



THE UNIVERSITY OF VERMONT  
COLLEGE OF ENGINEERING &  
MATHEMATICAL SCIENCES

# Representing Trees

CS 124 / Department of Computer Science

# How do we represent a tree?

## Choosing a data structure

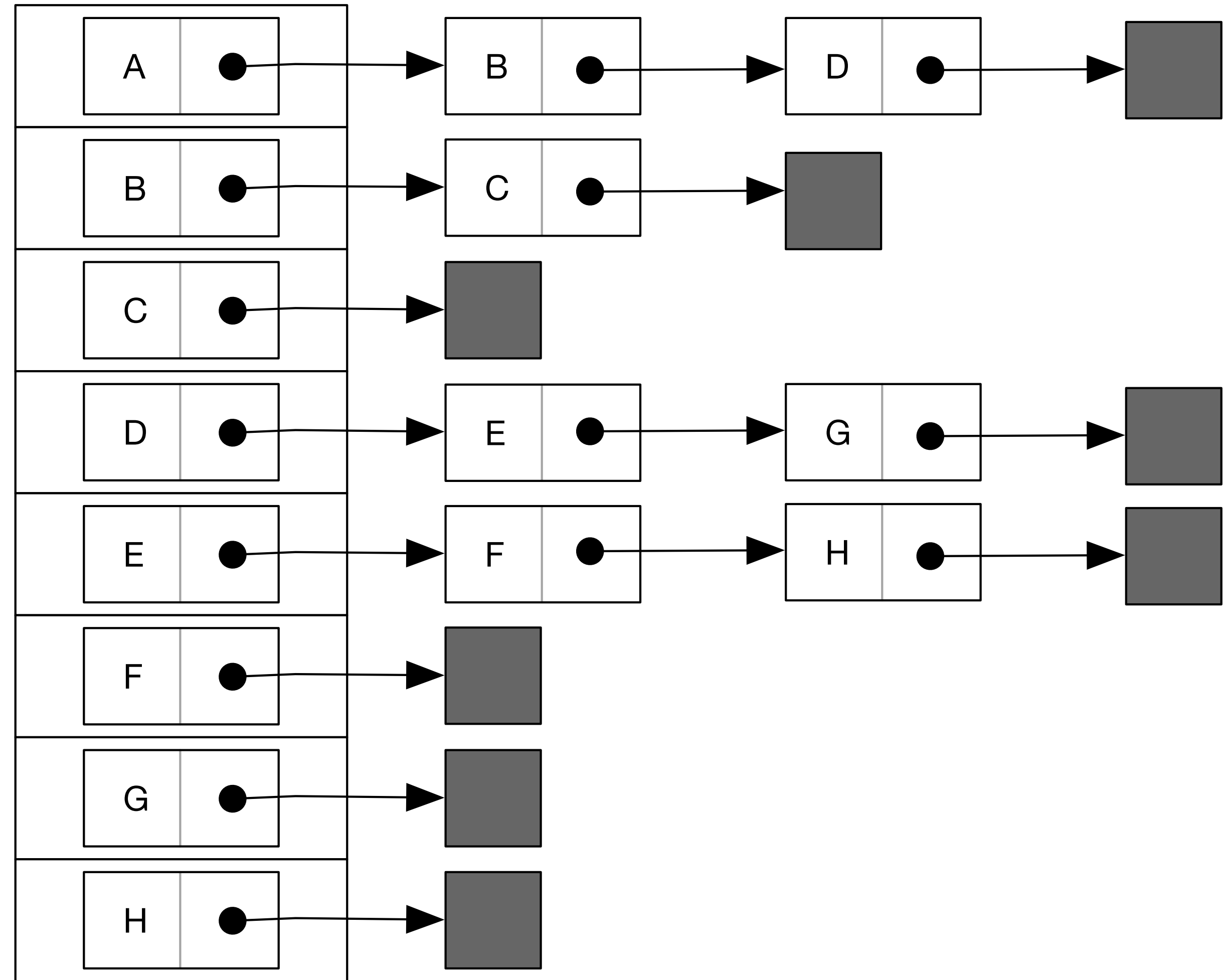
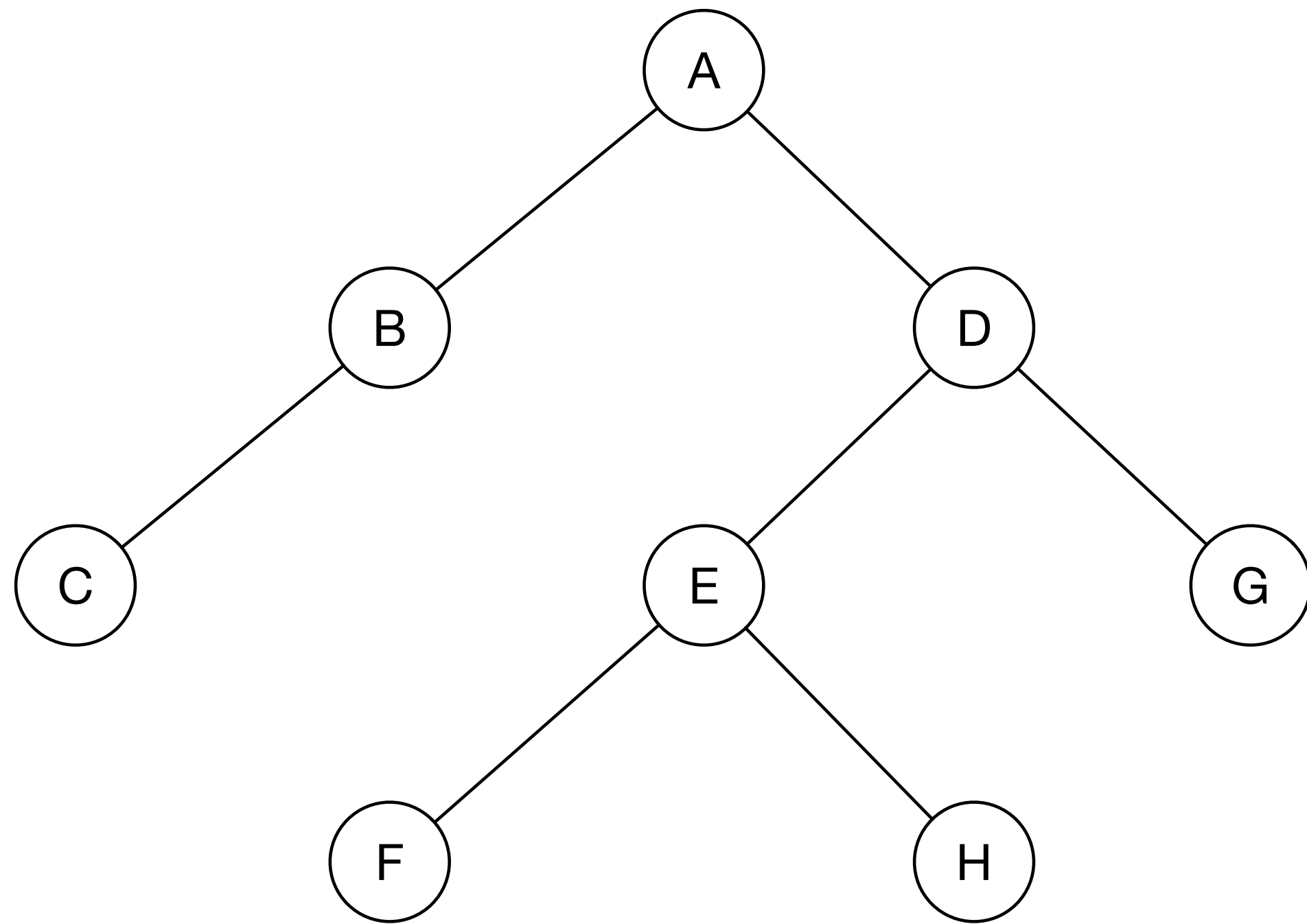
- Adjacency list
- Adjacency matrix
- Incidence matrix
- Simple L/R object-oriented approach

