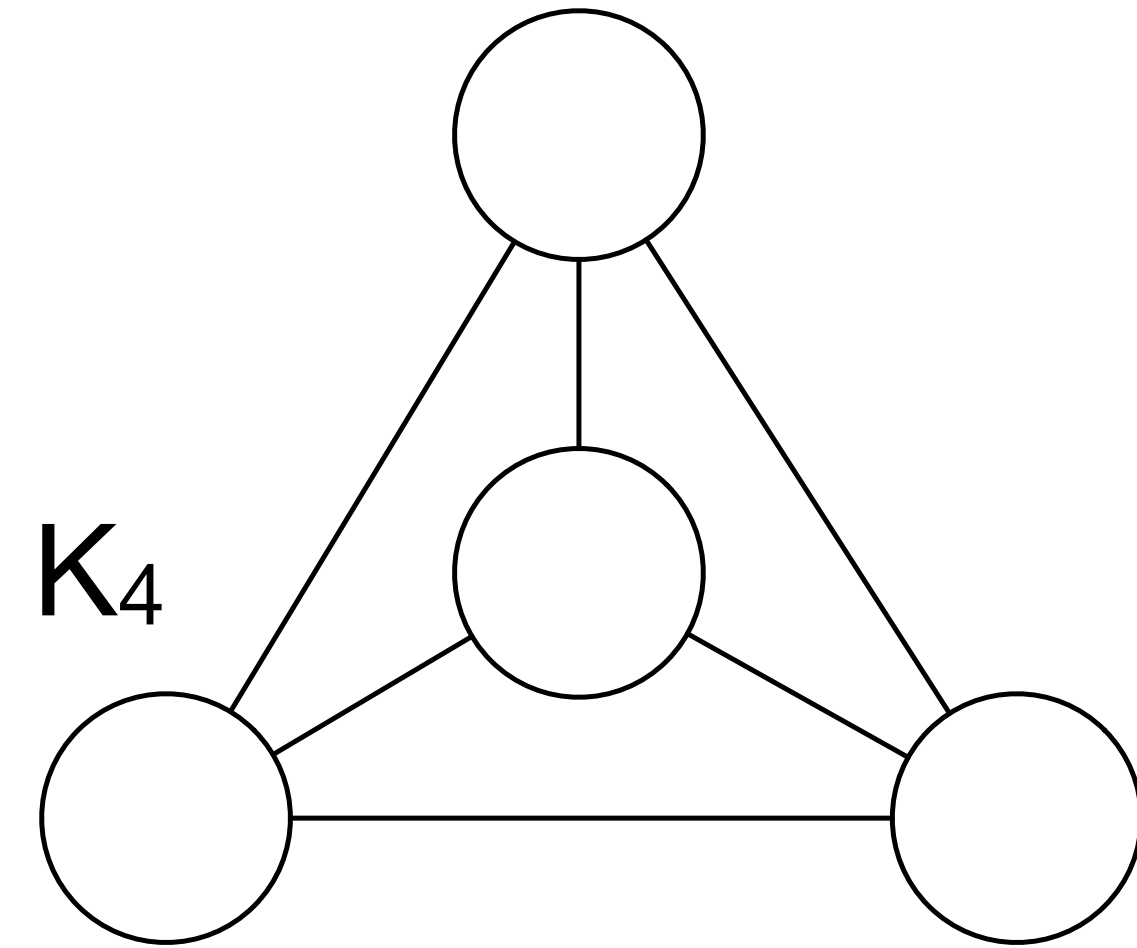
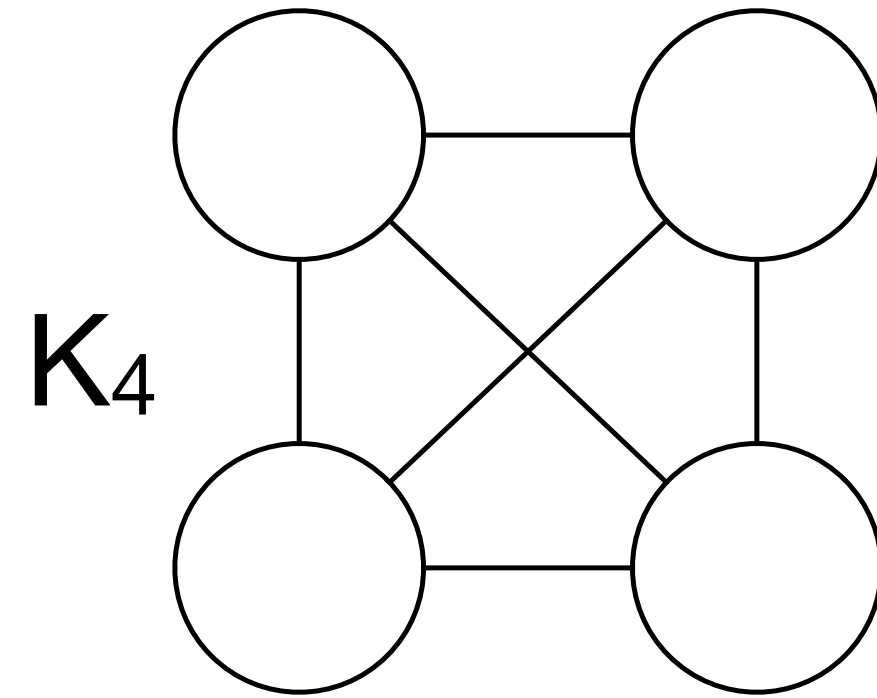
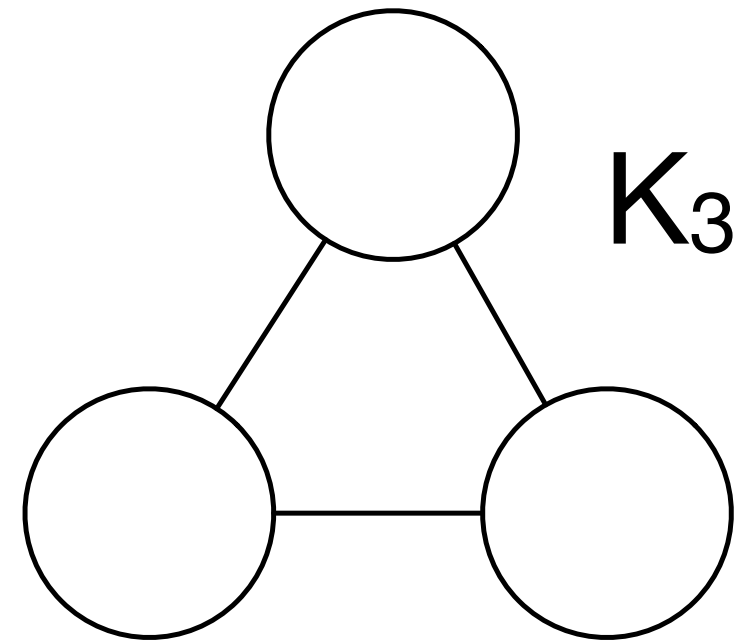
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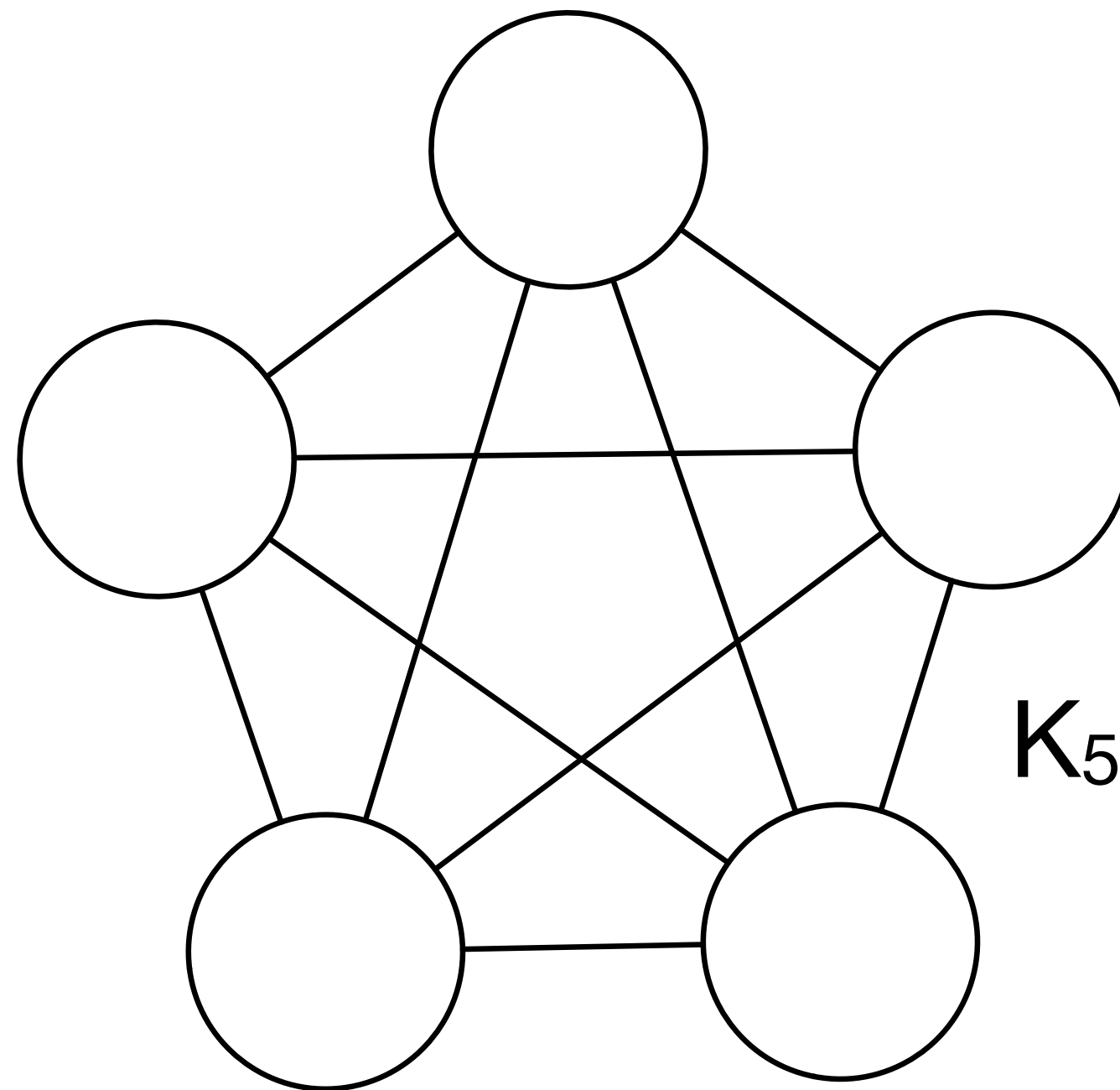
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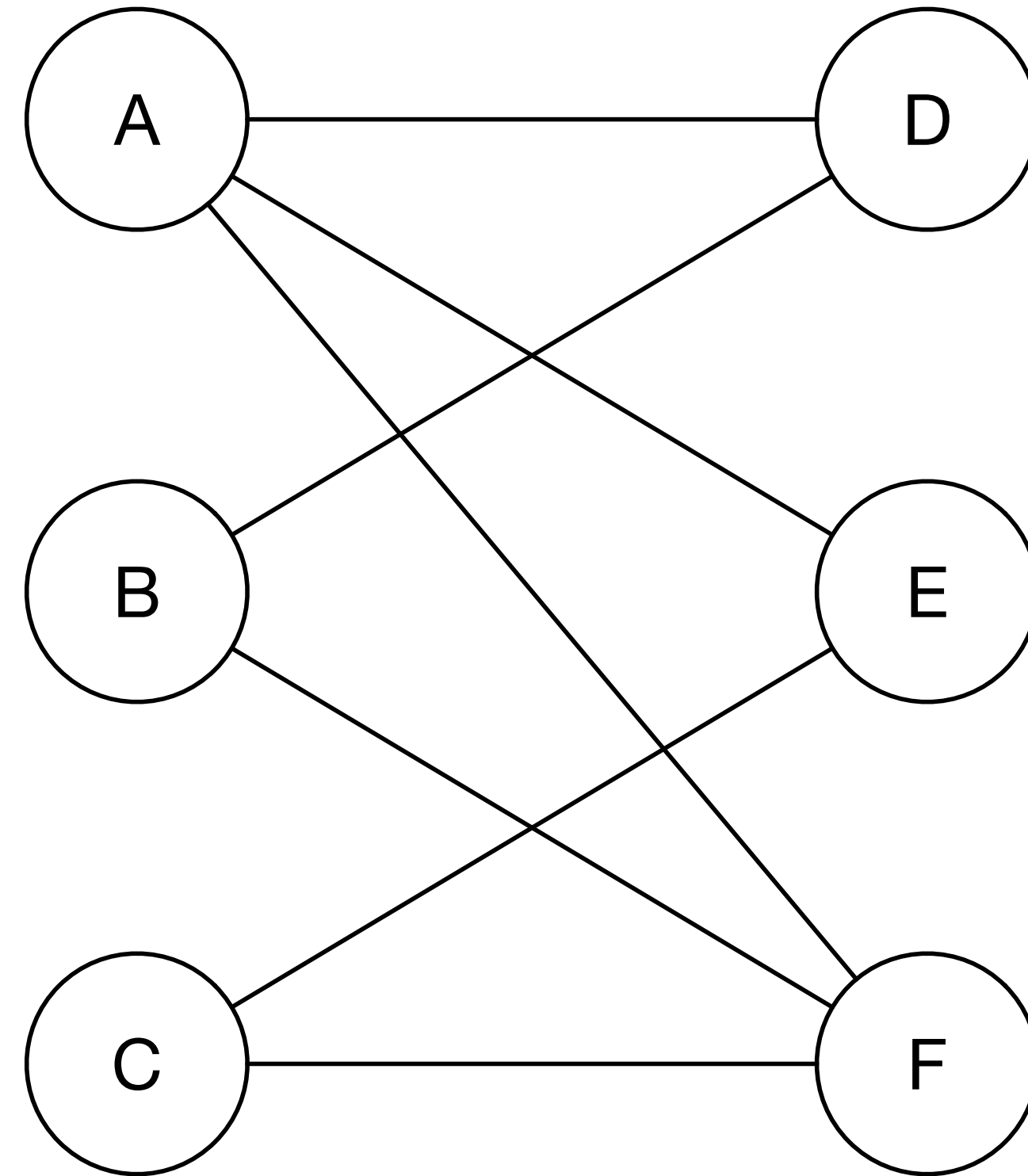
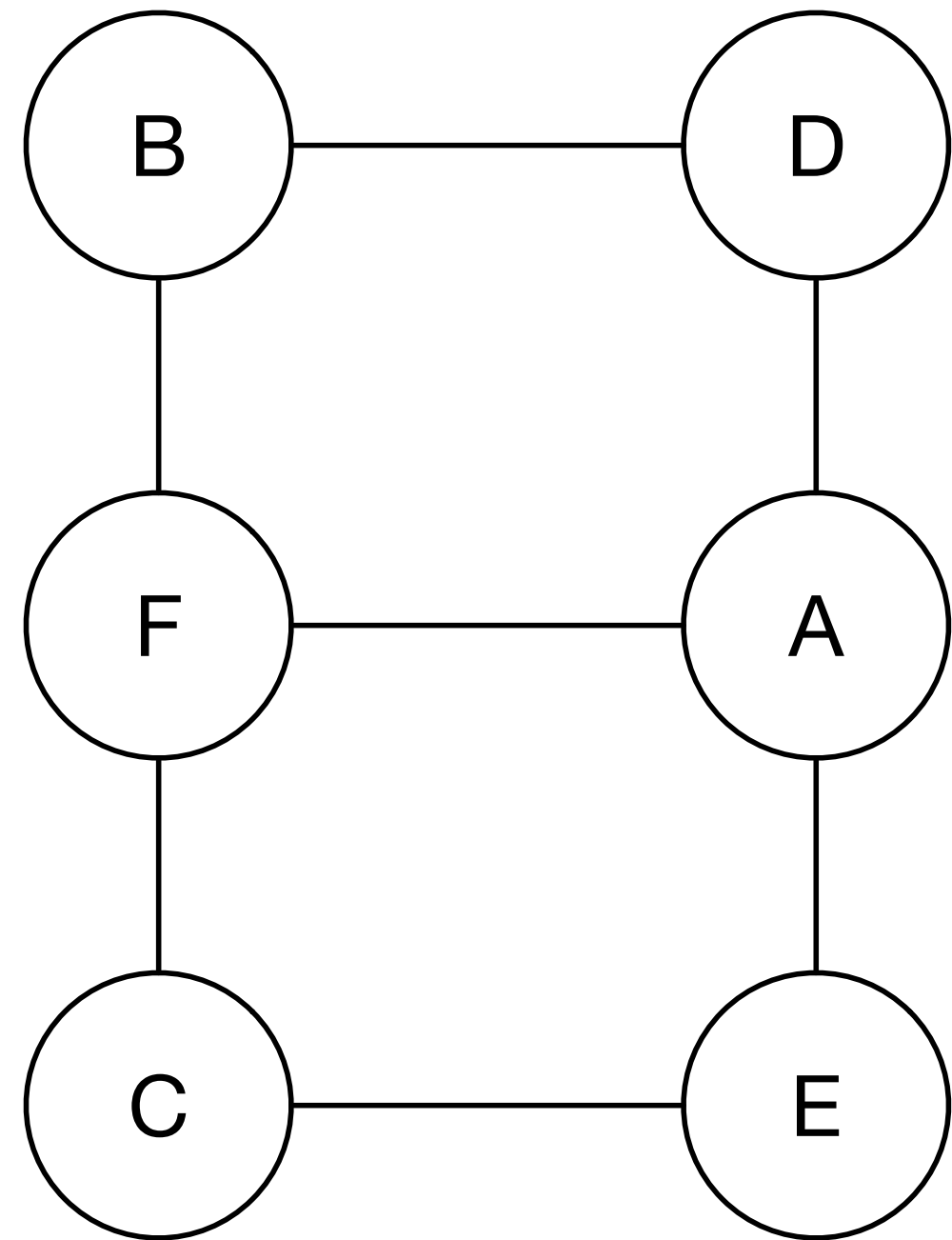


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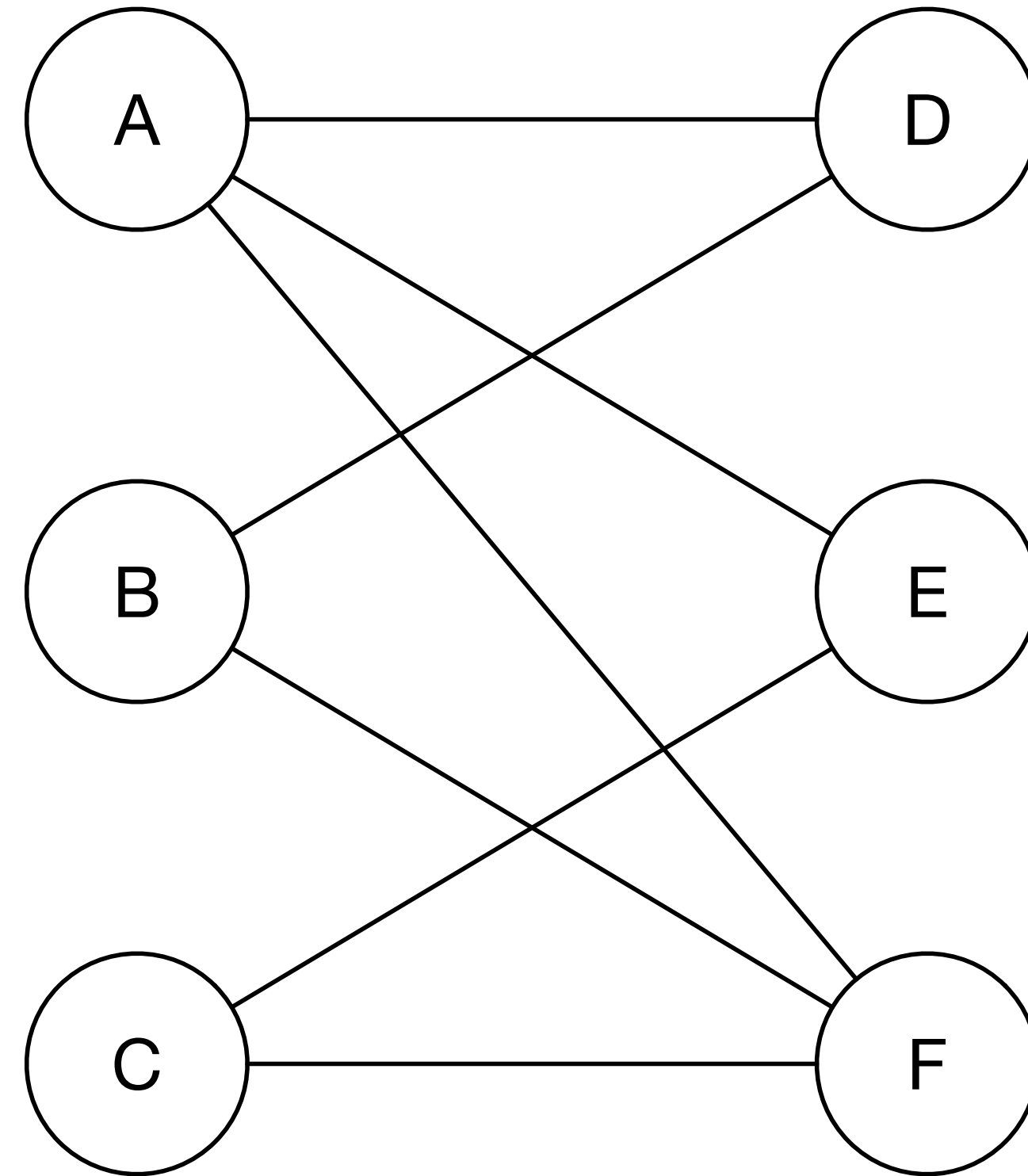
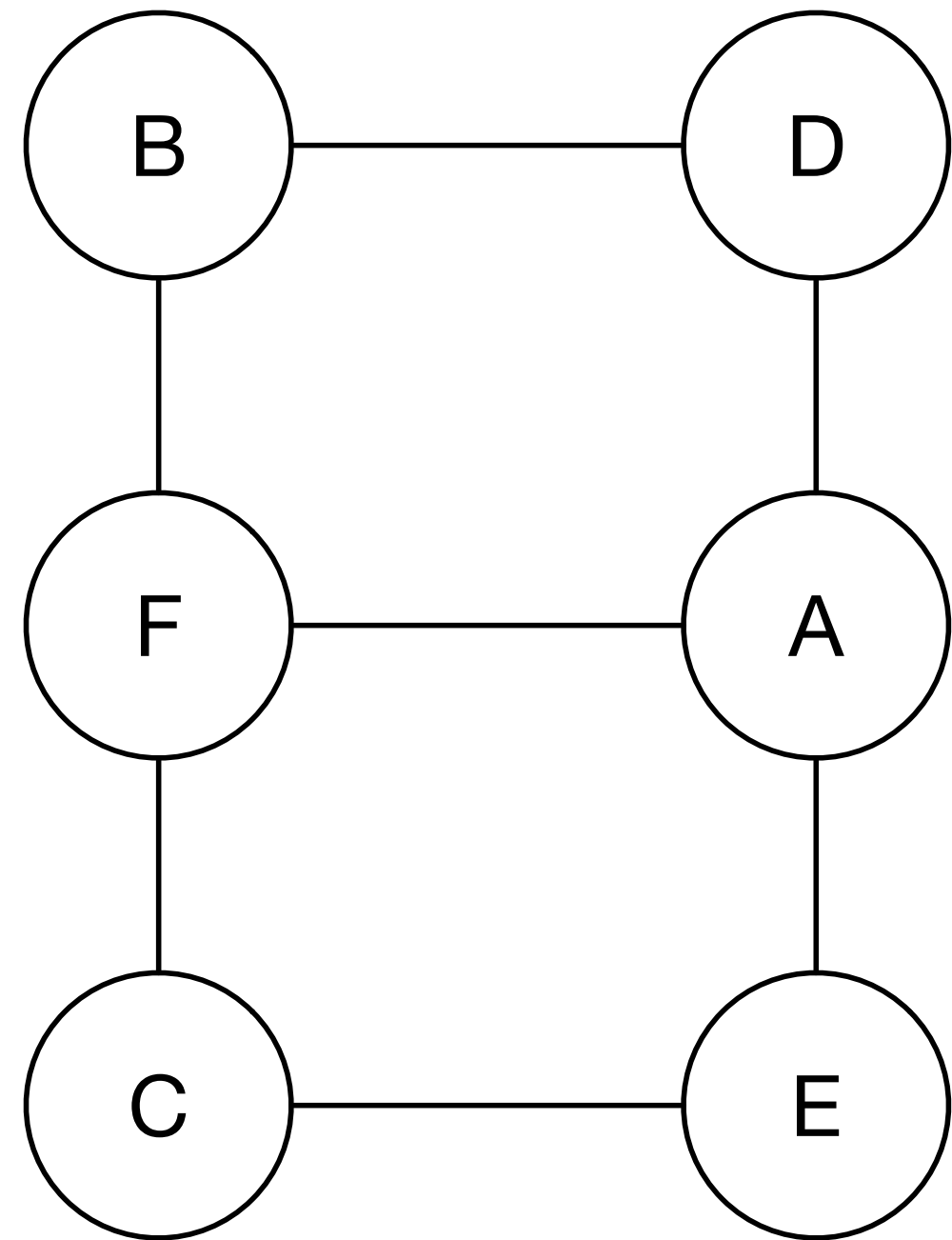


Definitely *not* planar!

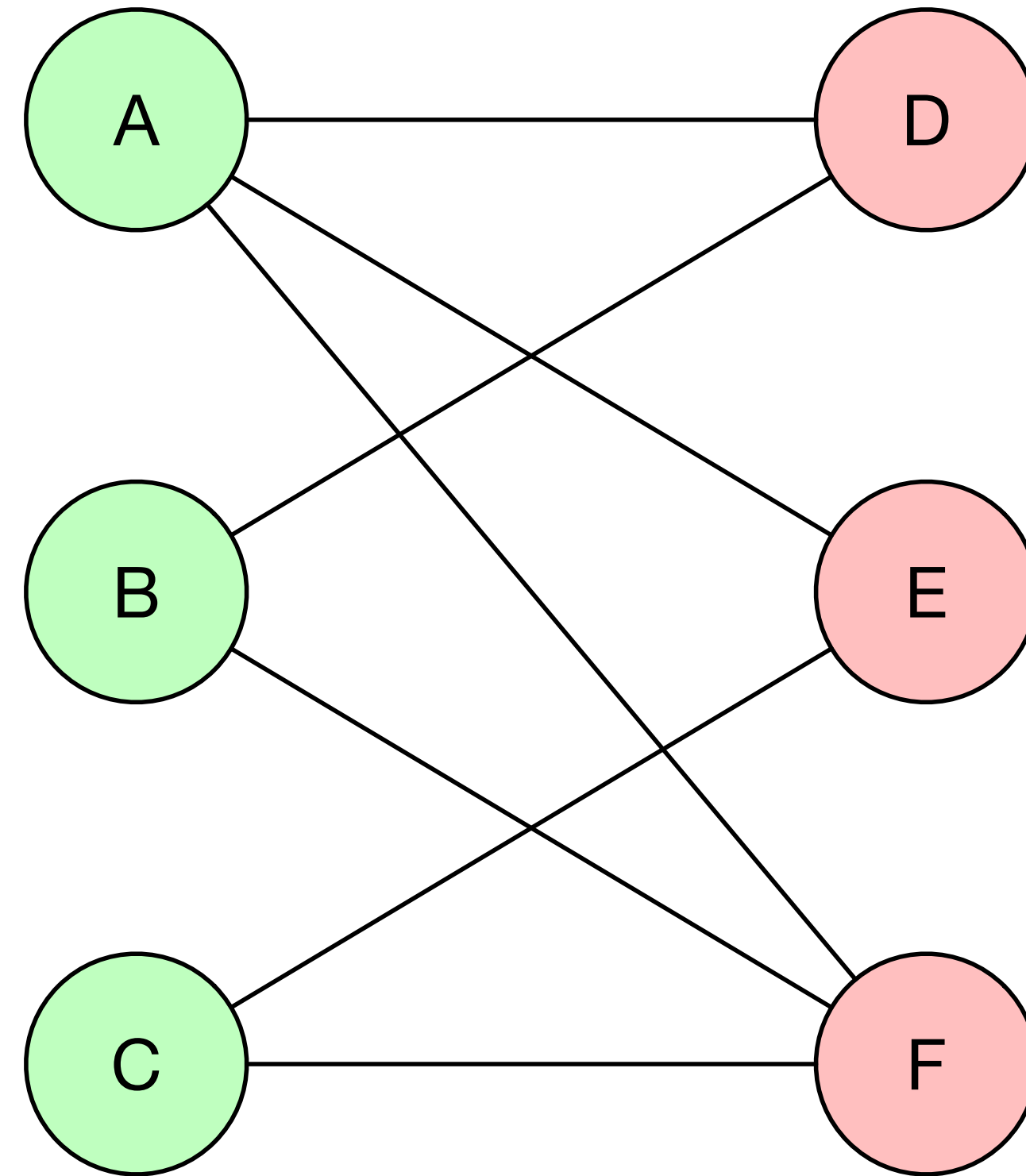
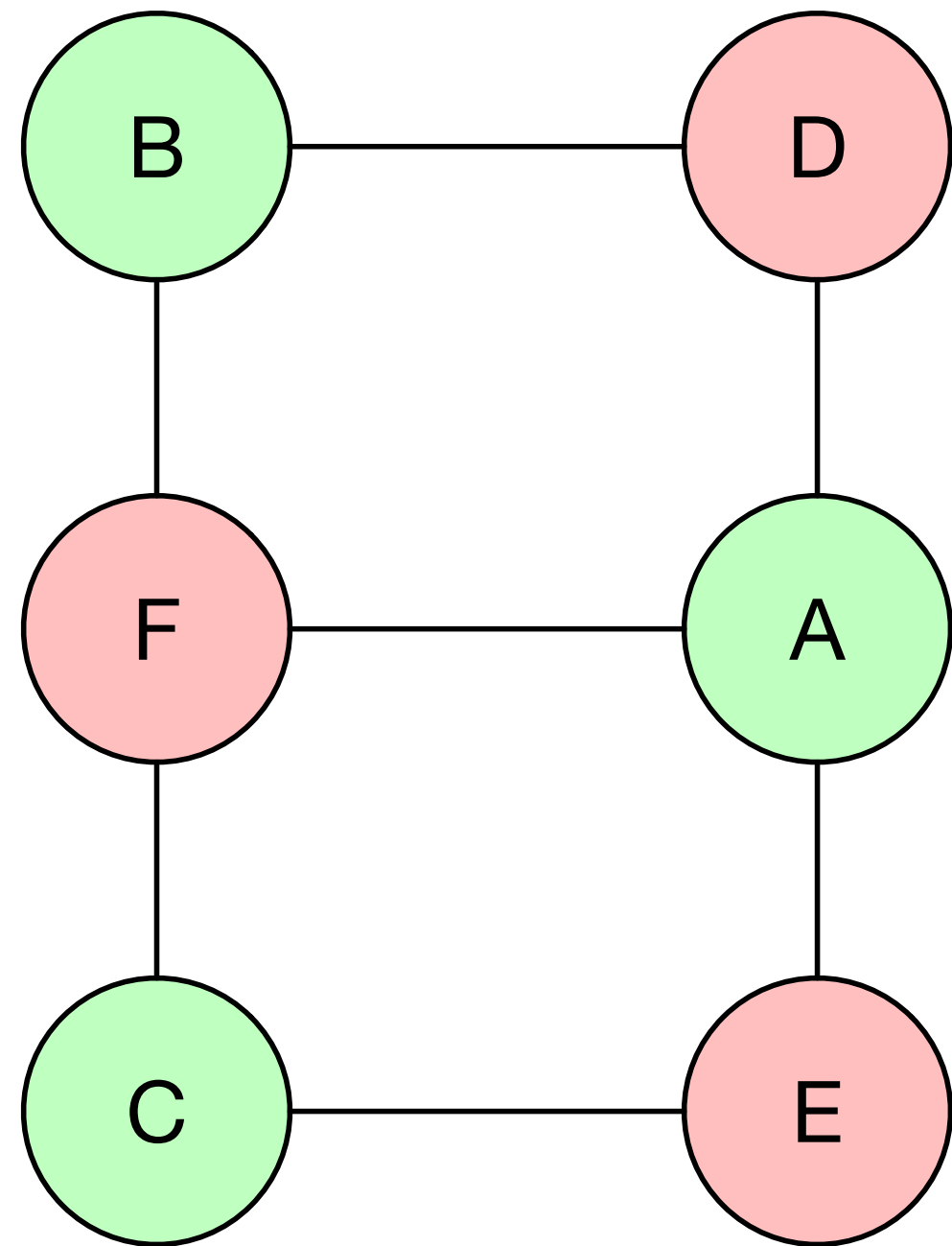
Introduction to graphs