# How do we represent a tree?

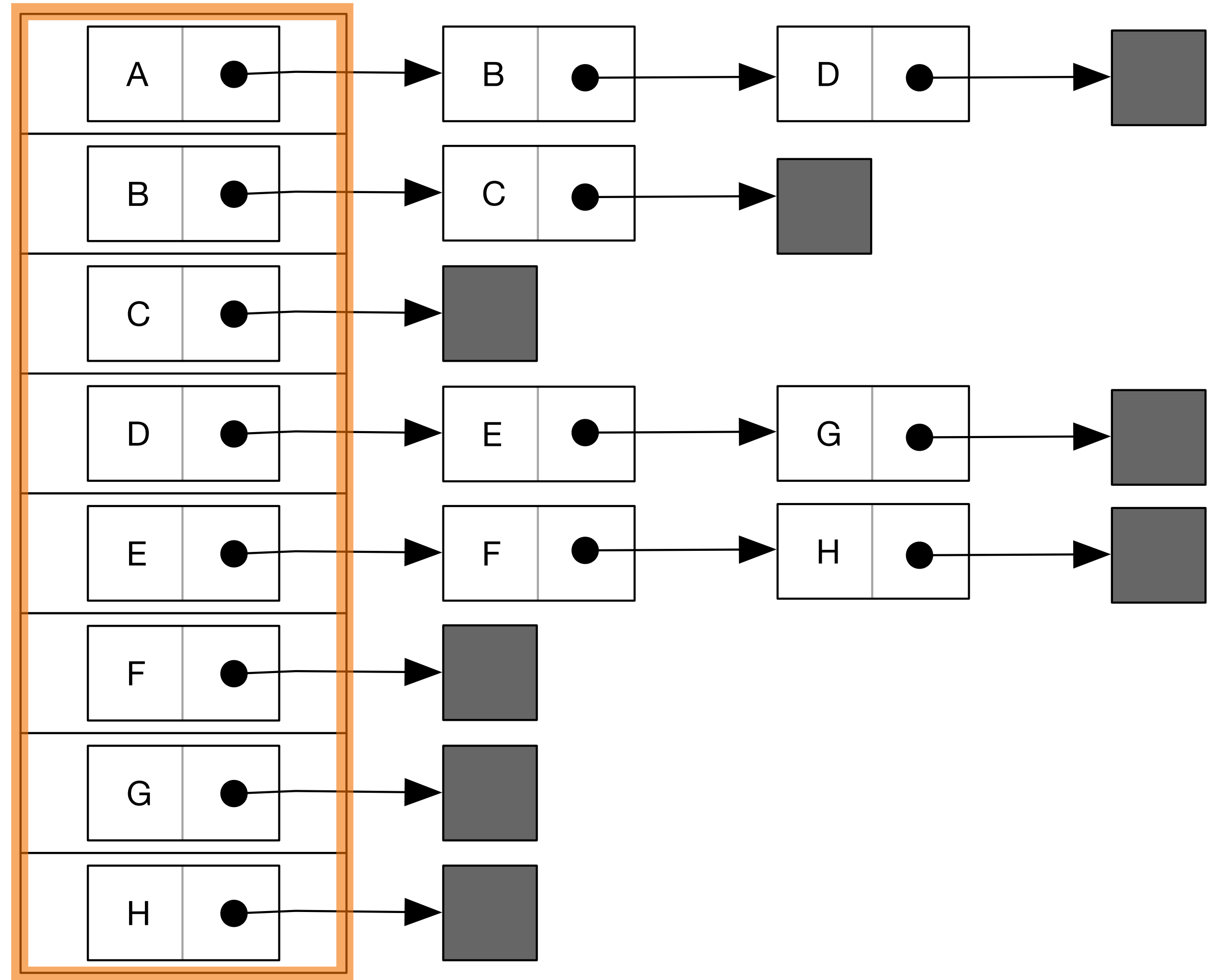
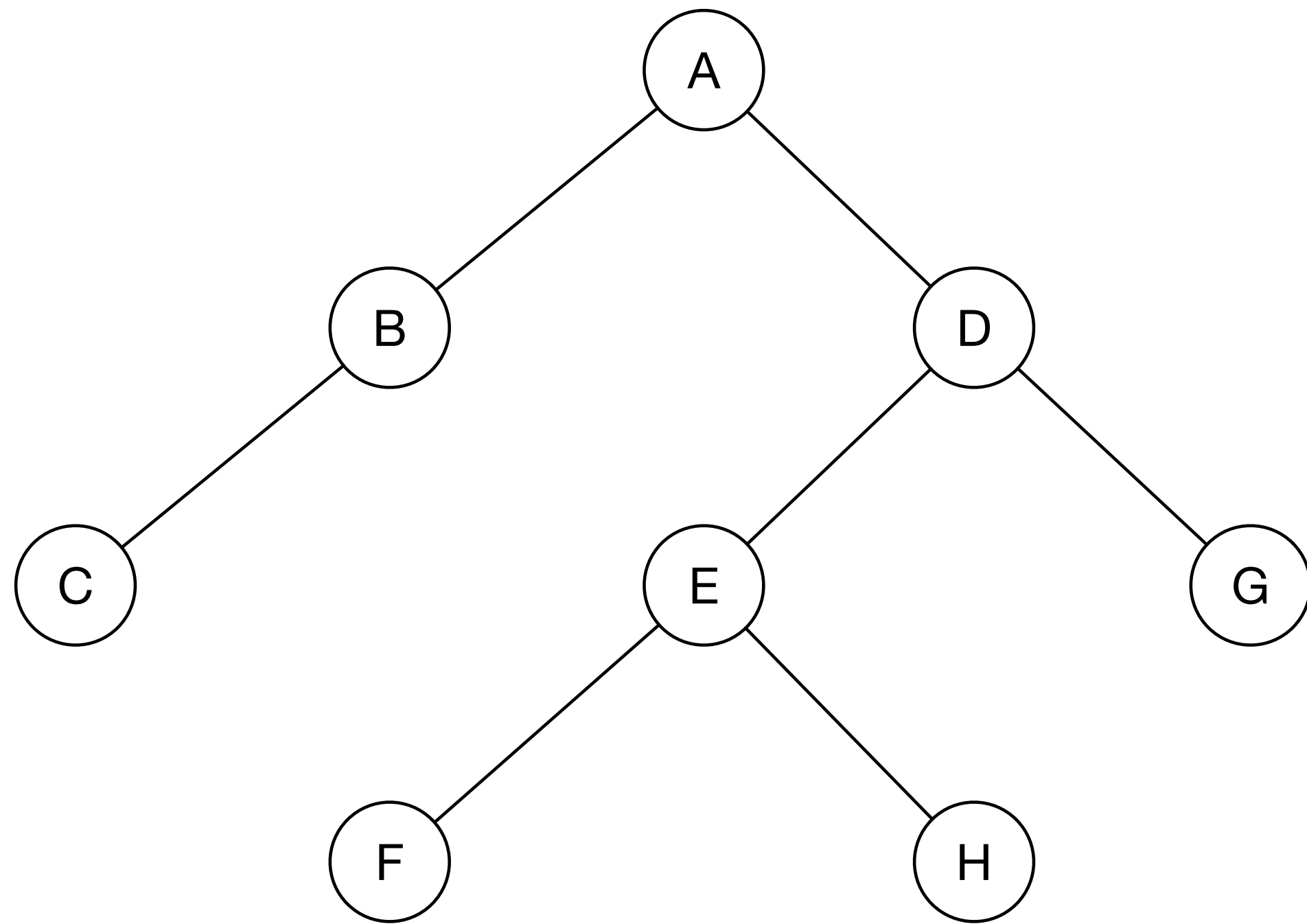
## Choosing a data structure

- Adjacency list
- Adjacency matrix
- Incidence matrix
- Simple L/R object-oriented approach

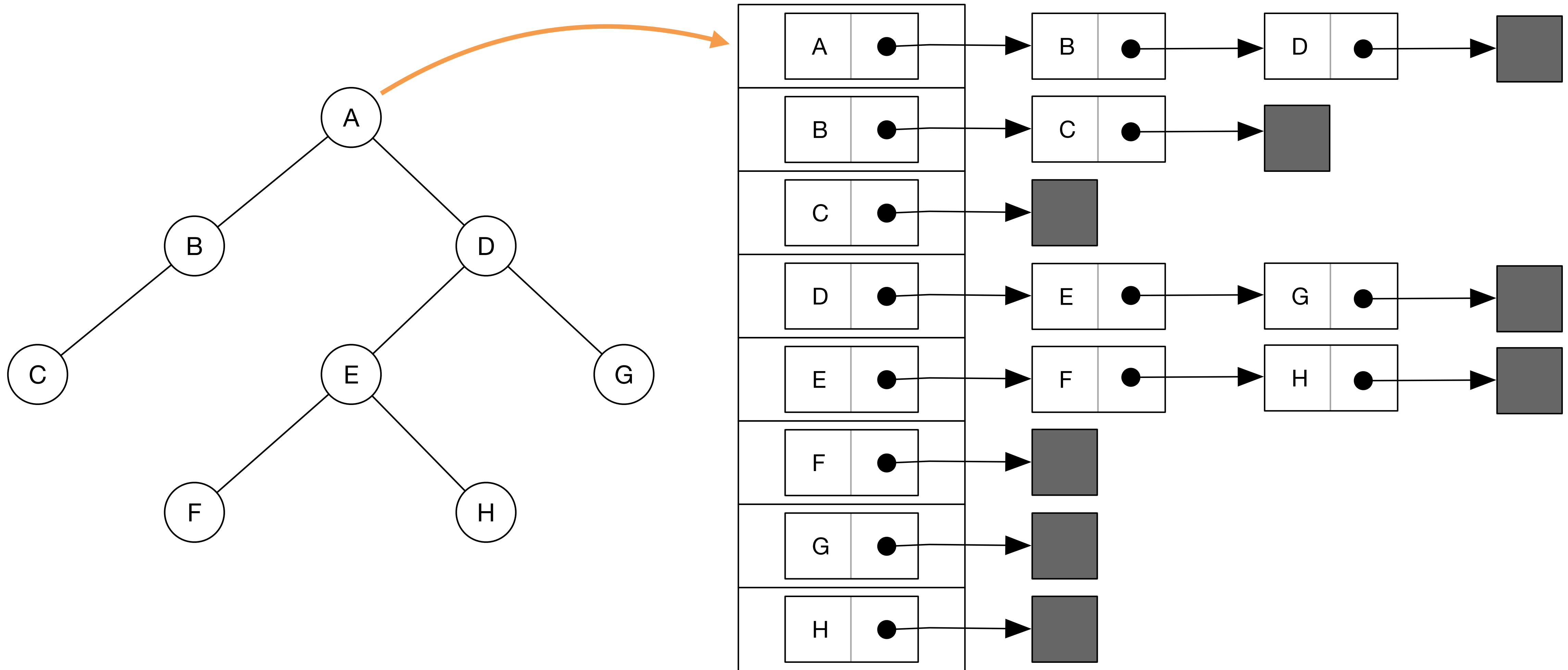
# Adjacency list



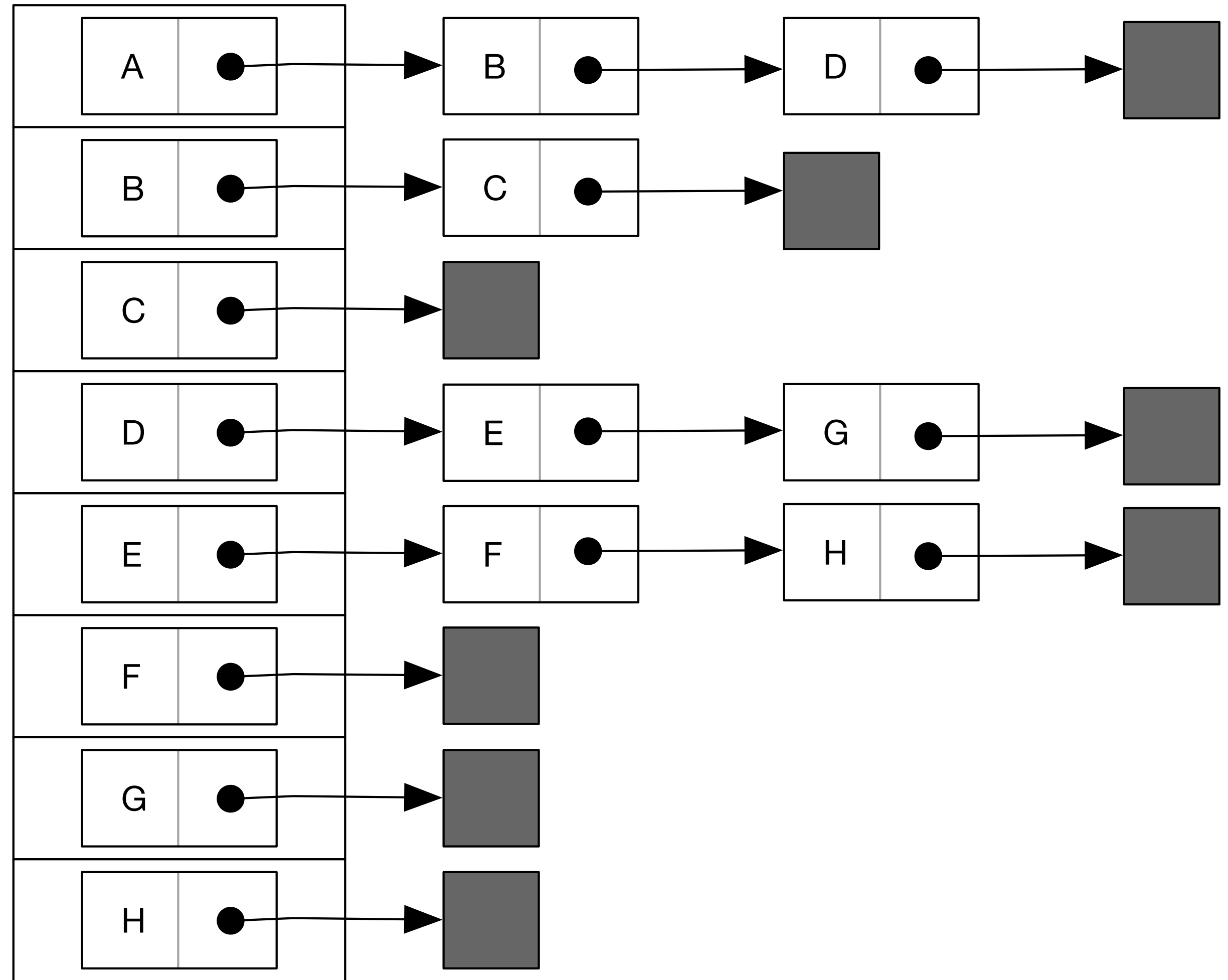
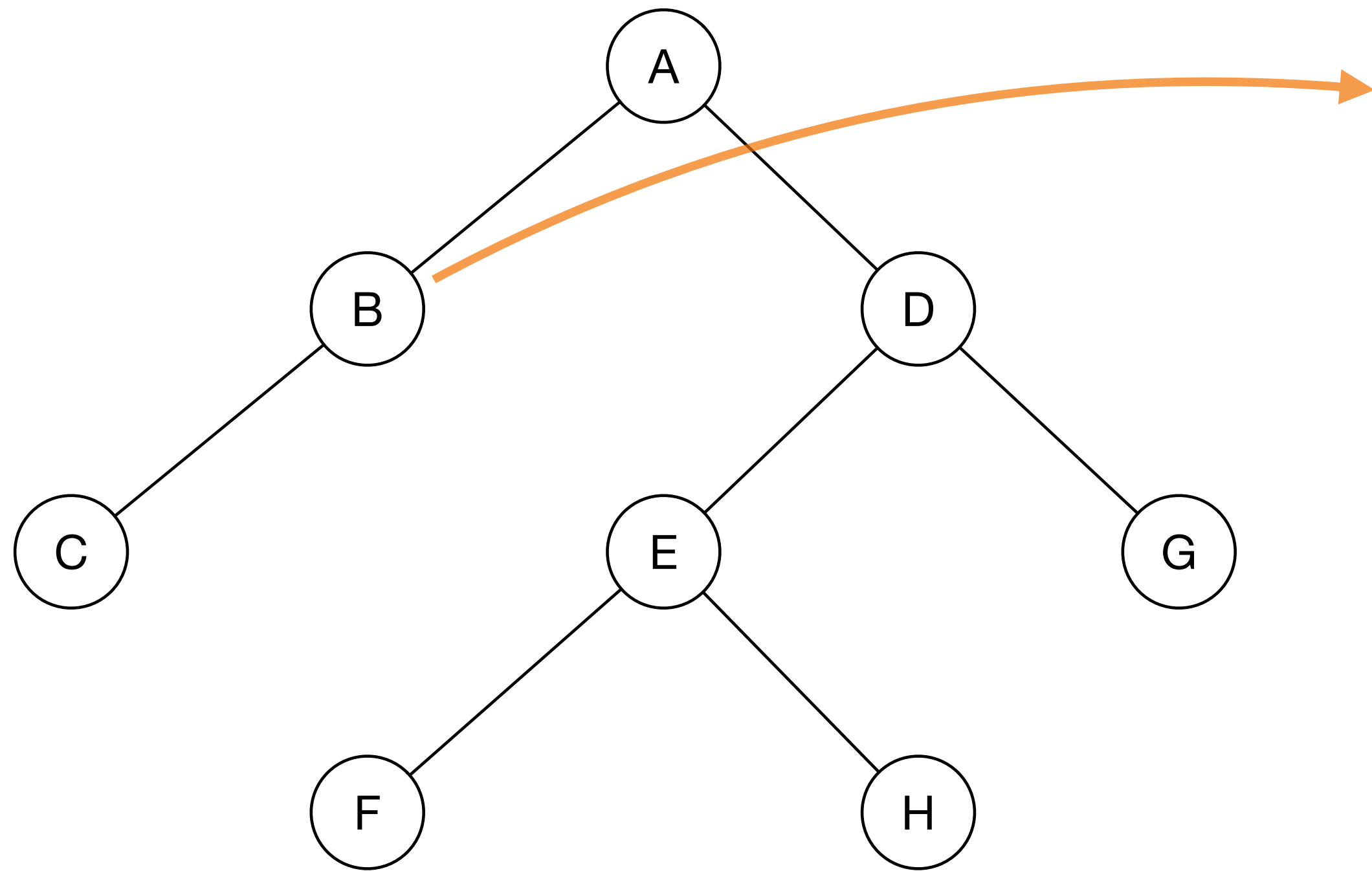
# Adjacency list