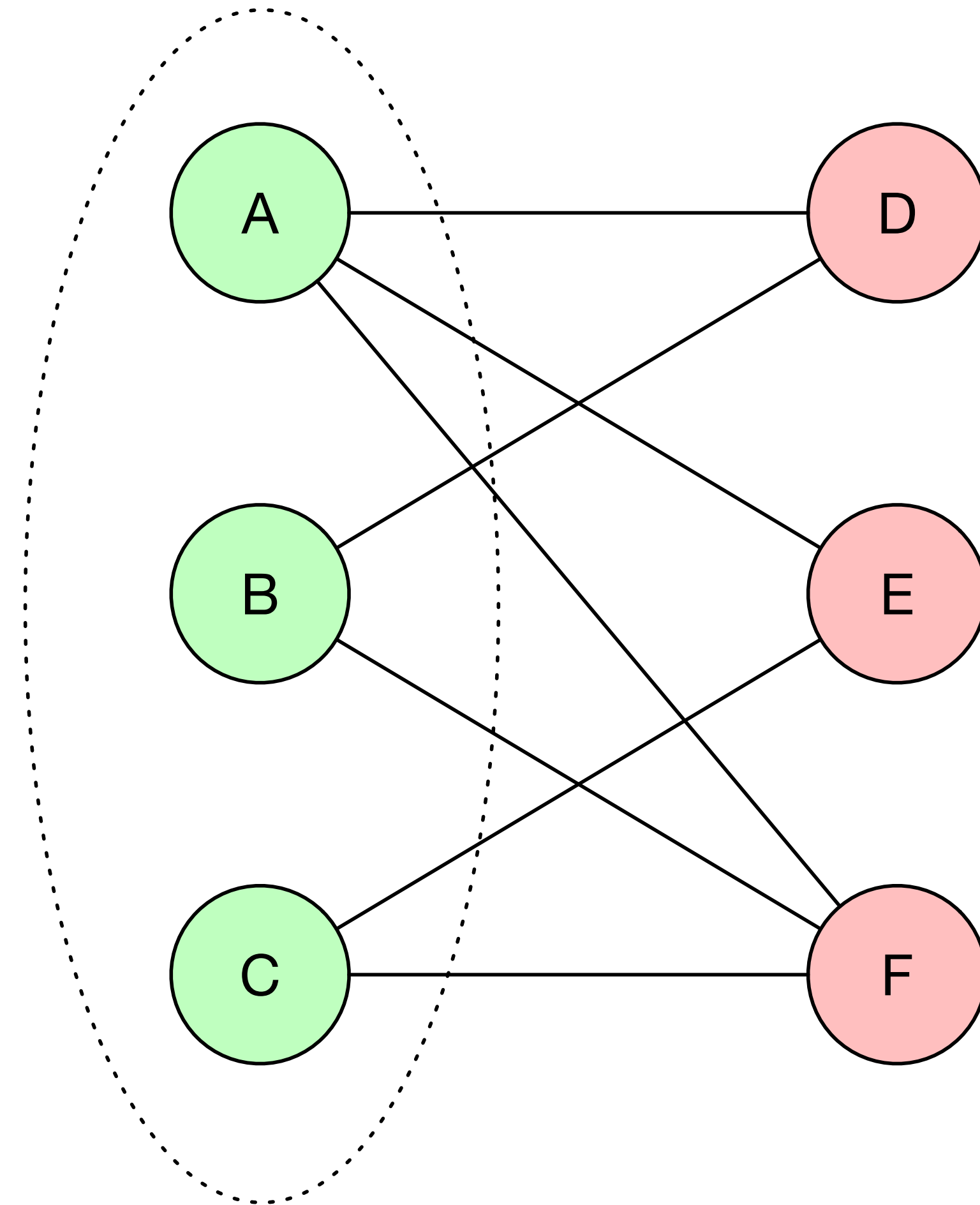
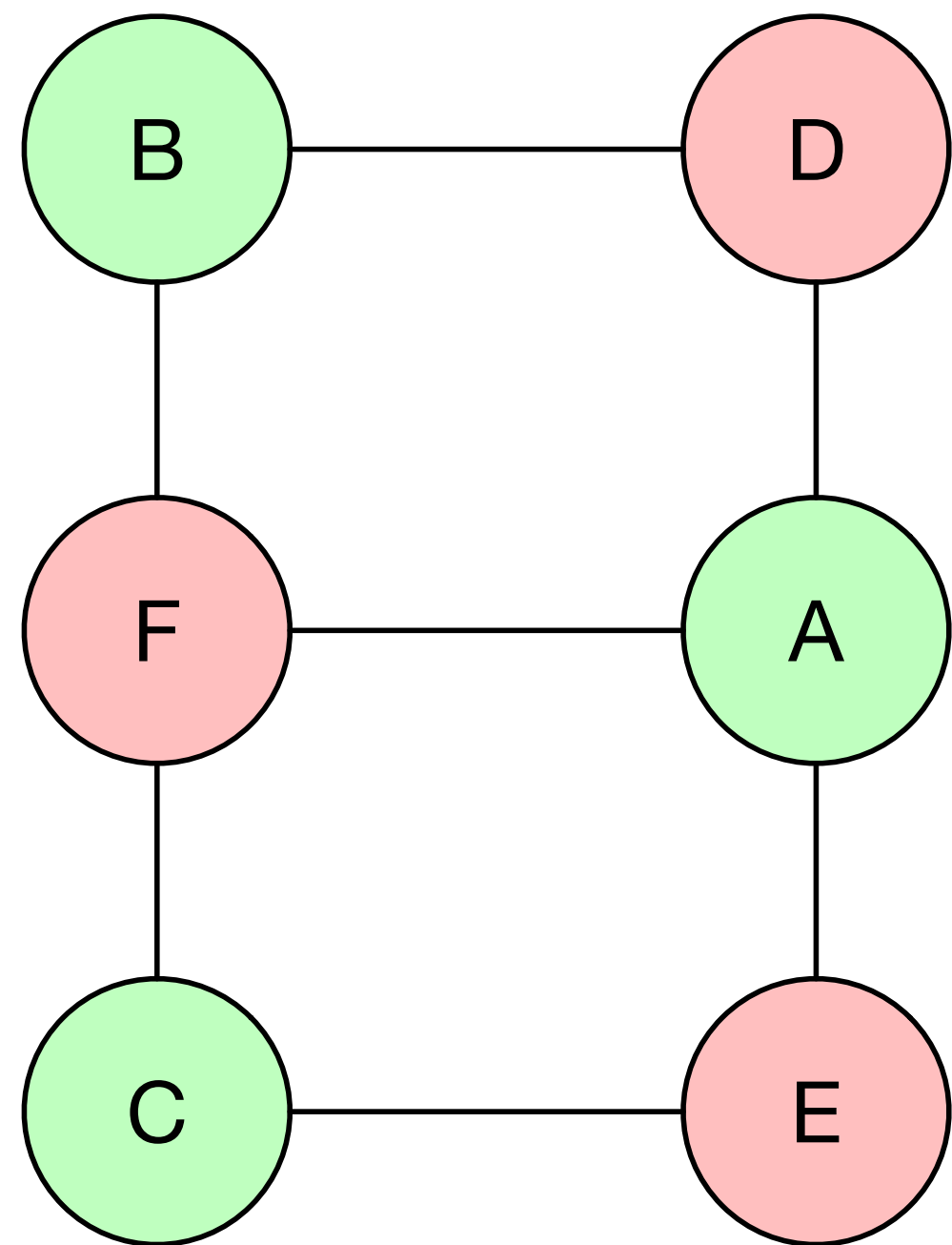
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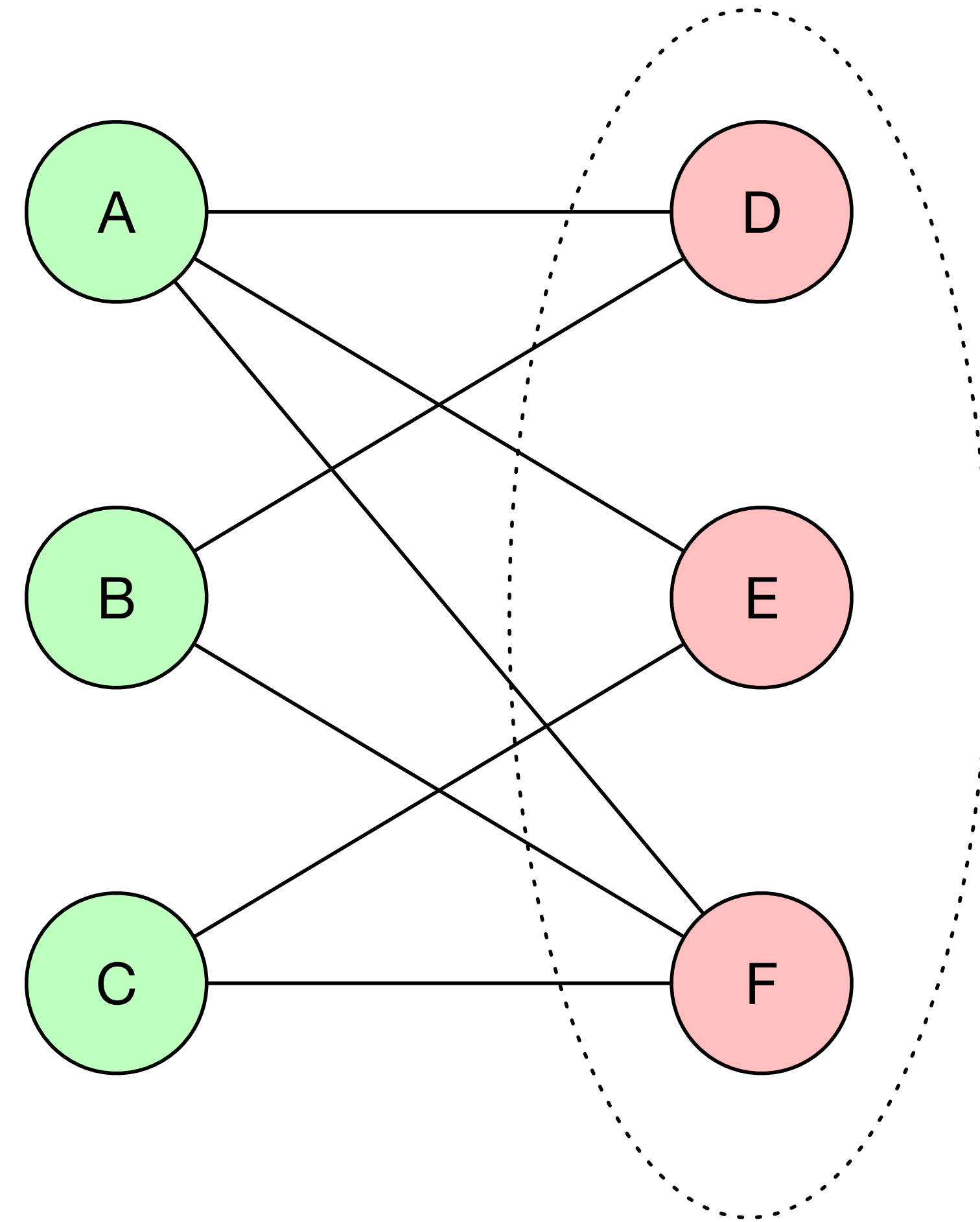
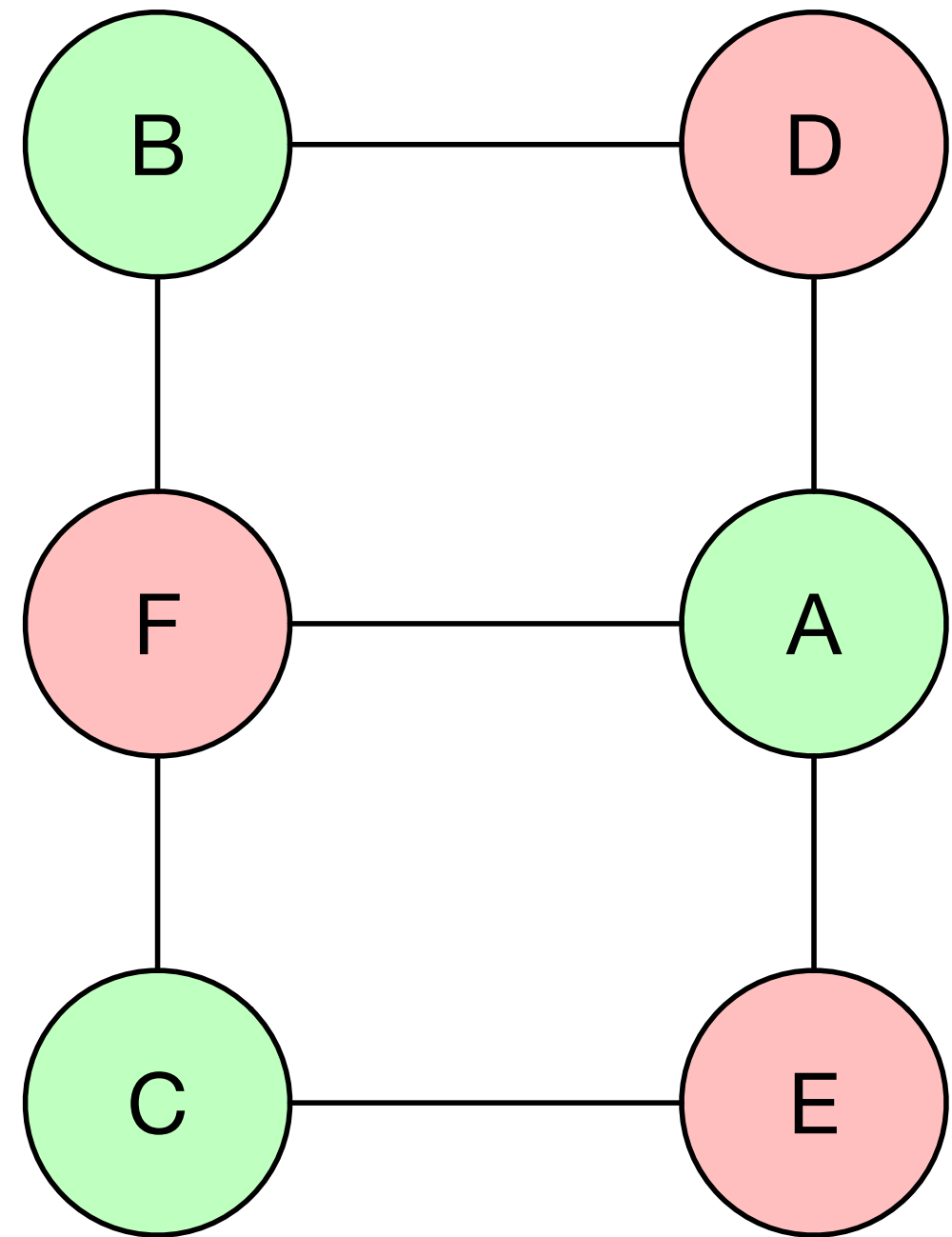
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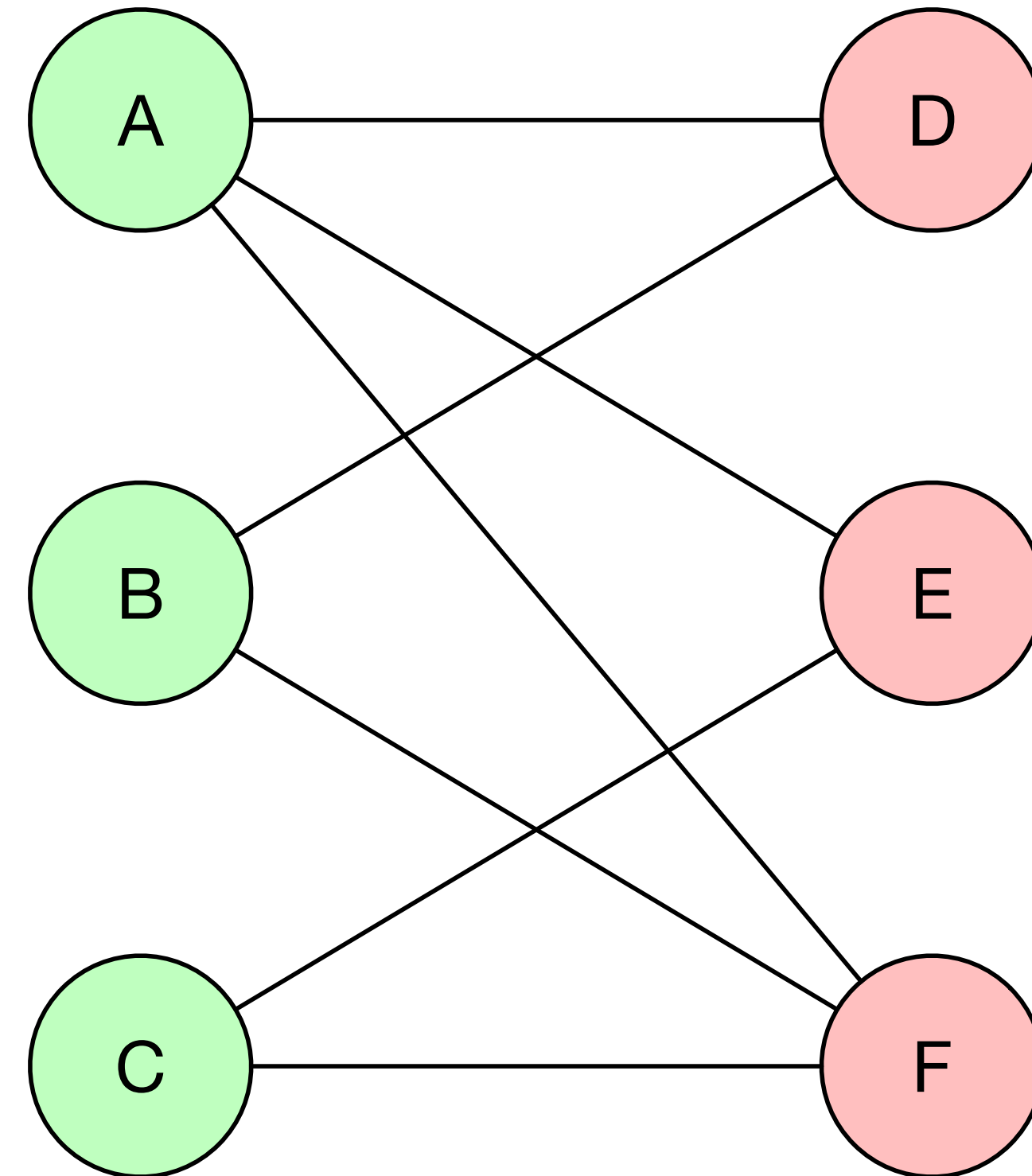
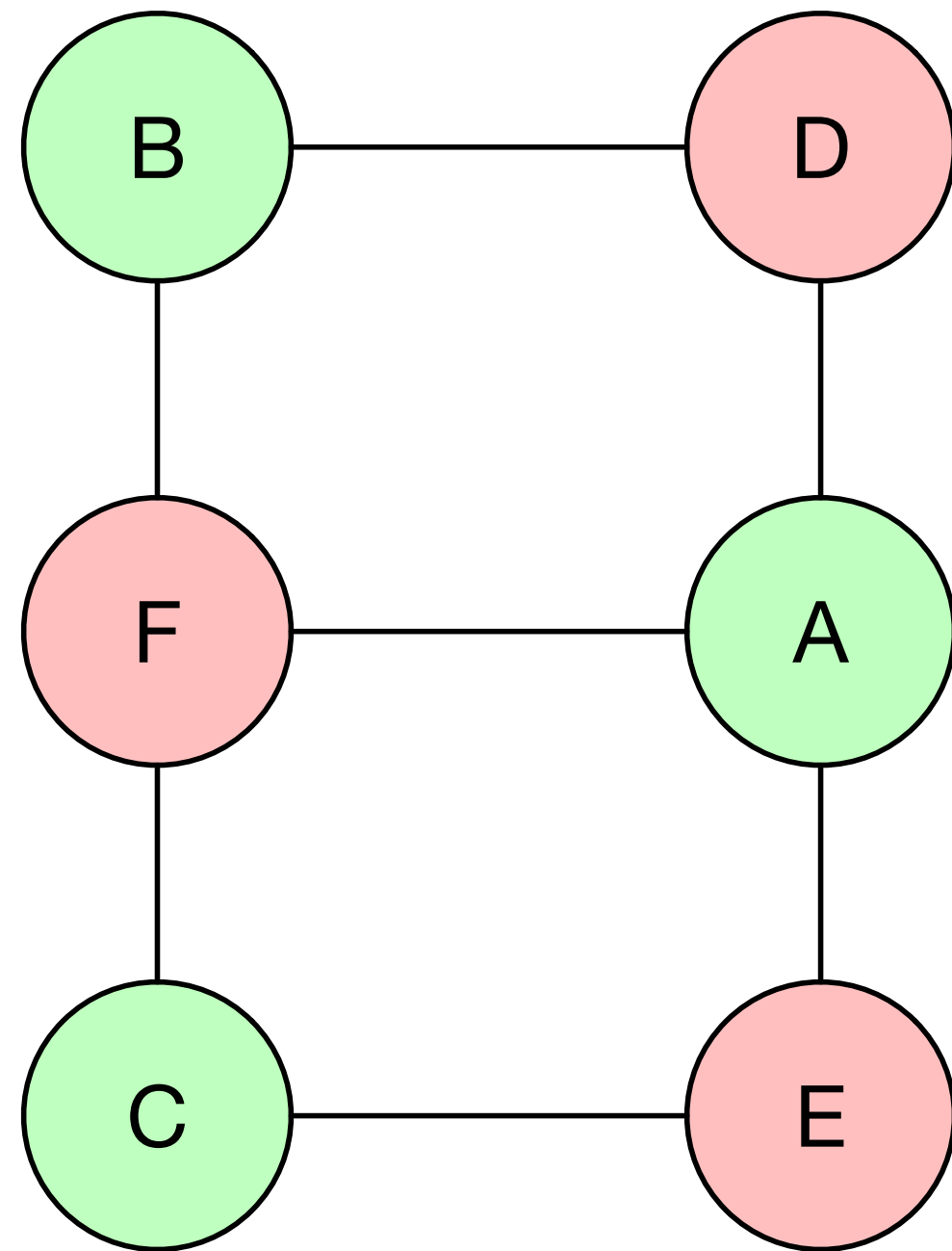