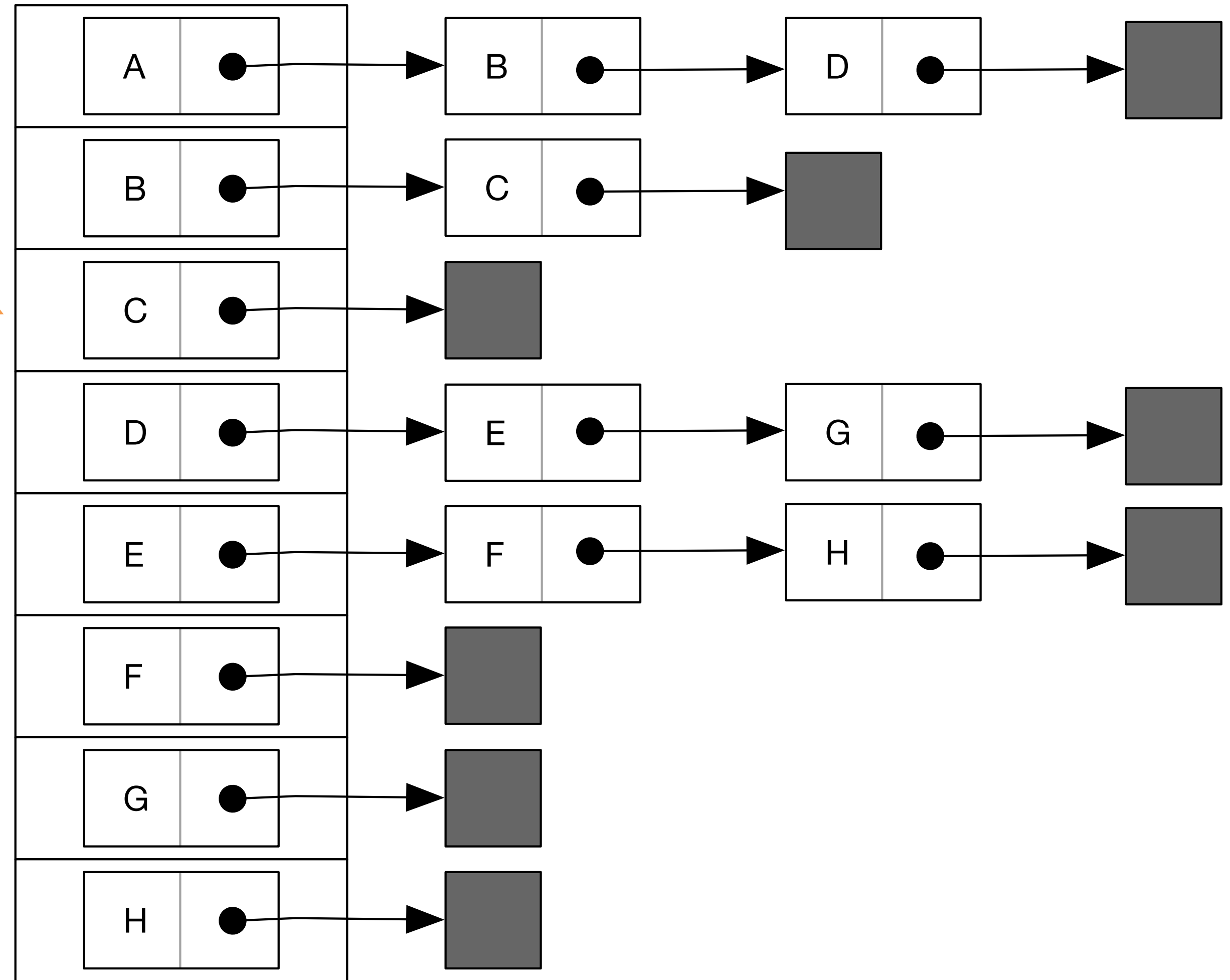
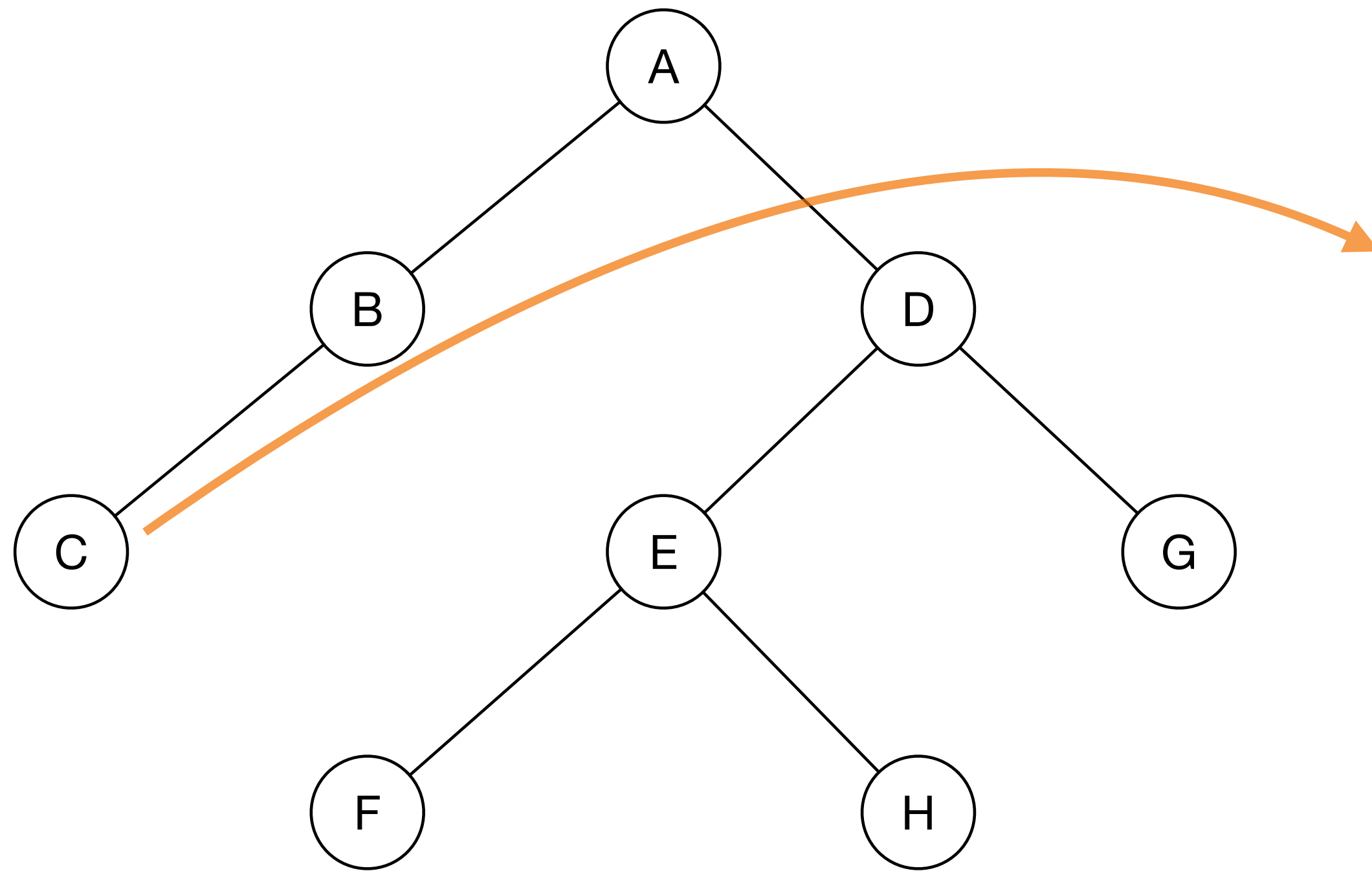
# Adjacency list



# Adjacency list

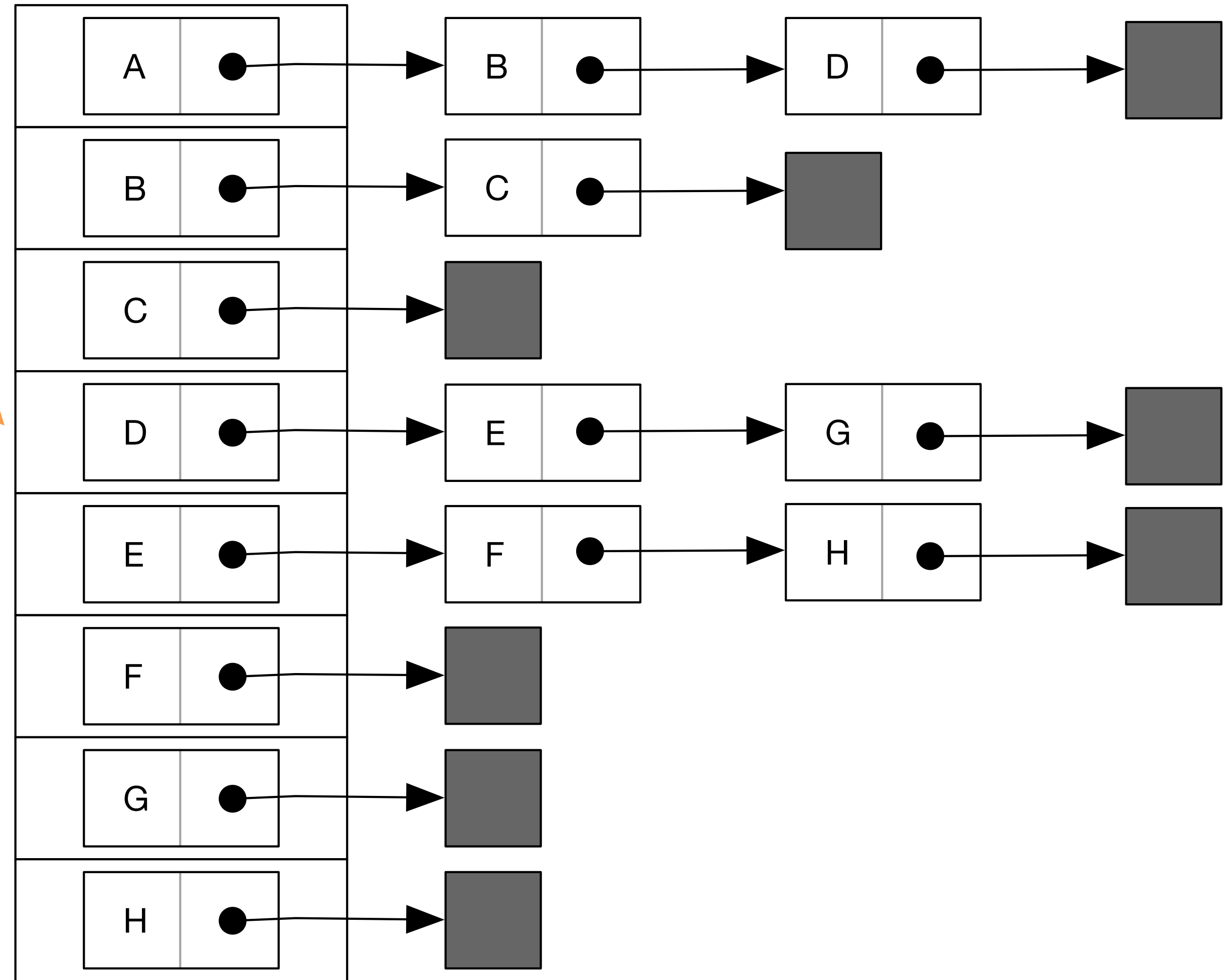
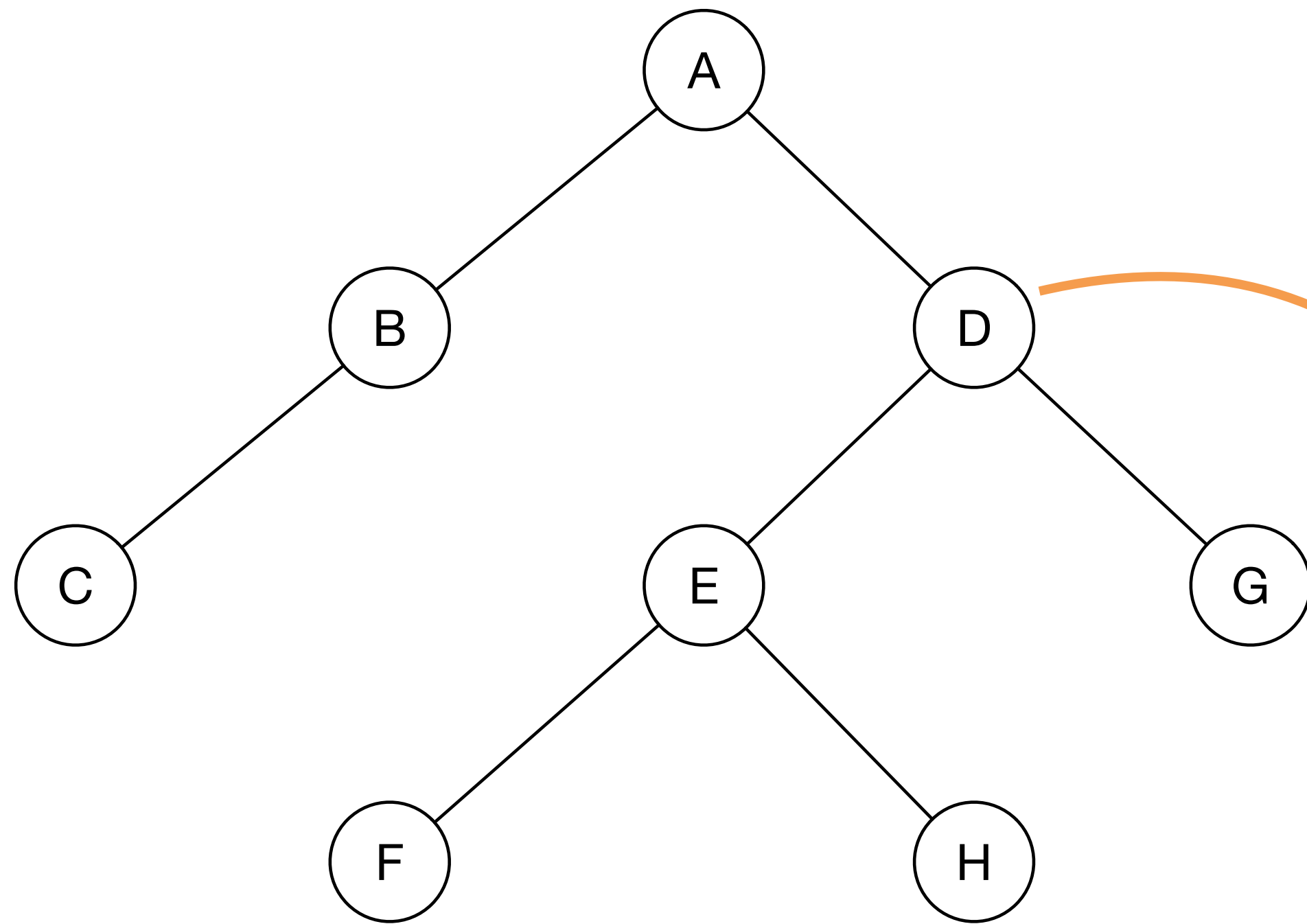