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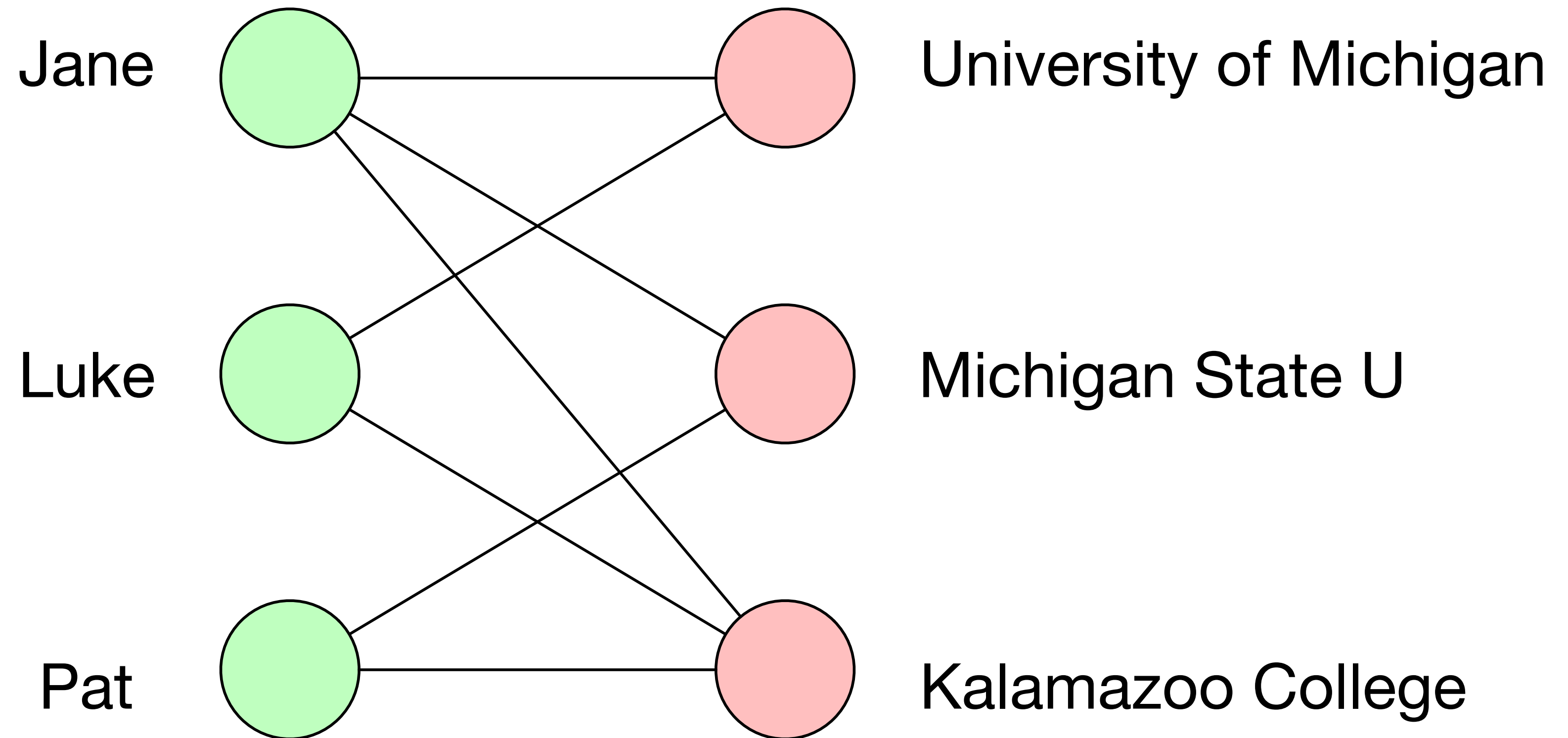


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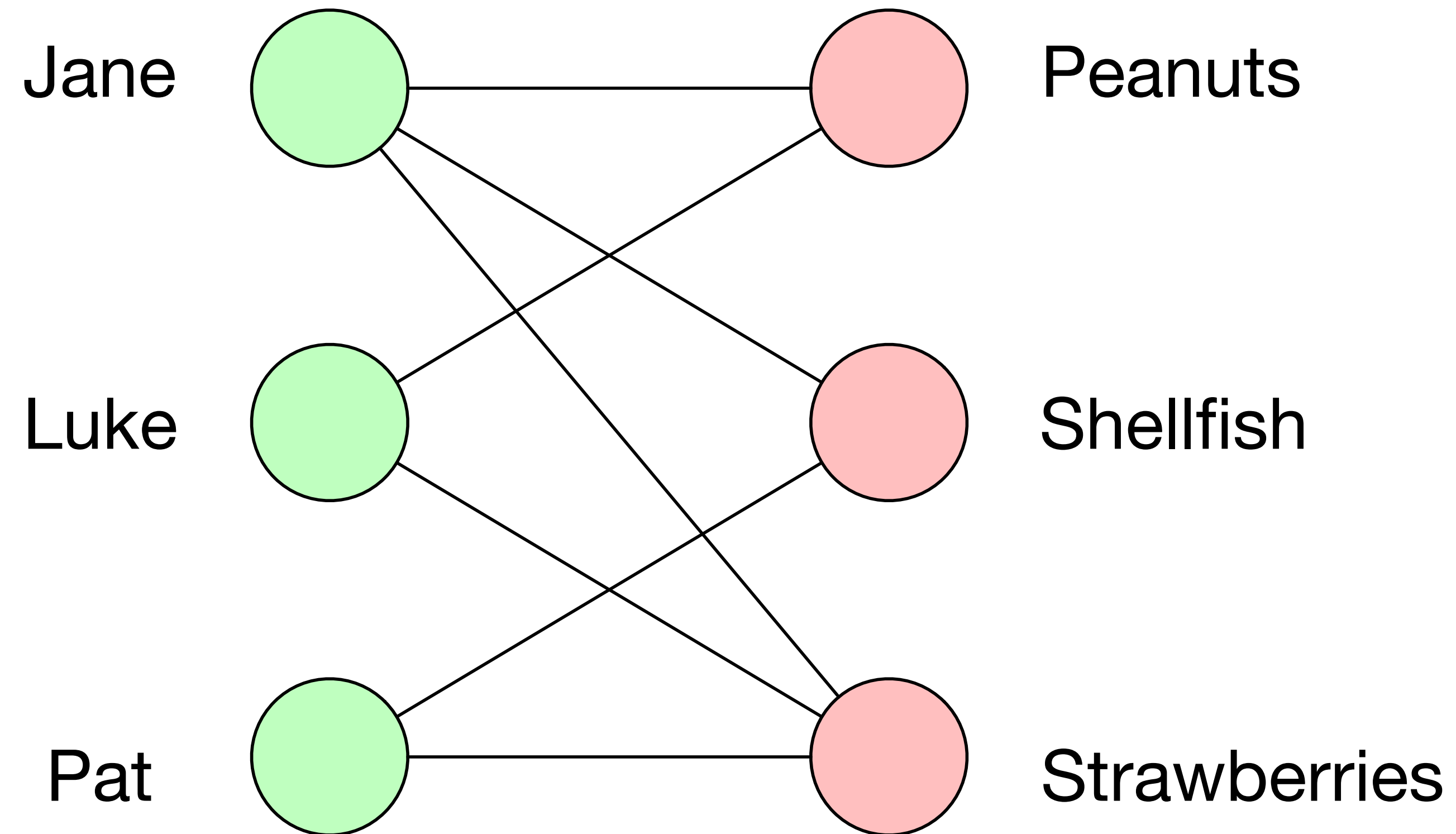
bipartite graph



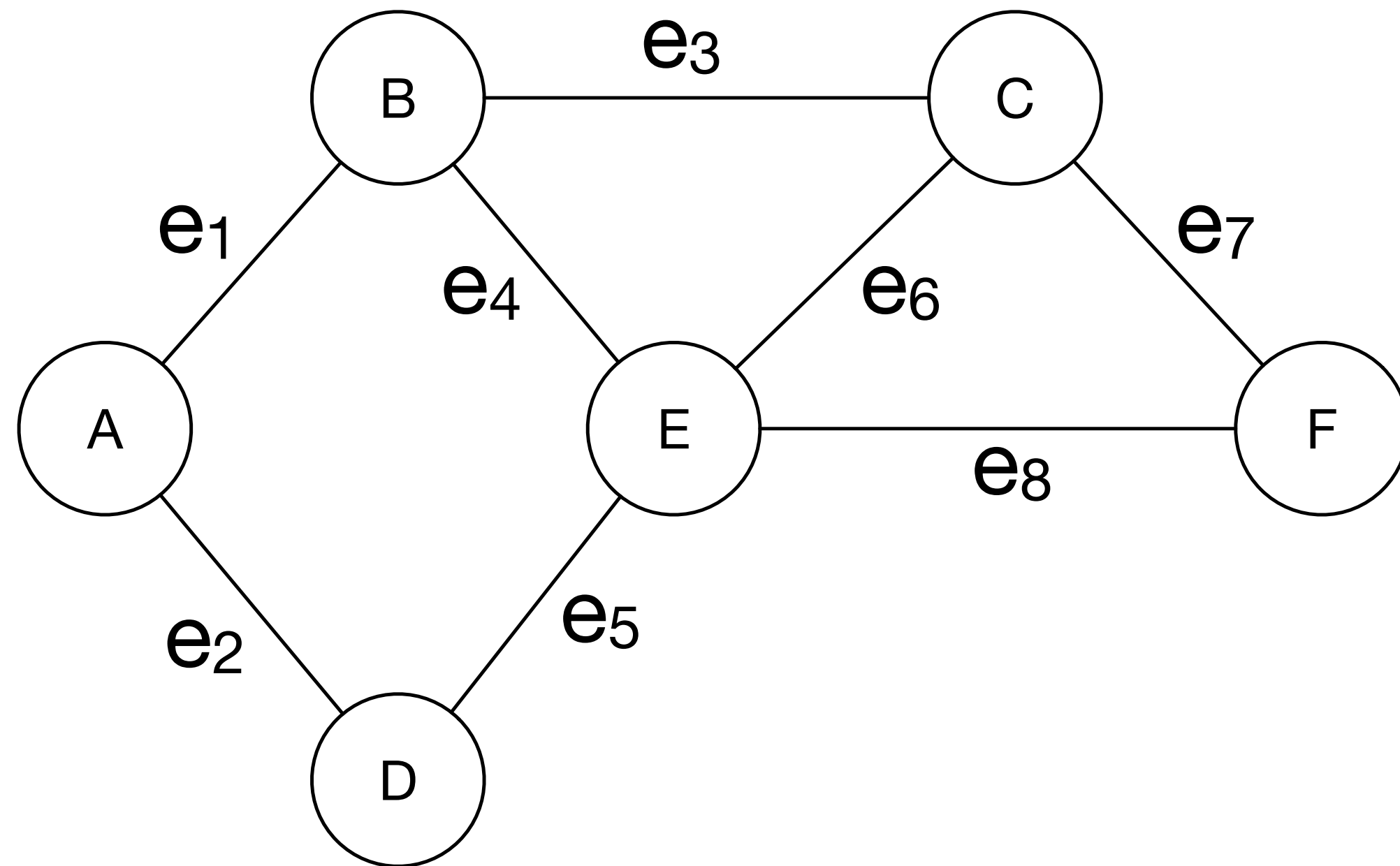
Introduction to graphs



Introduction to graphs



Introduction to graphs

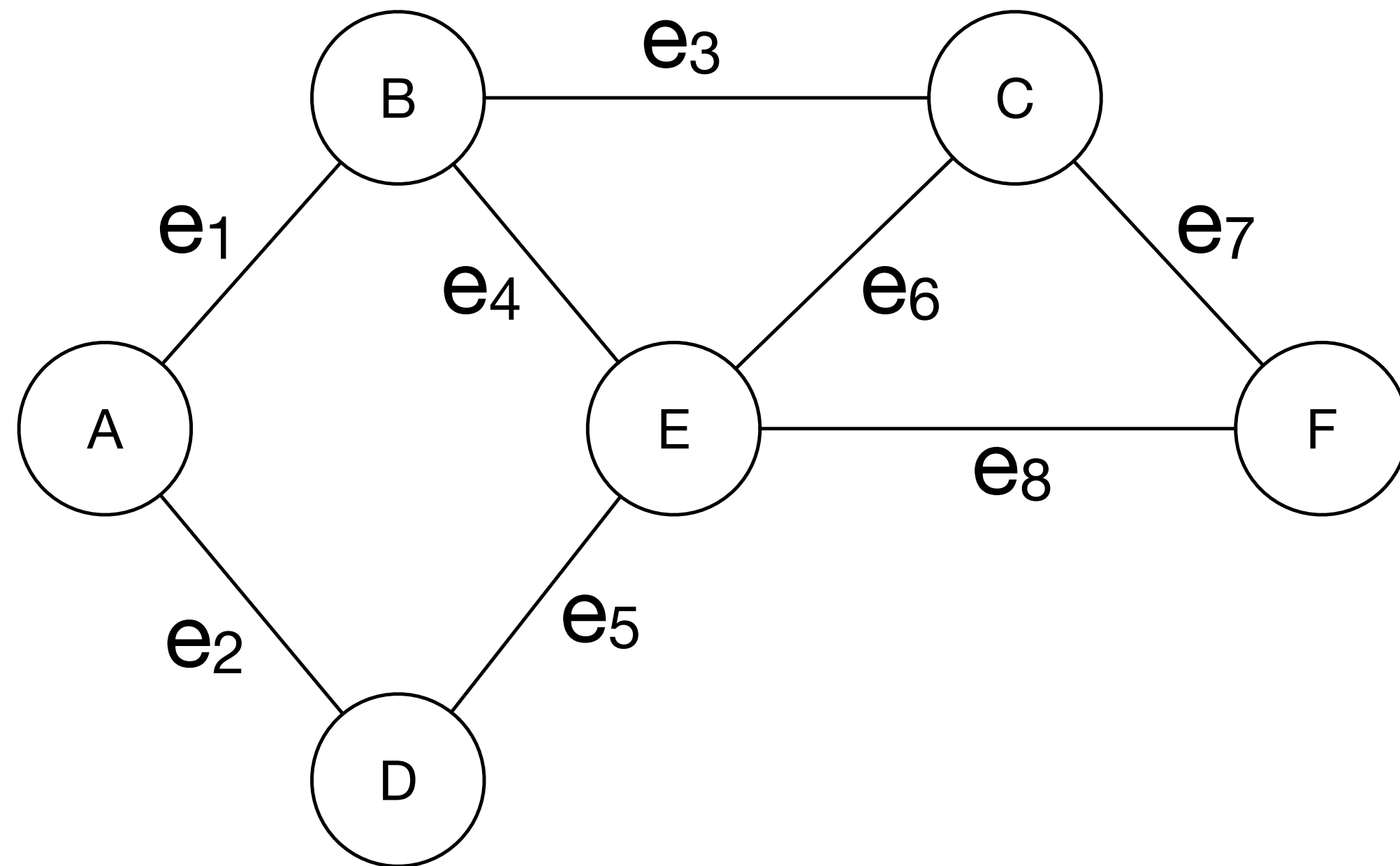


Incidence matrix

	1	2	3	4	5	6	7	8
A								
B								
C								
D								
E								
F								

$$M_{v,e} = \begin{cases} 1 & \text{if node } v \text{ is an endpoint of edge } e \\ 0 & \text{otherwise} \end{cases}$$

Introduction to graphs

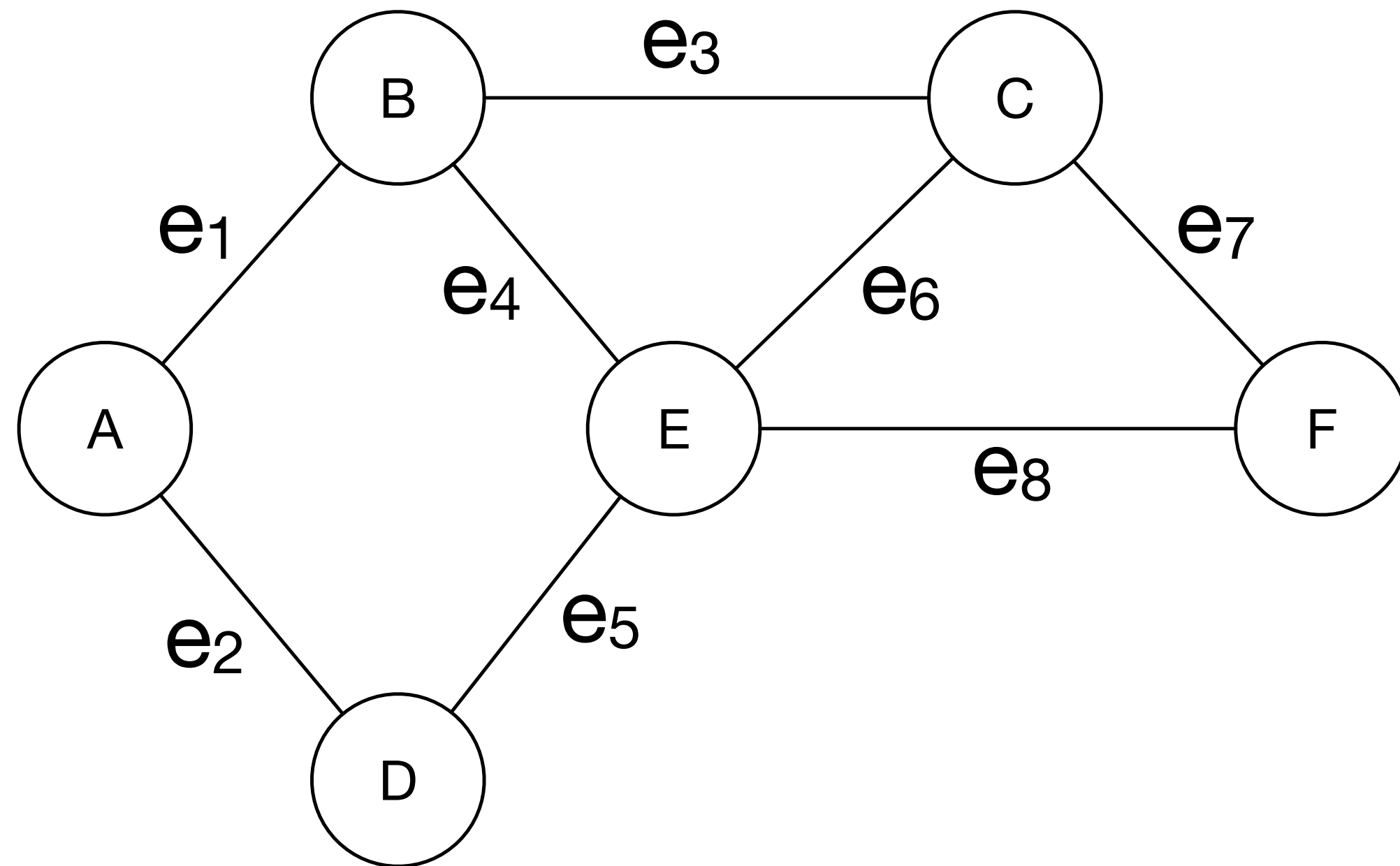


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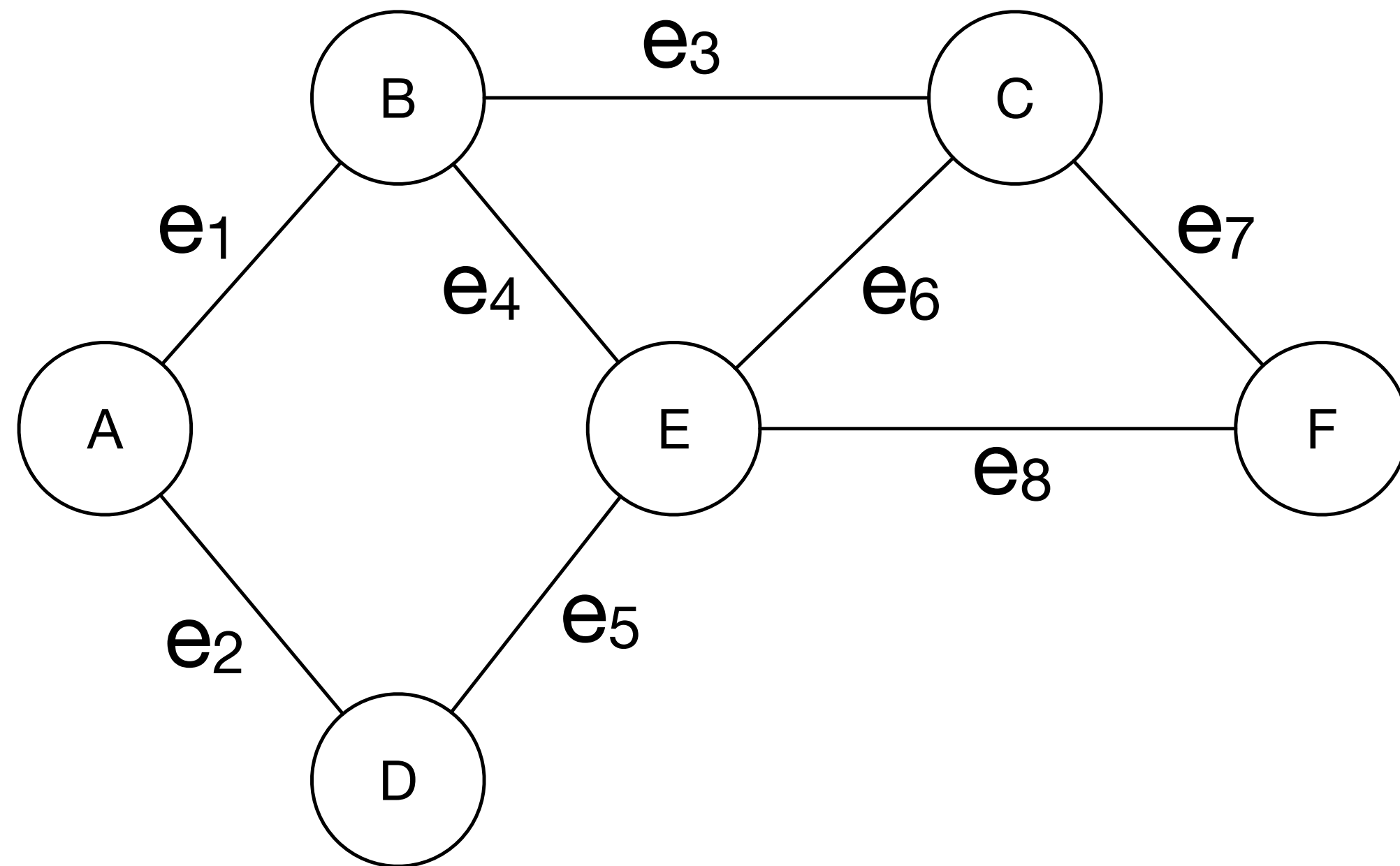


Incidence matrix

	1	2	3	4	5	6	7	8
A	1							
B	1							
C	0							
D	0							
E	0							
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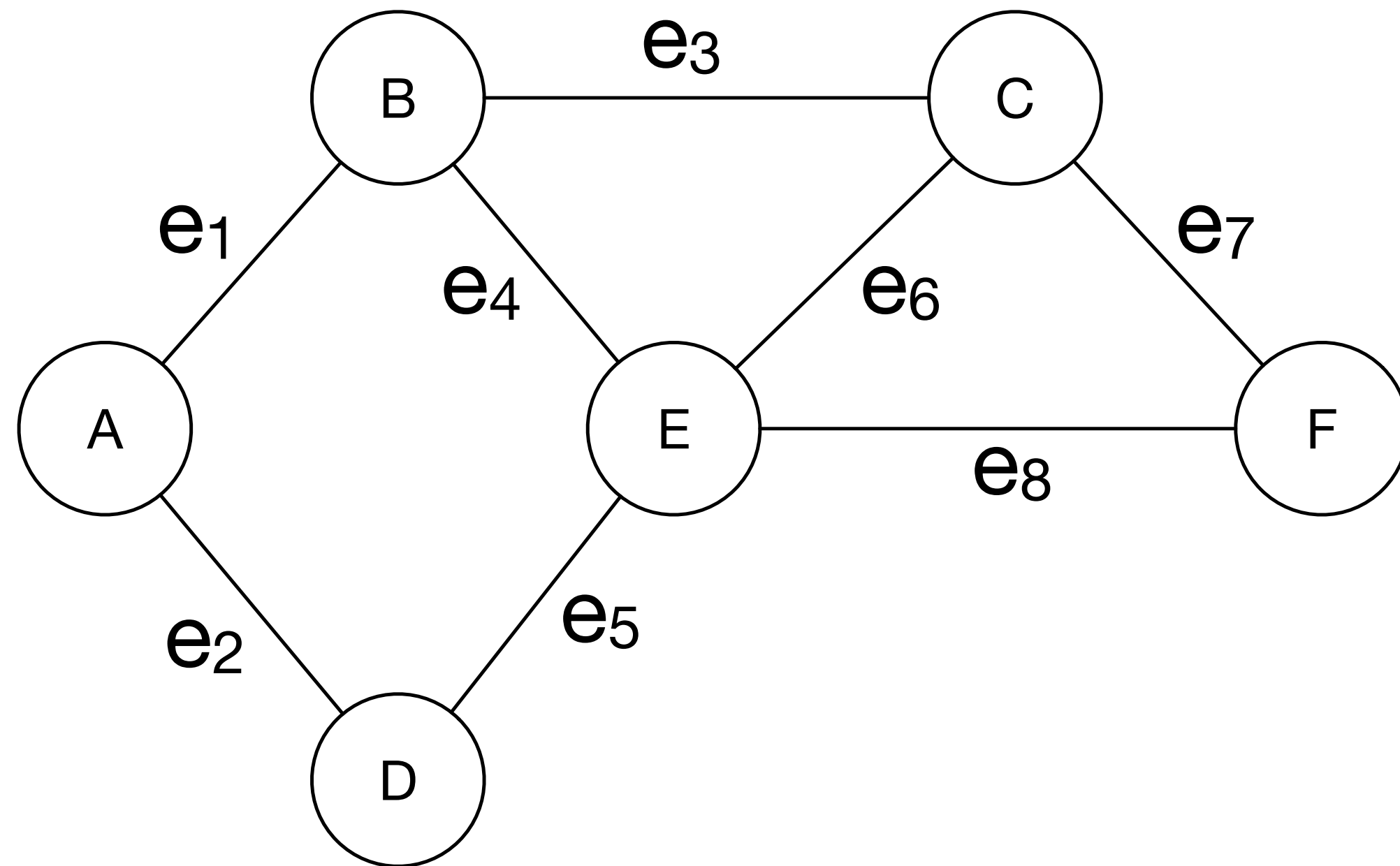


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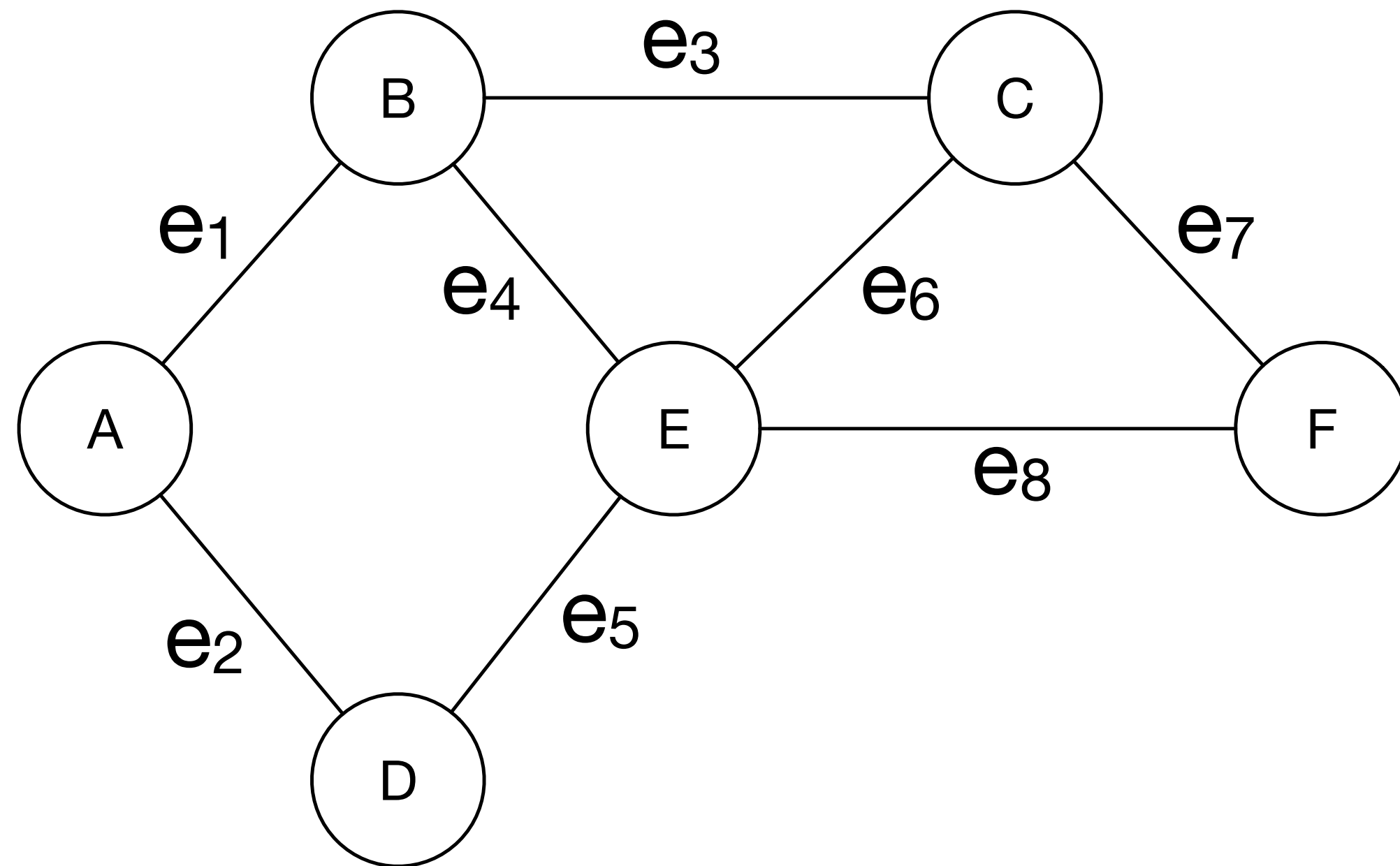


Incidence matrix

	1	2	3	4	5	6	7	8
A	1	1	0					
B	1	0	1					
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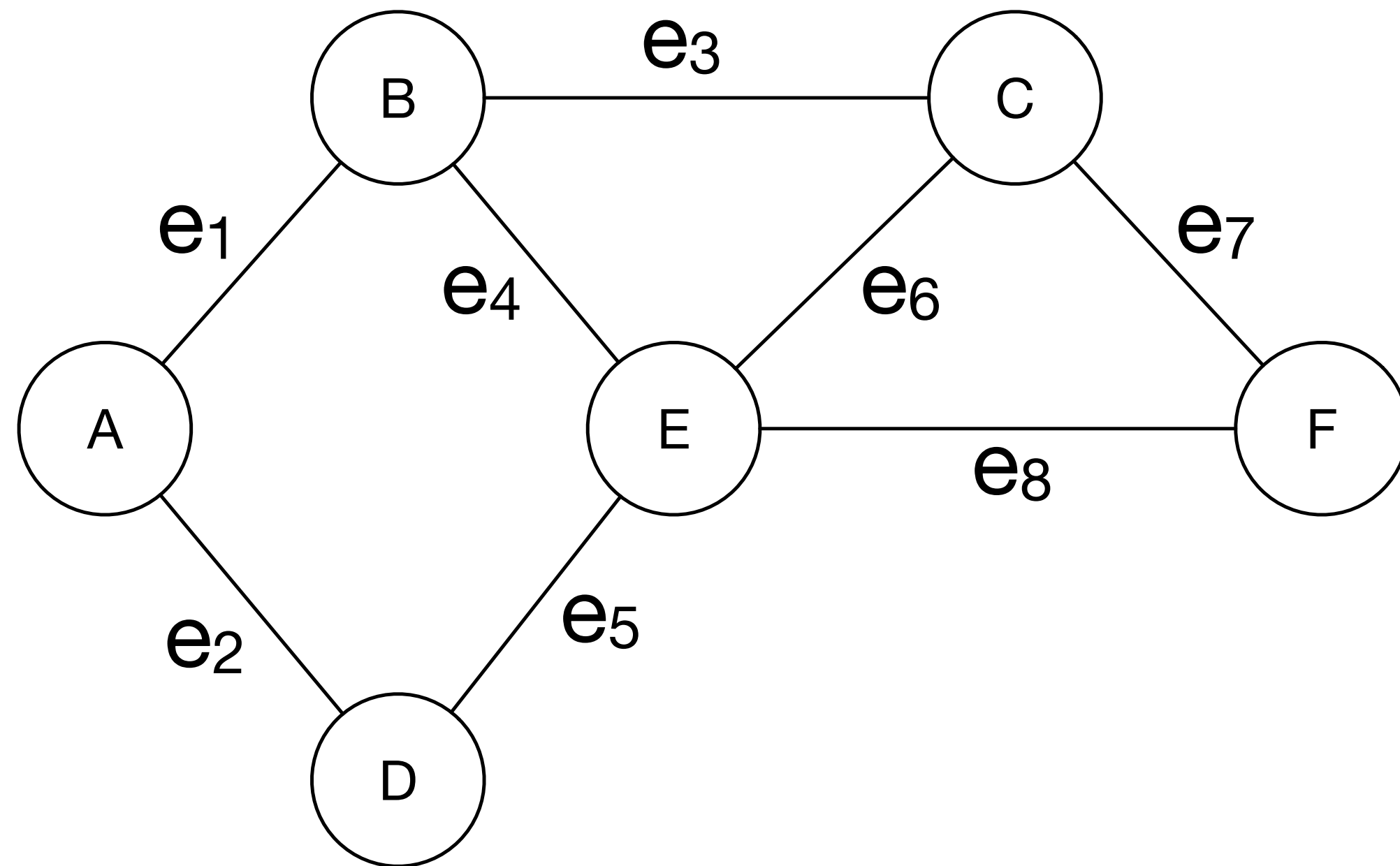


Incidence matrix

	1	2	3	4	5	6	7	8
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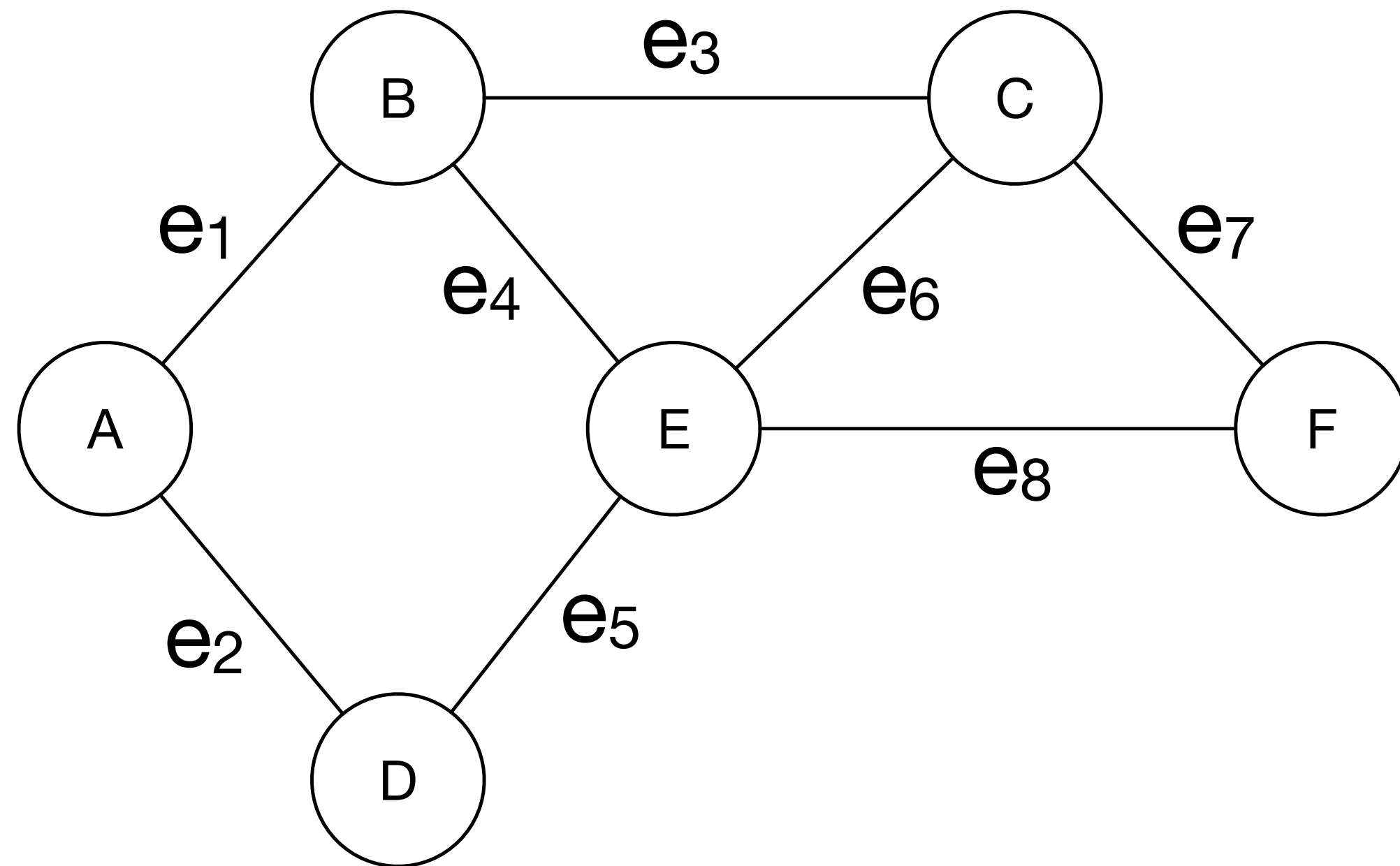