# Adjacency list

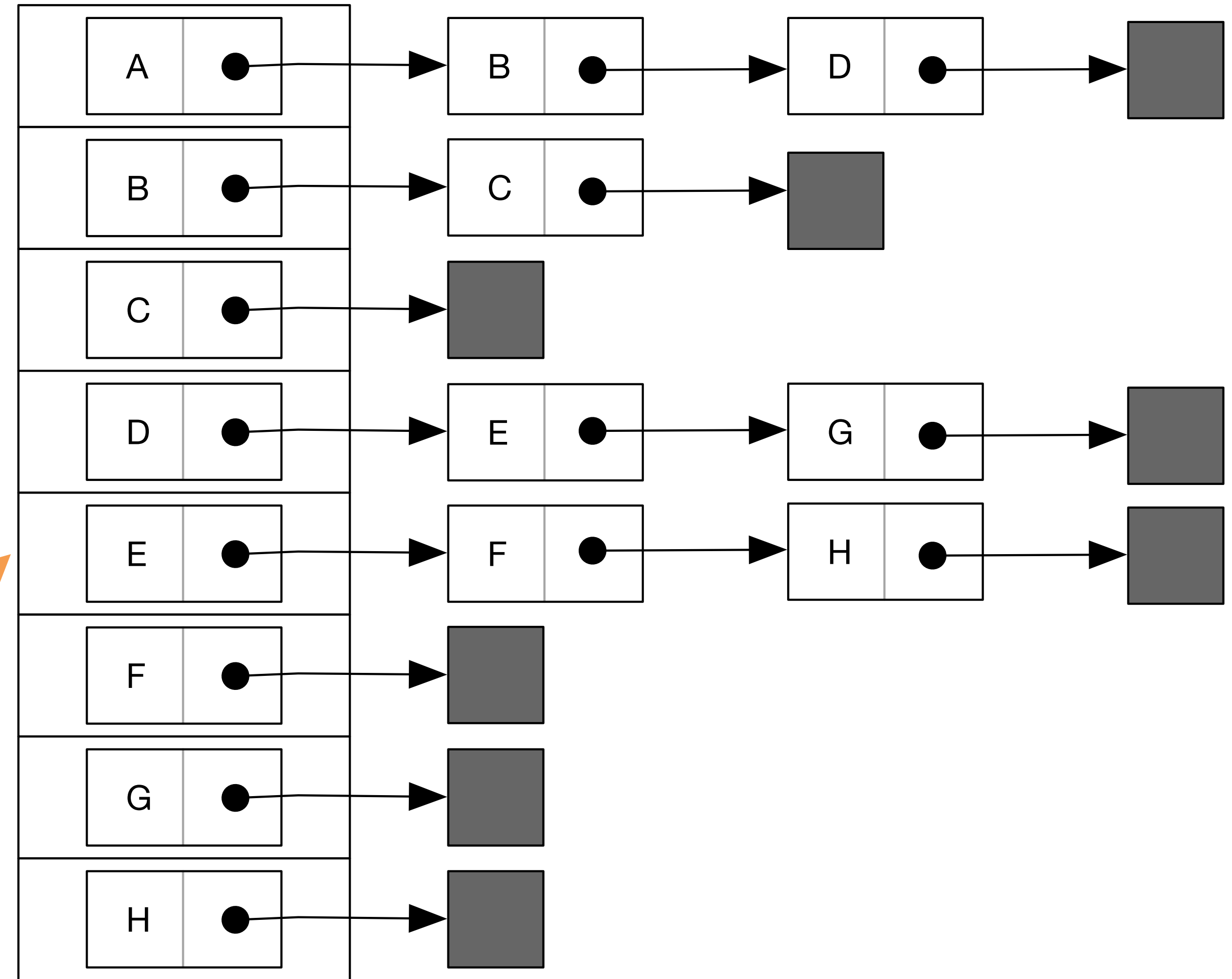
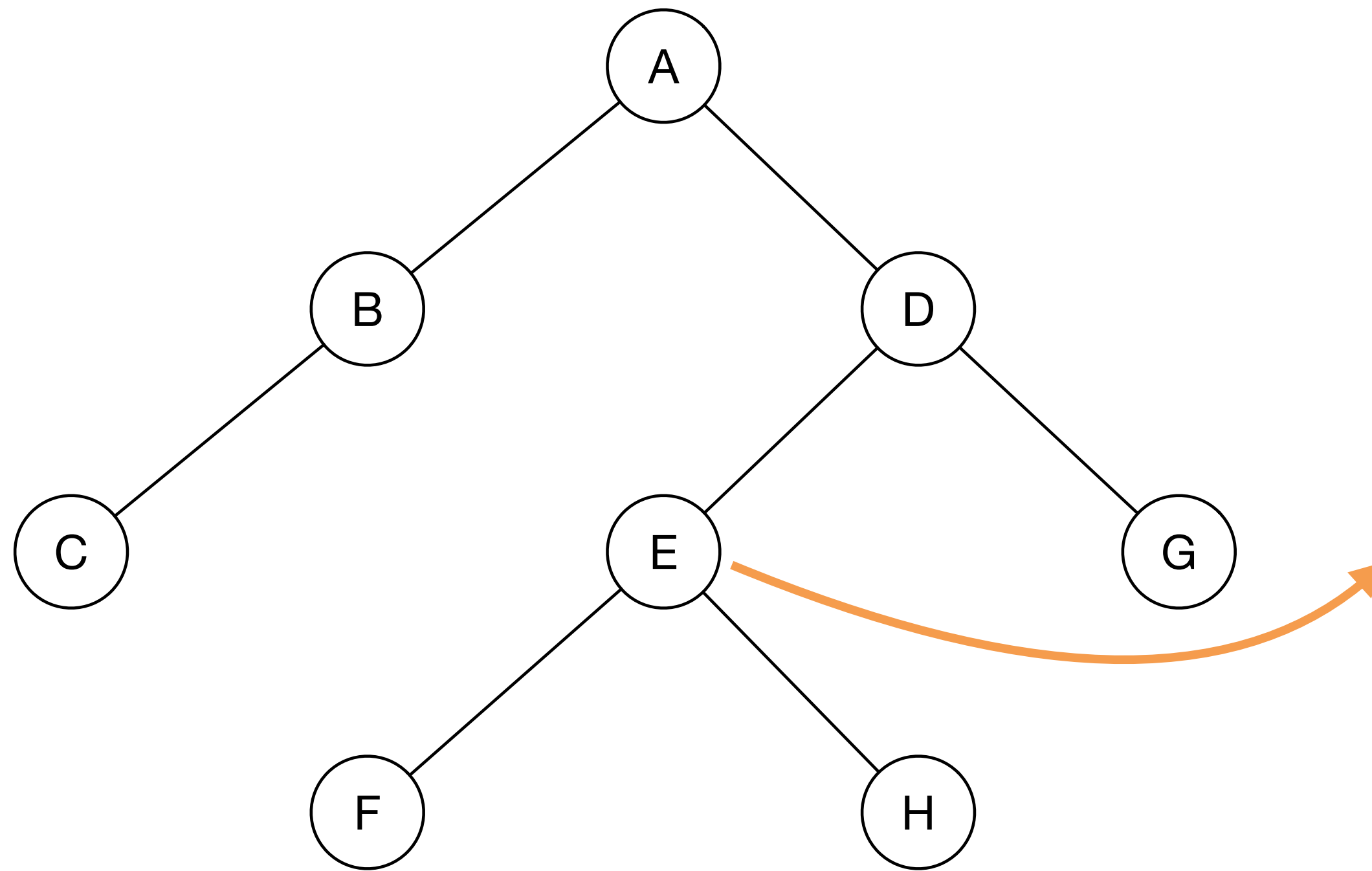




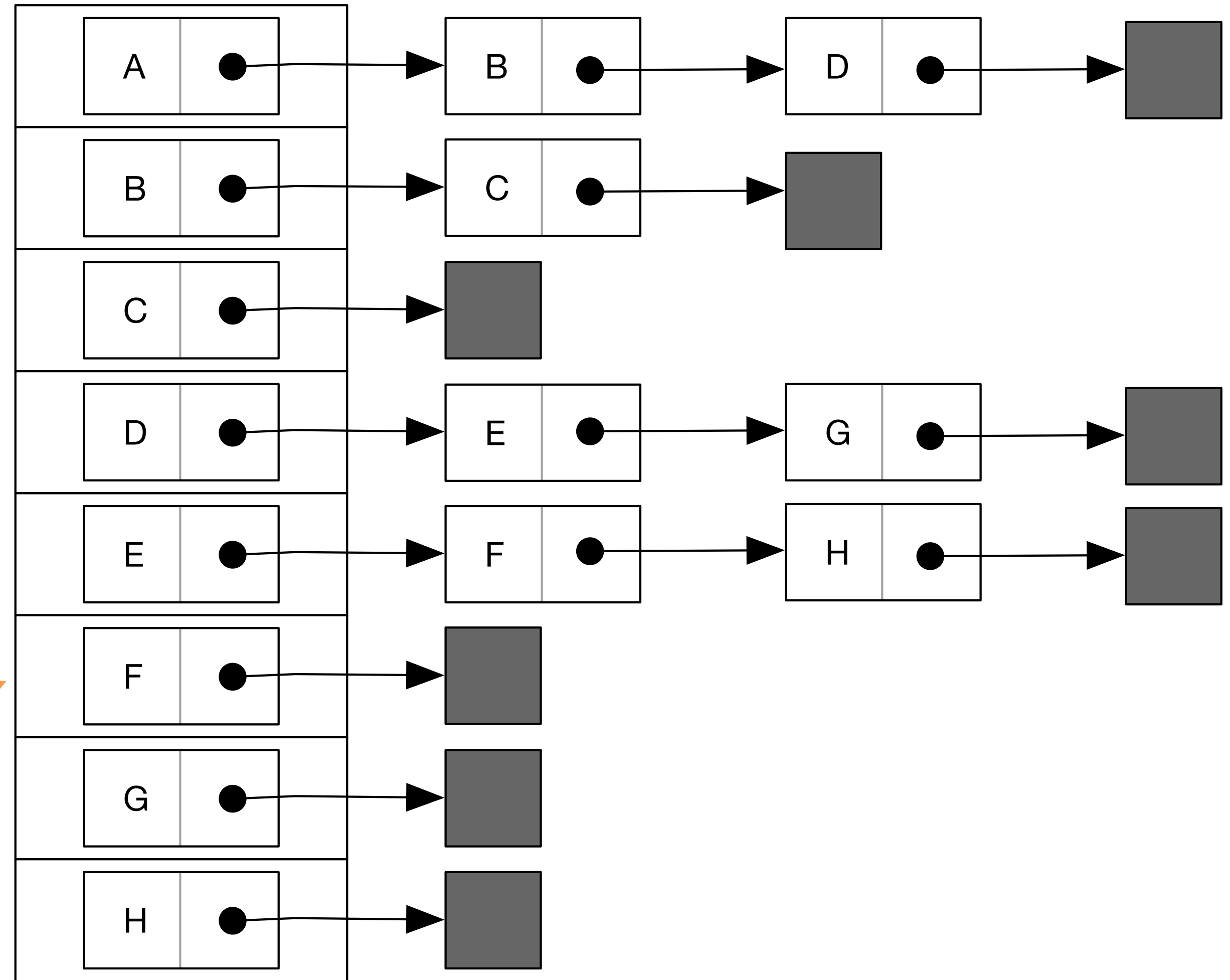
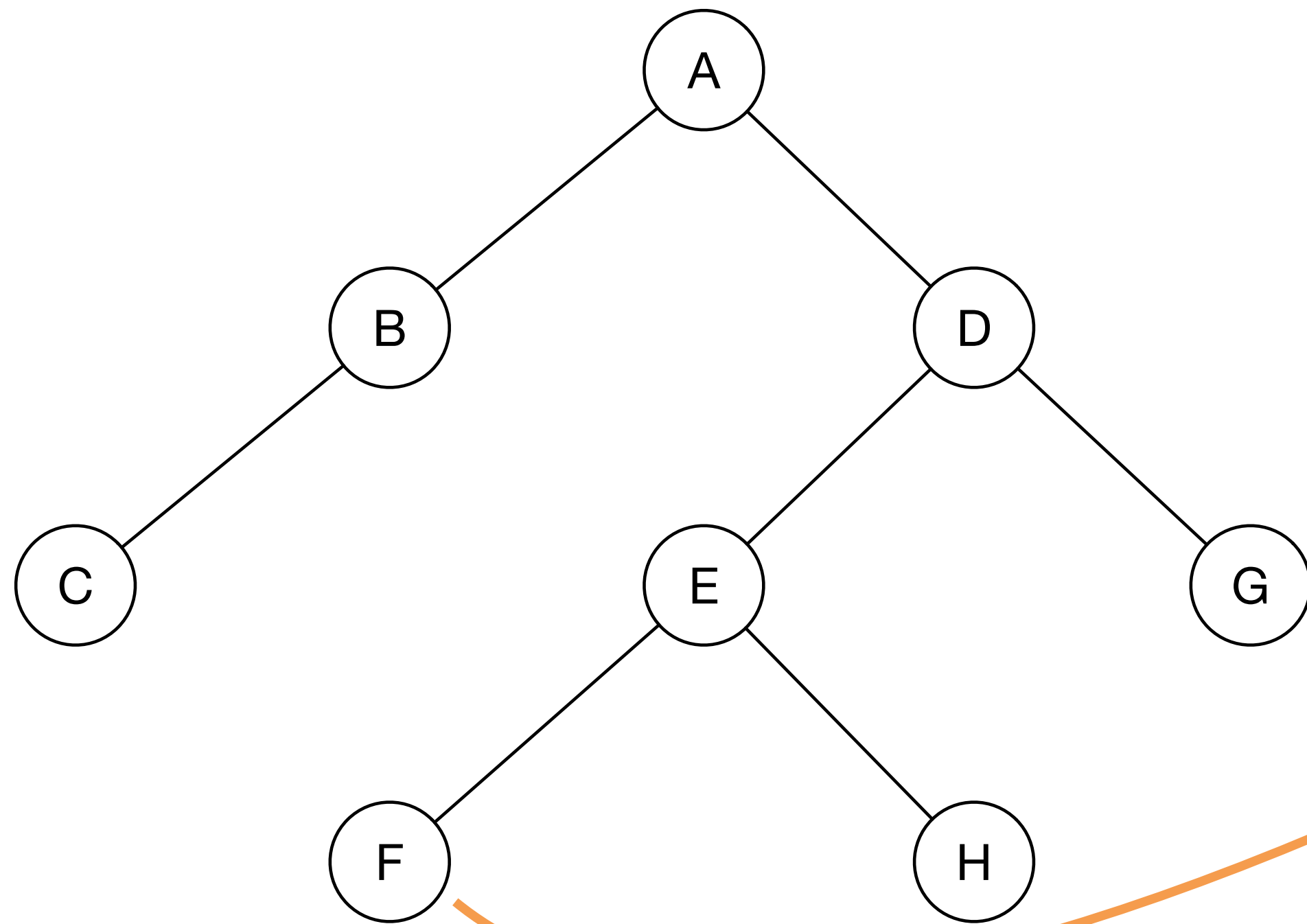