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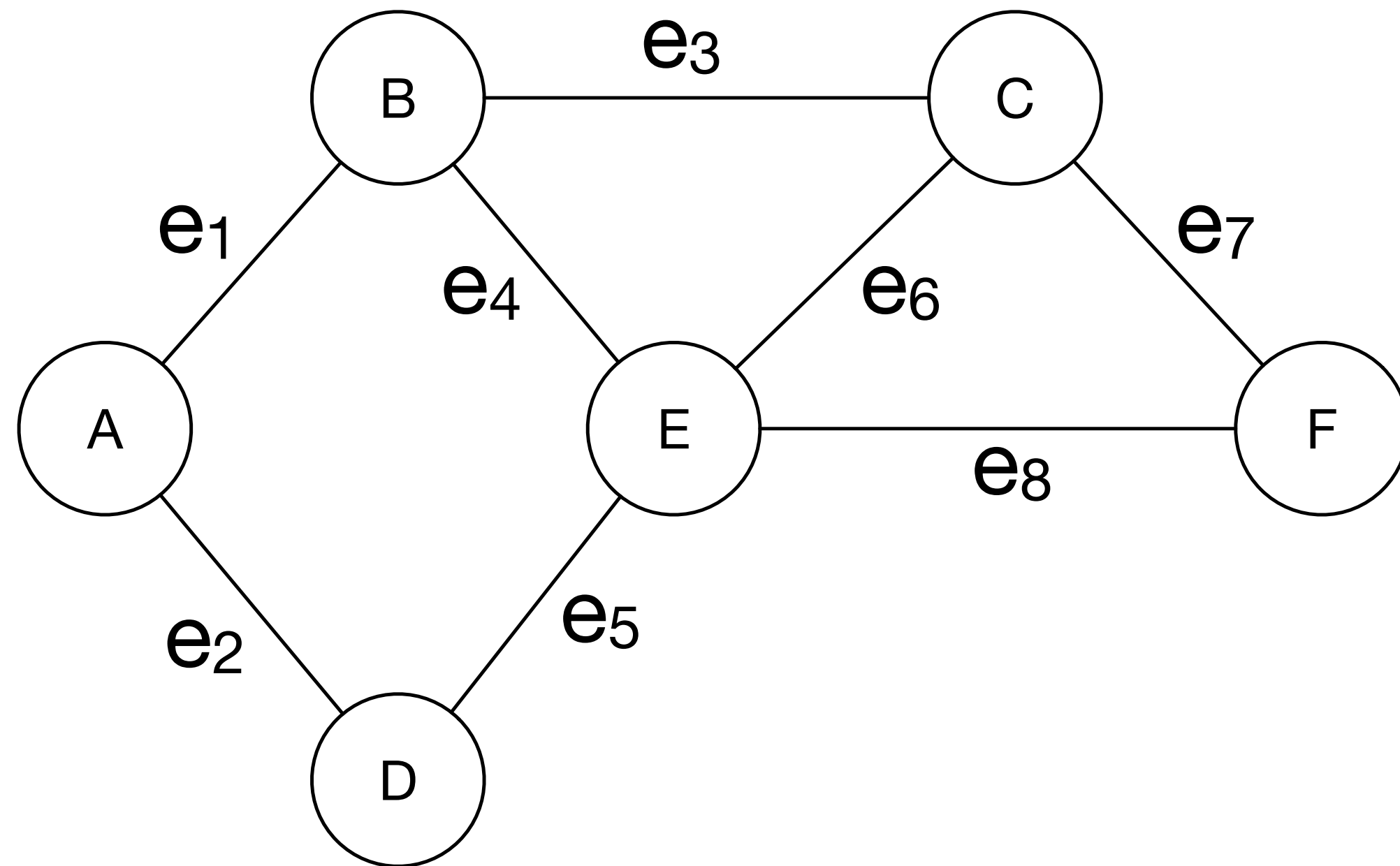


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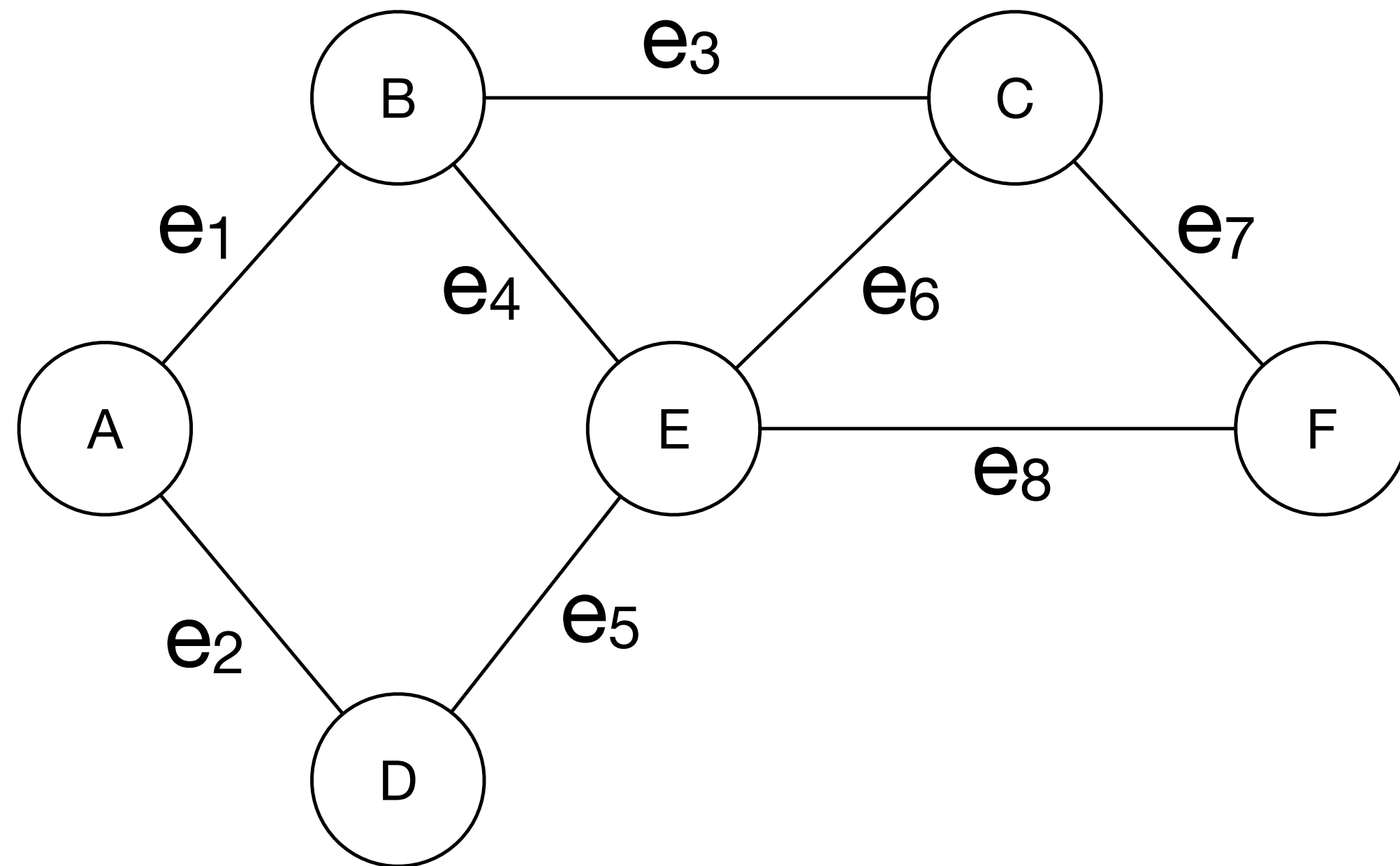


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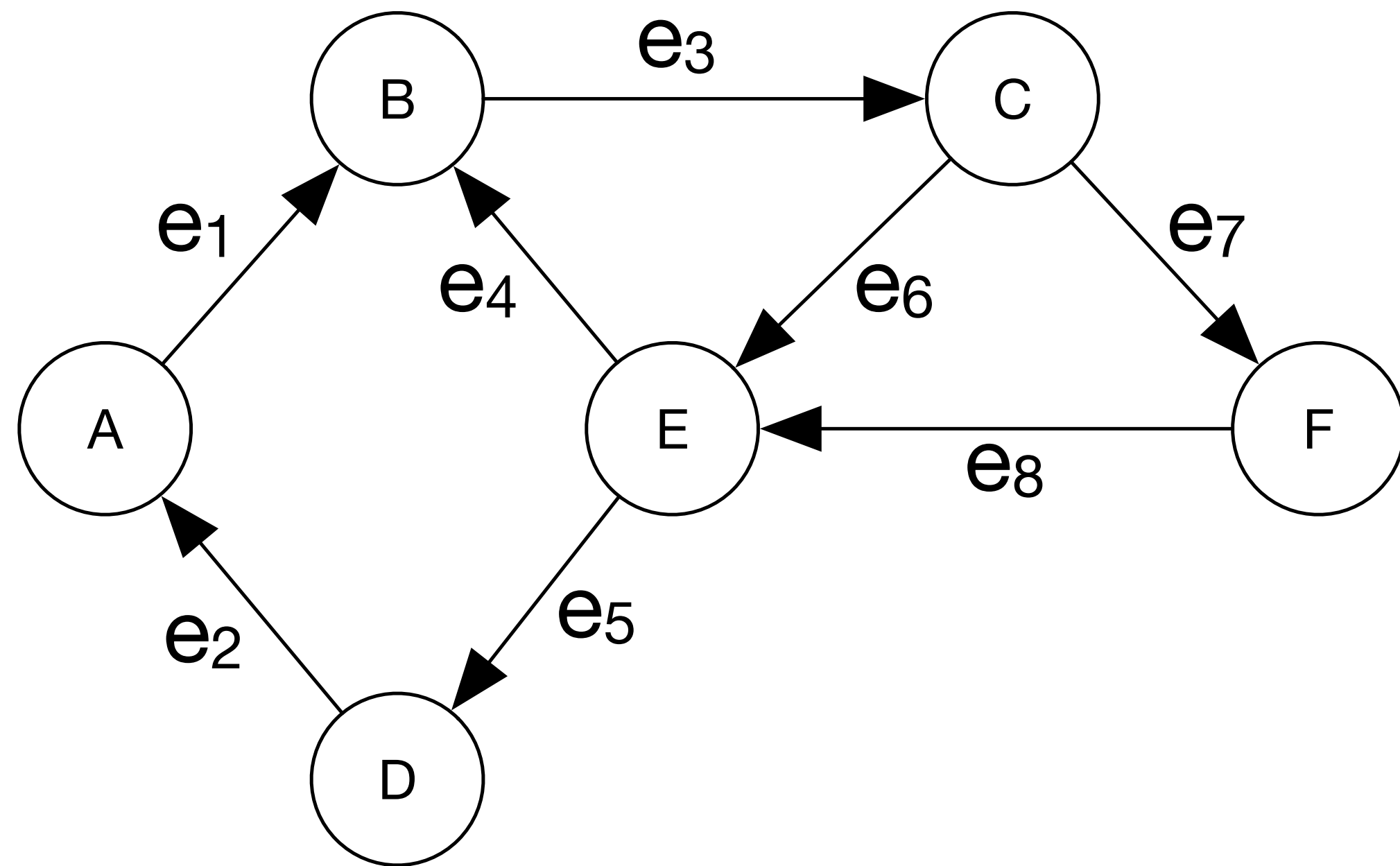


Incidence matrix

	1	2	3	4	5	6	7	8
A	1	1	0	0	0	0	0	0
B	1	0	1	1	0	0	0	0
C	0	0	1	0	0	1	1	0
D	0	1	0	0	1	0	0	0
E	0	0	0	1	1	1	0	1
F	0	0	0	0	0	0	1	1

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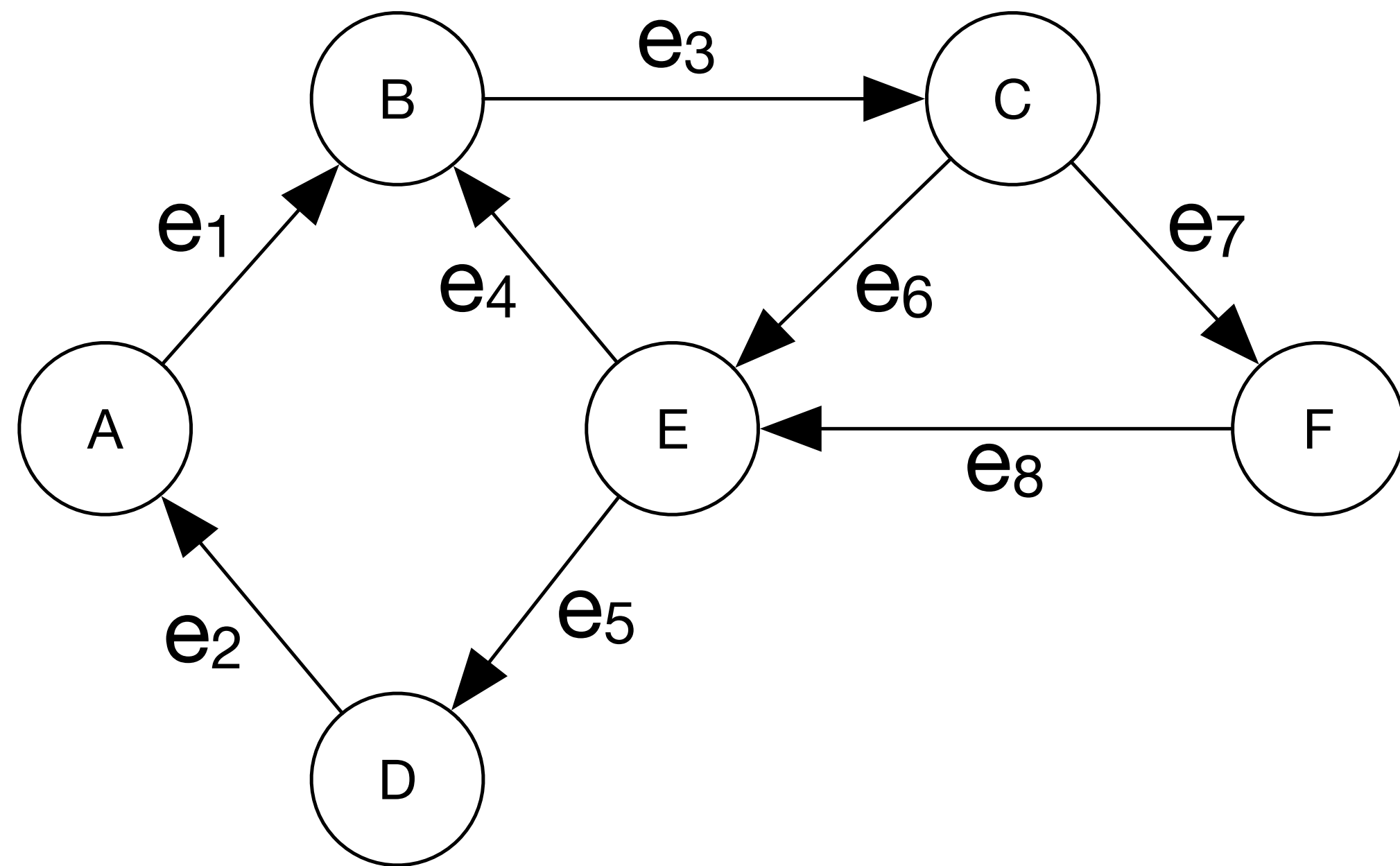
Introduction to graphs



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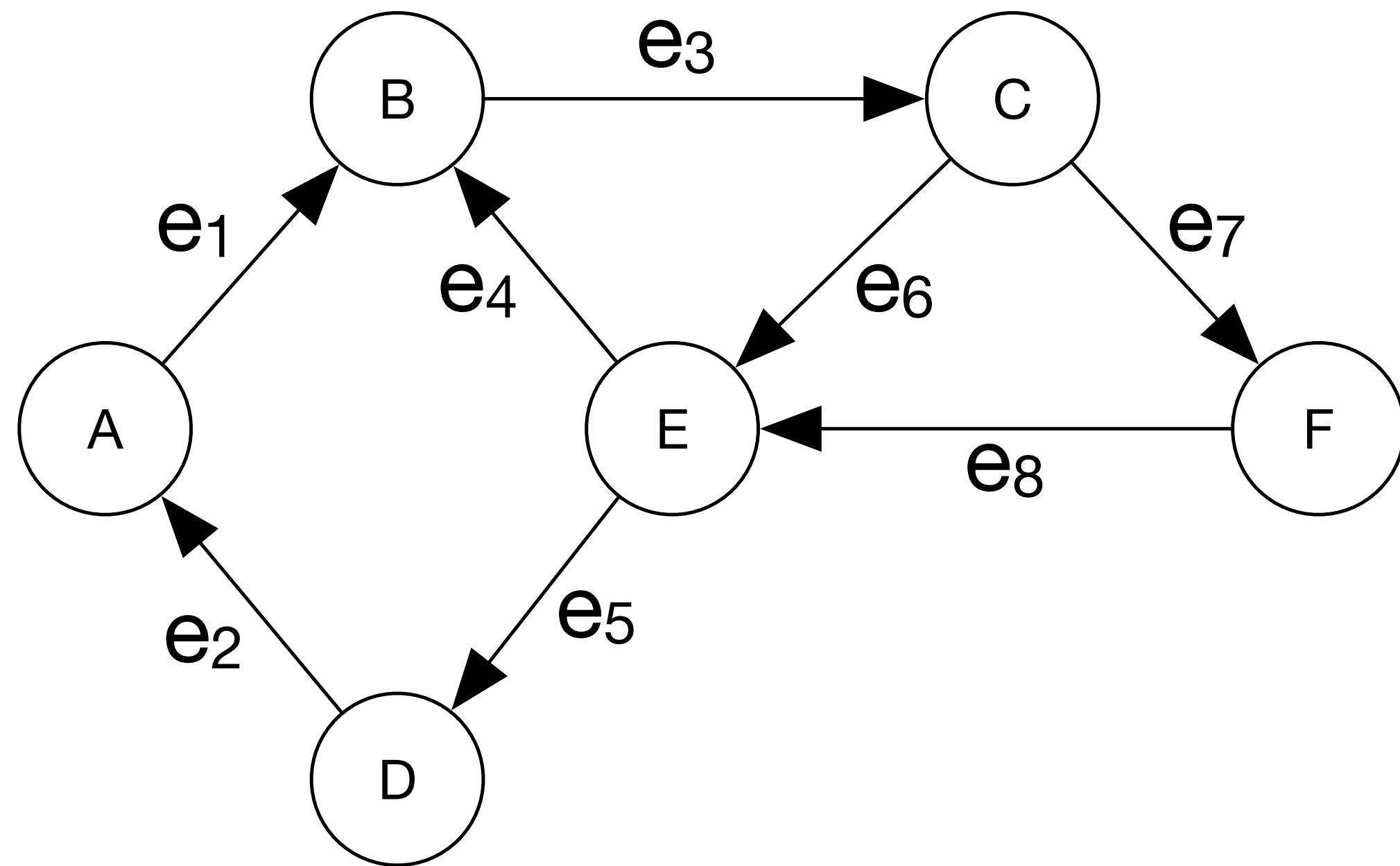


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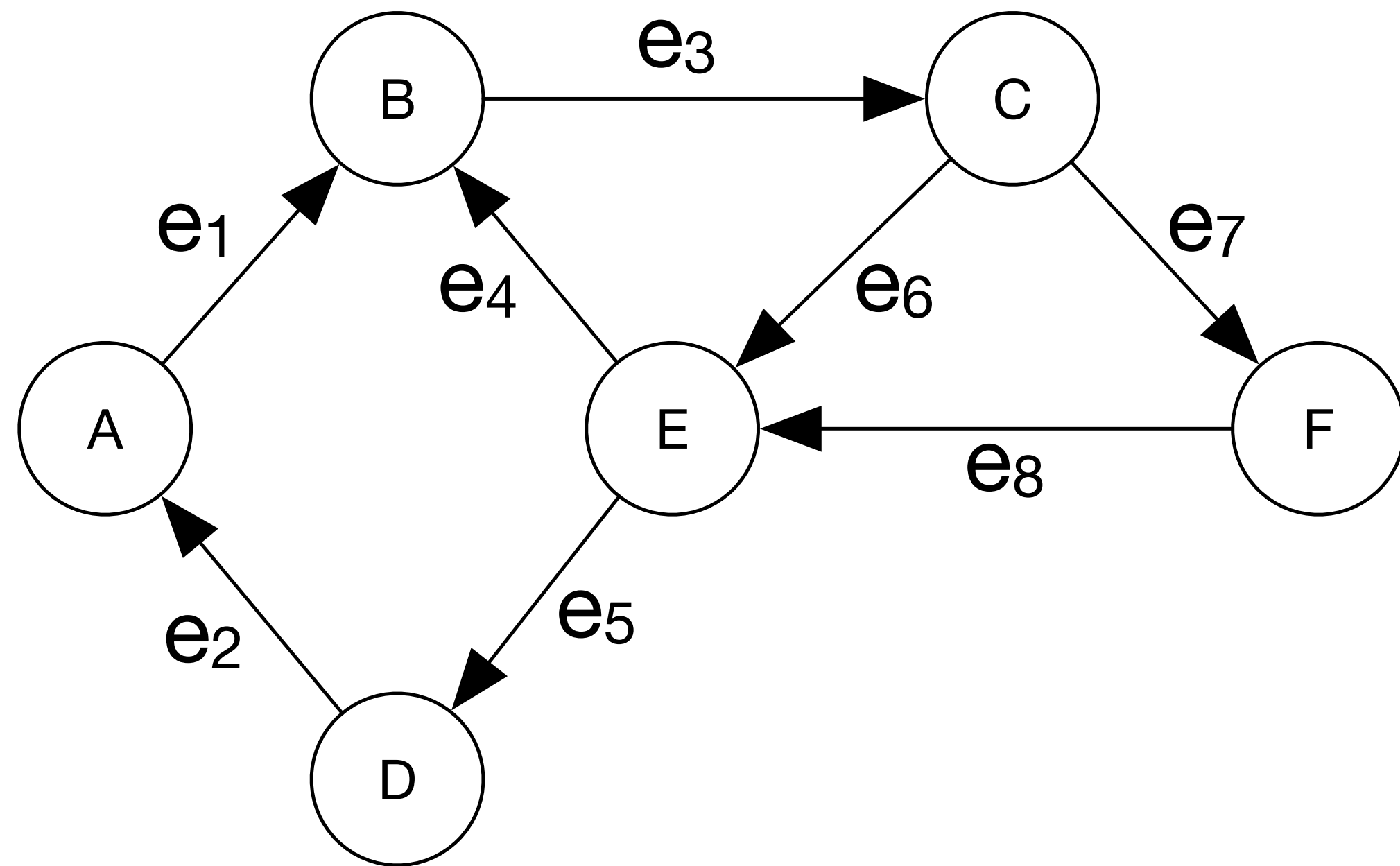


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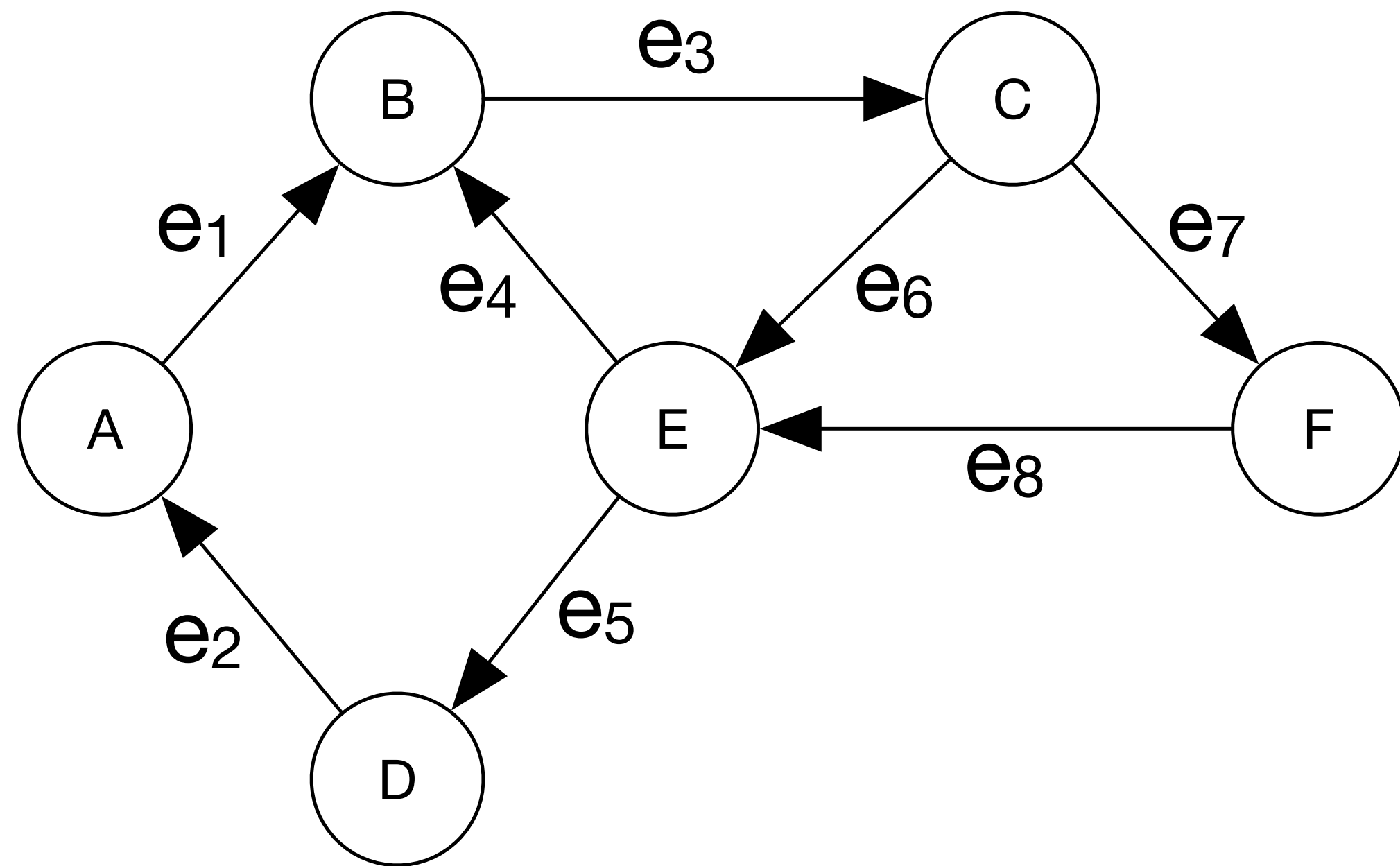


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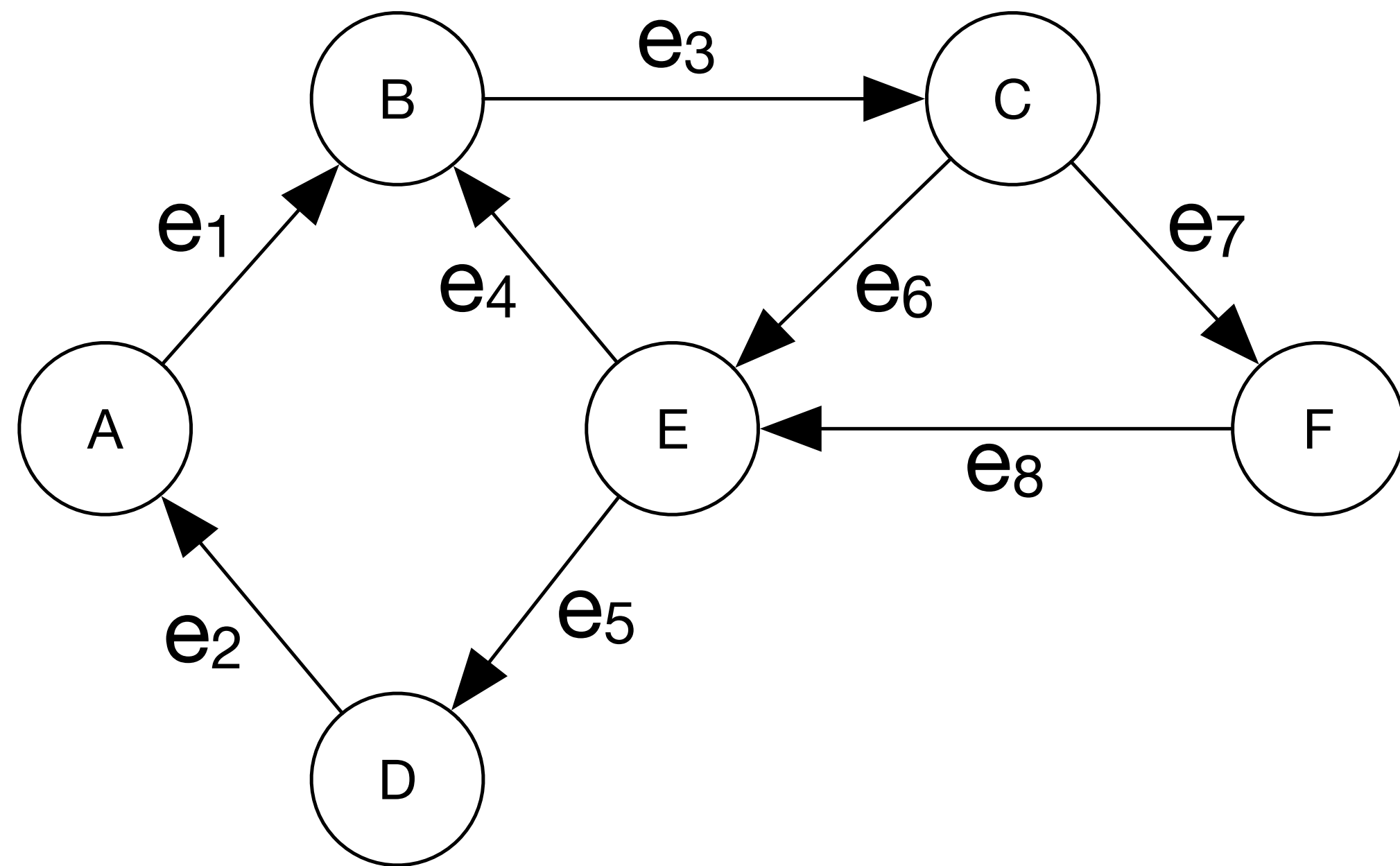


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Incidence matrix

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A	-1	1						
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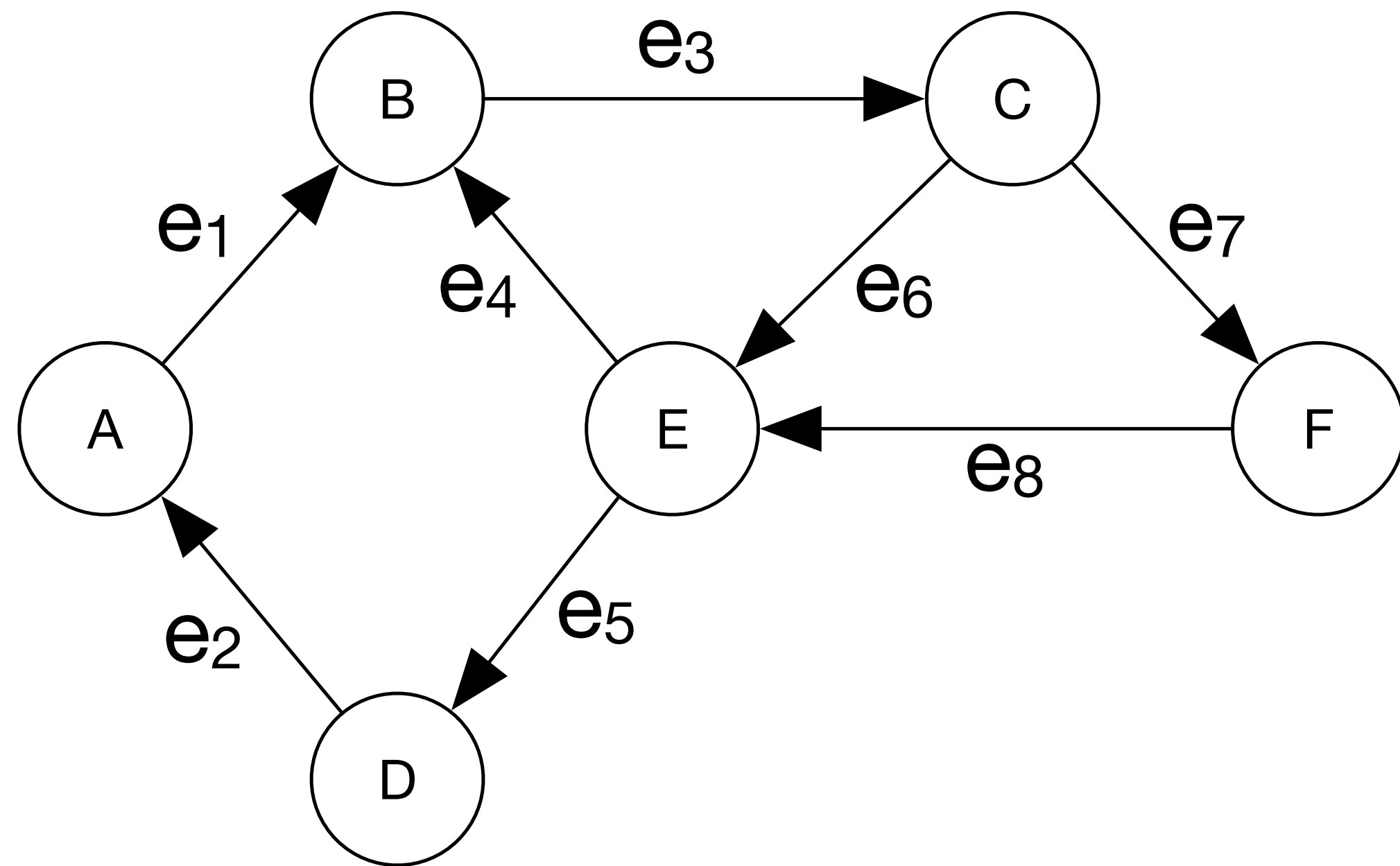


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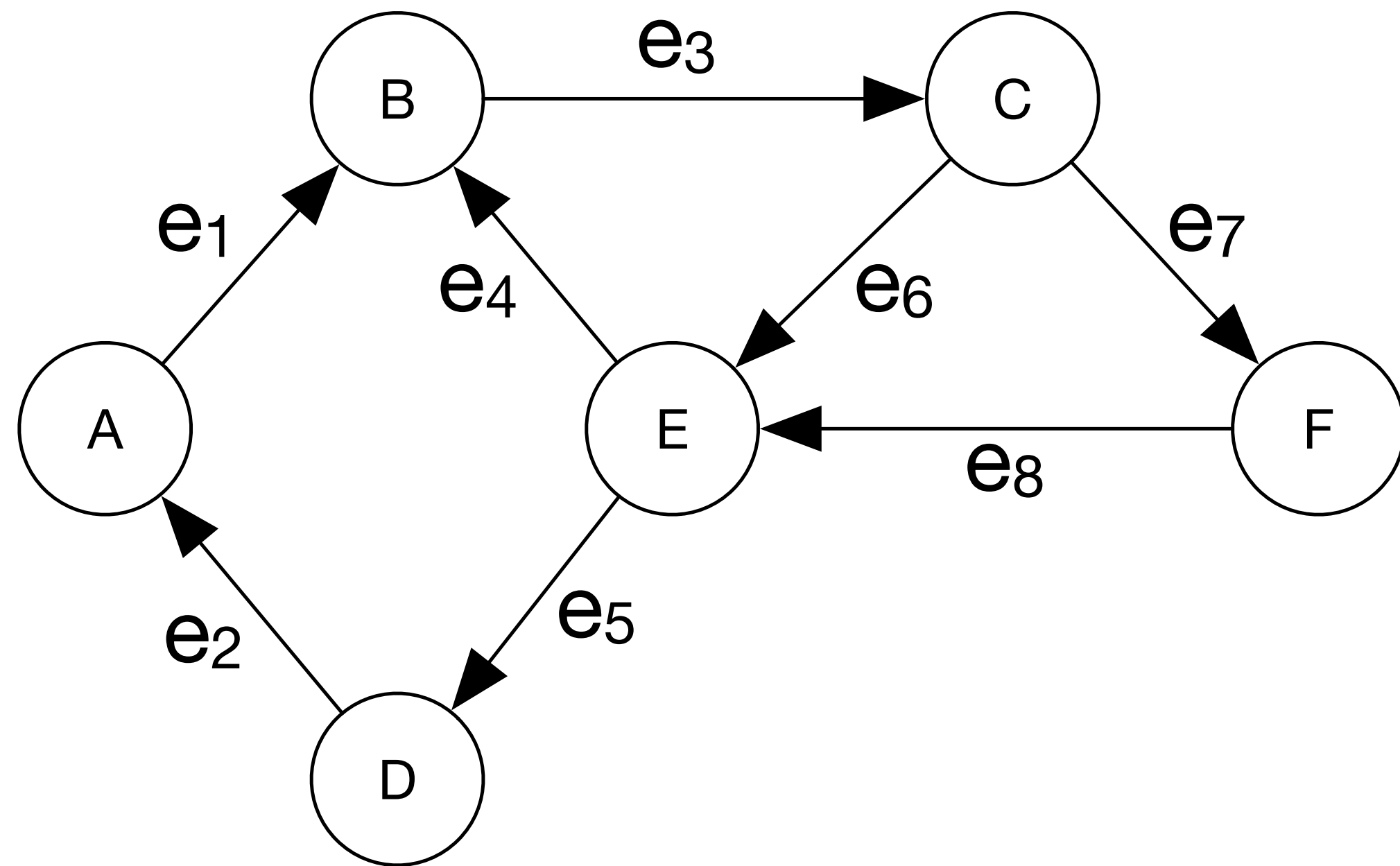


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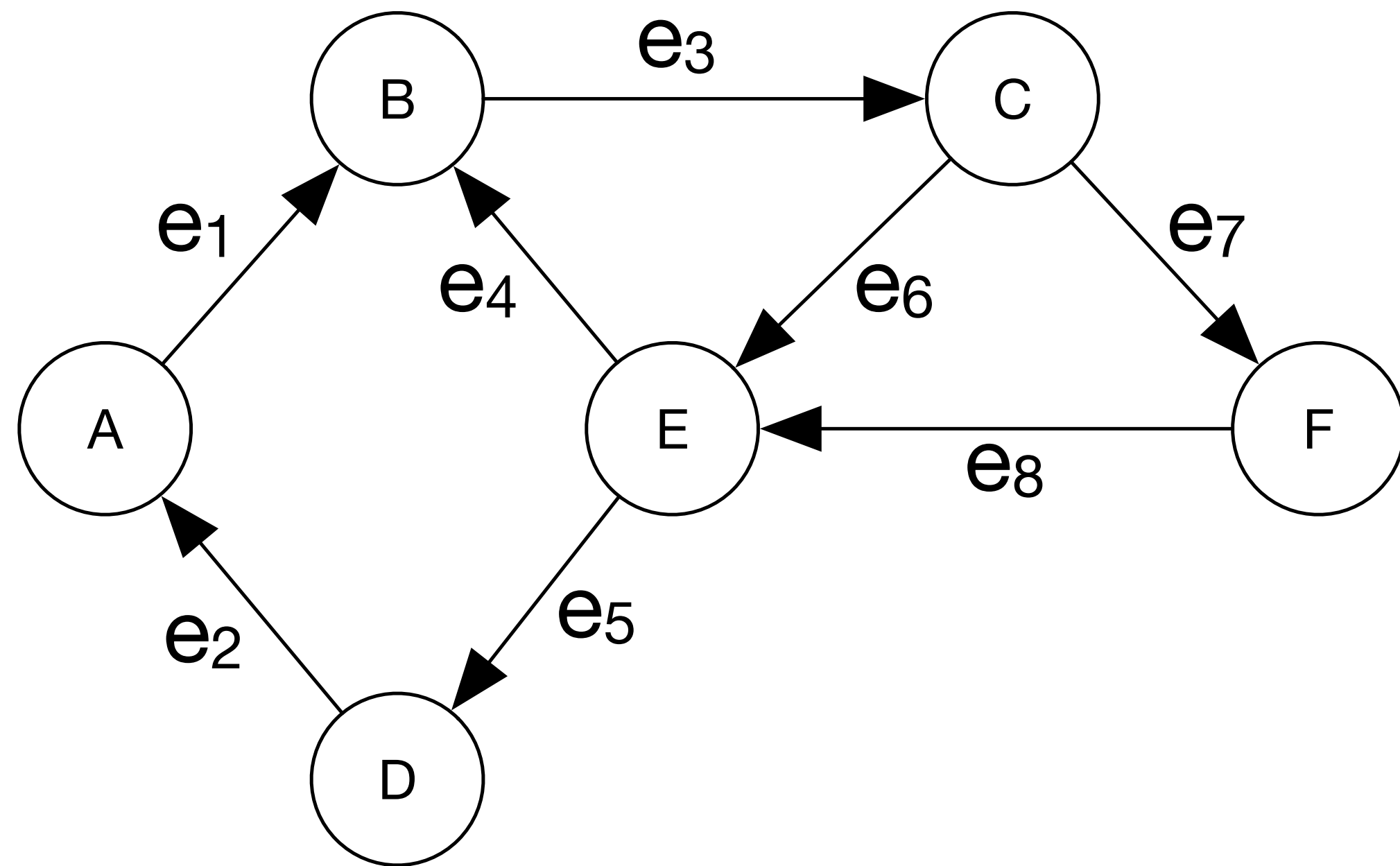


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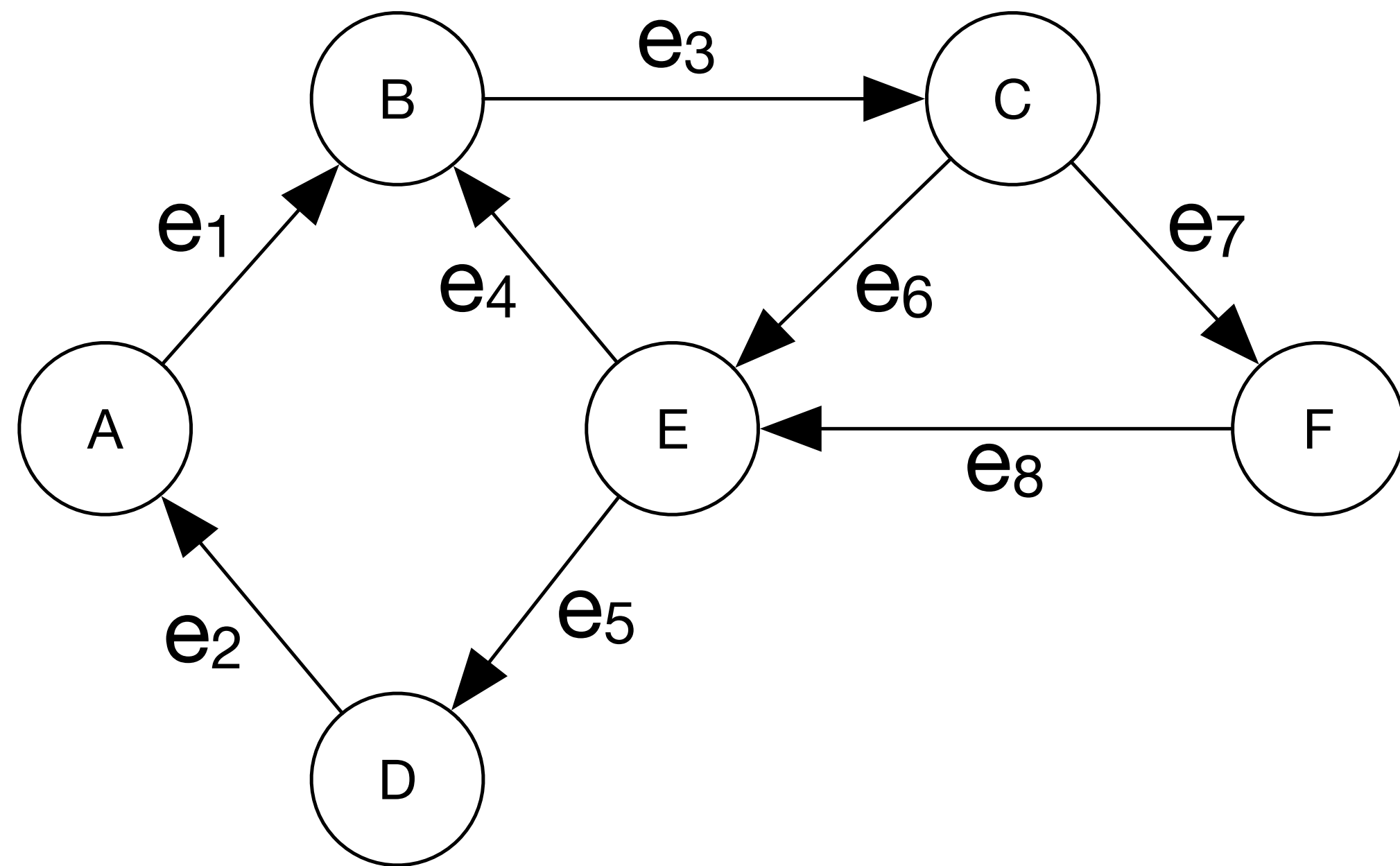


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C	0	0	1	0	0	-1		
D	0	-1	0	0	1	0		
E	0	0	0	-1	-1	1		
F	0	0	0	0	0	0		

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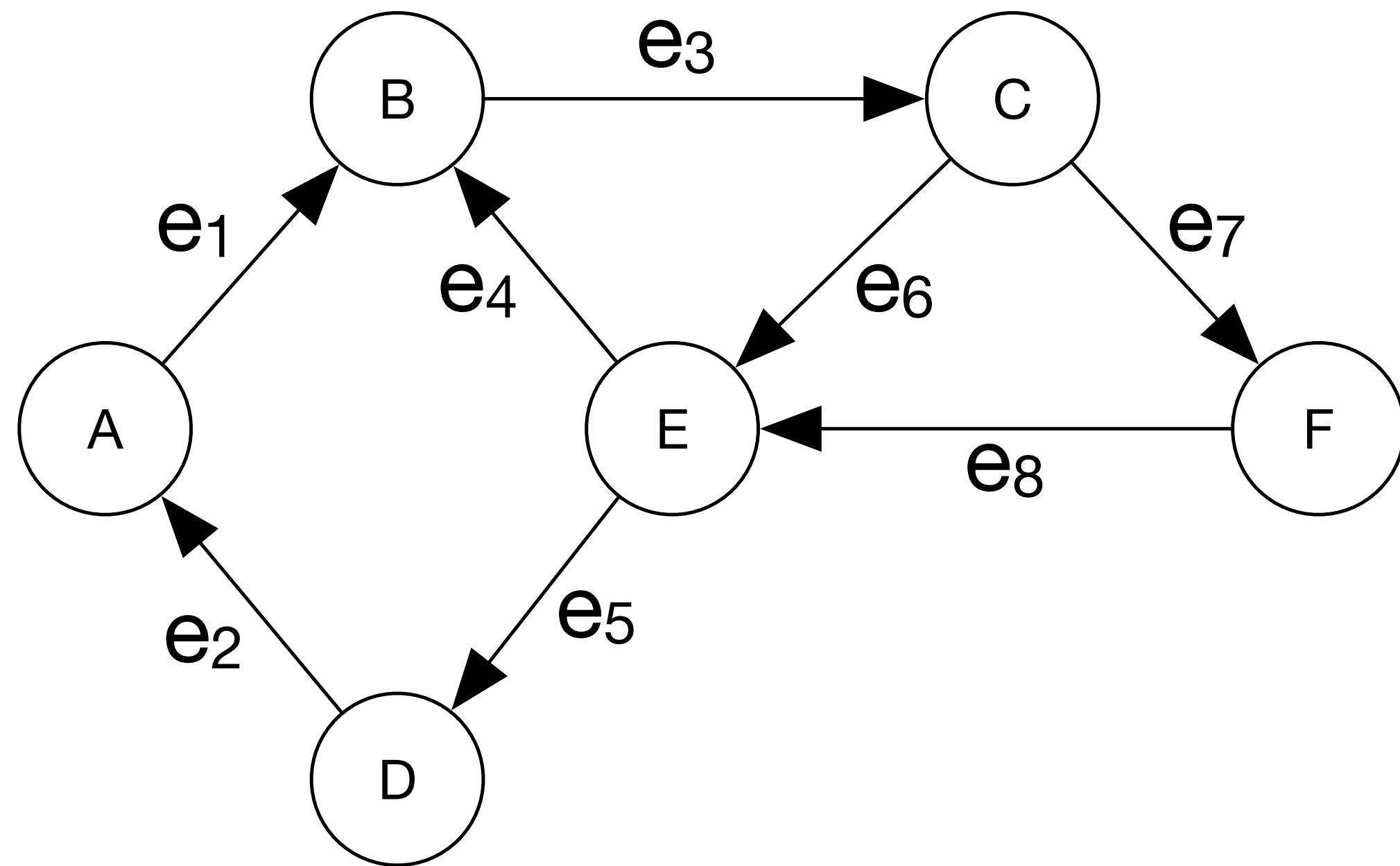


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B	1	0	-1	1	0	0	0	
C	0	0	1	0	0	-1	-1	
D	0	-1	0	0	1	0	0	
E	0	0	0	-1	-1	1	0	
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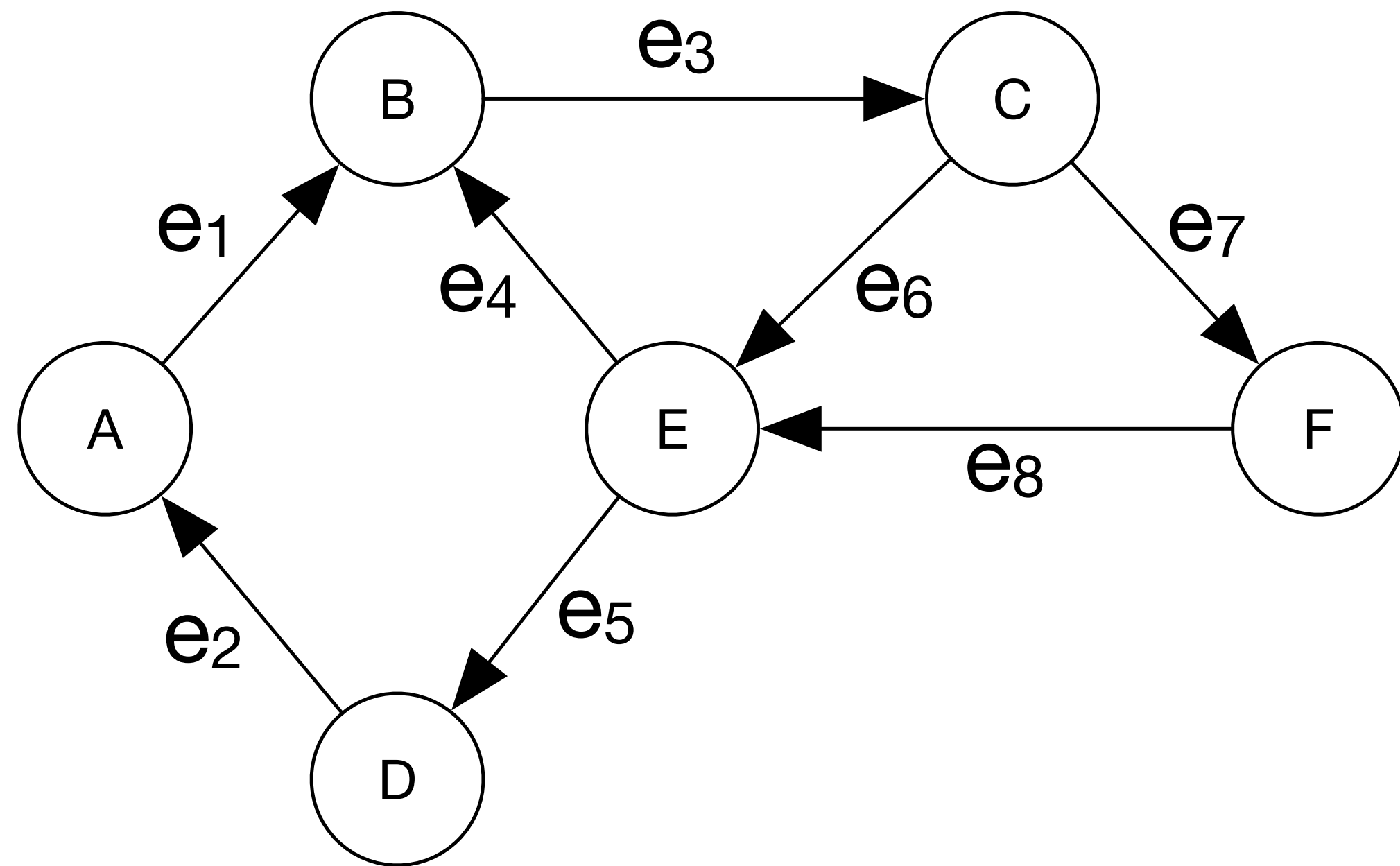


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	1	2	3	4	5	6	7	8
A	-1	1	0	0	0	0	0	0
B	1	0	-1	1	0	0	0	0
C	0	0	1	0	0	-1	-1	0
D	0	-1	0	0	1	0	0	0
E	0	0	0	-1	-1	1	0	1
F	0	0	0	0	0	0	1	-1

Introduction to graphs



$$M_{v,e} = \begin{cases} -1 & \text{if edge } e \text{ leaves node } v \\ 1 & \text{if edge } e \text{ enters node } v \\ 0 & \text{otherwise} \end{cases}$$

Incidence matrix

	1	2	3	4	5	6	7	8
A	-1	1	0	0	0	0	0	0
B	1	0	-1	1	0	0	0	0
C	0	0	1	0	0	-1	-1	0
D	0	-1	0	0	1	0	0	0
E	0	0	0	-1	-1	1	0	1
F	0	0	0	0	0	0	1	-1

Introduction to graphs

Which data structure is "best"? It depends!

	Adjacency list	Adjacency matrix	Incidence matrix
Space	$O(V + E)$	$O(V ^2)$	$O(V \times E)$
Add vertex	$O(1)$	$O(V ^2)$	$O(V \times E)$
Add edge	$O(1)$	$O(1)$	$O(V \times E)$
Adjacency query	$O(V)$	$O(1)$	$O(V \times E)$