# Adjacency list



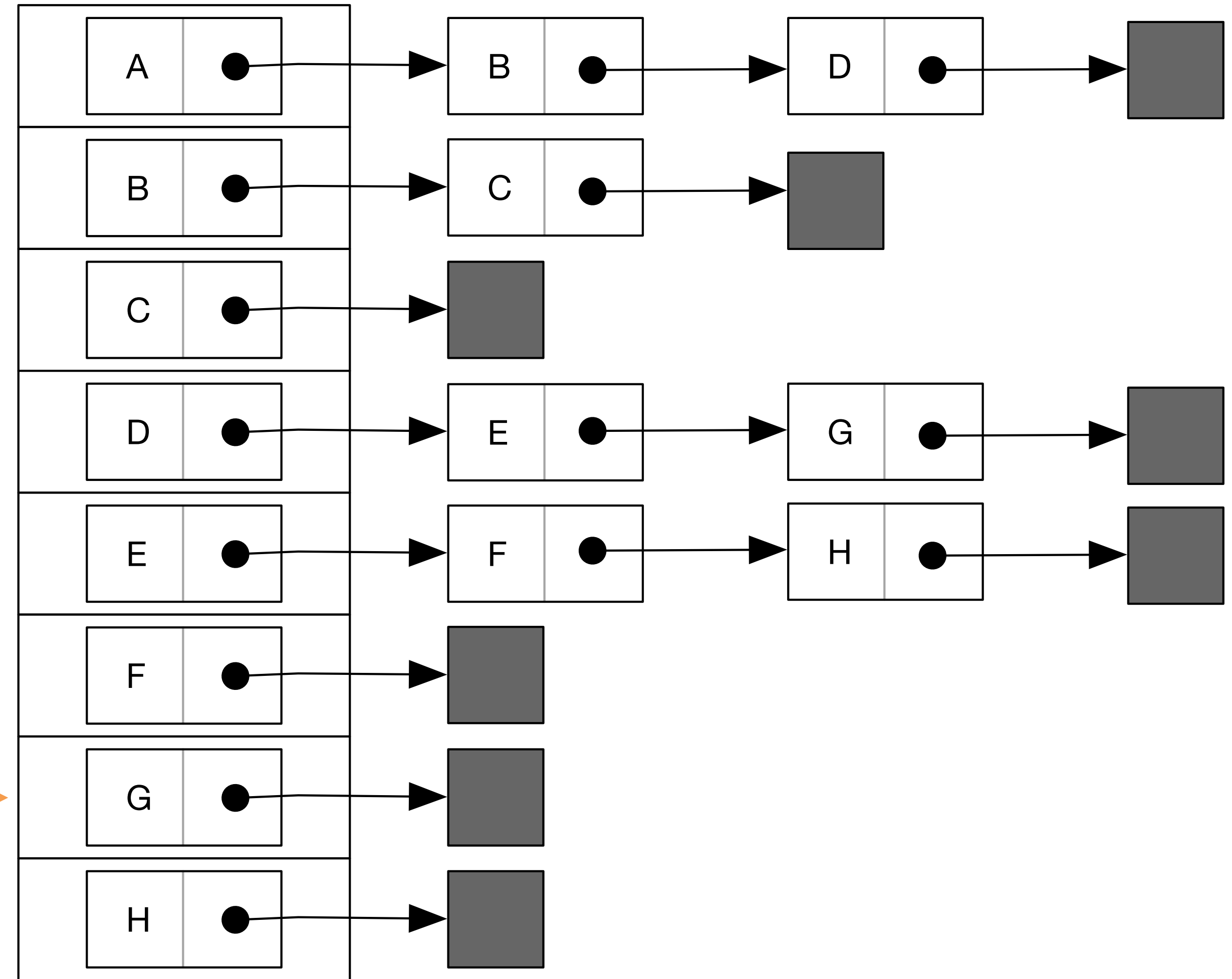
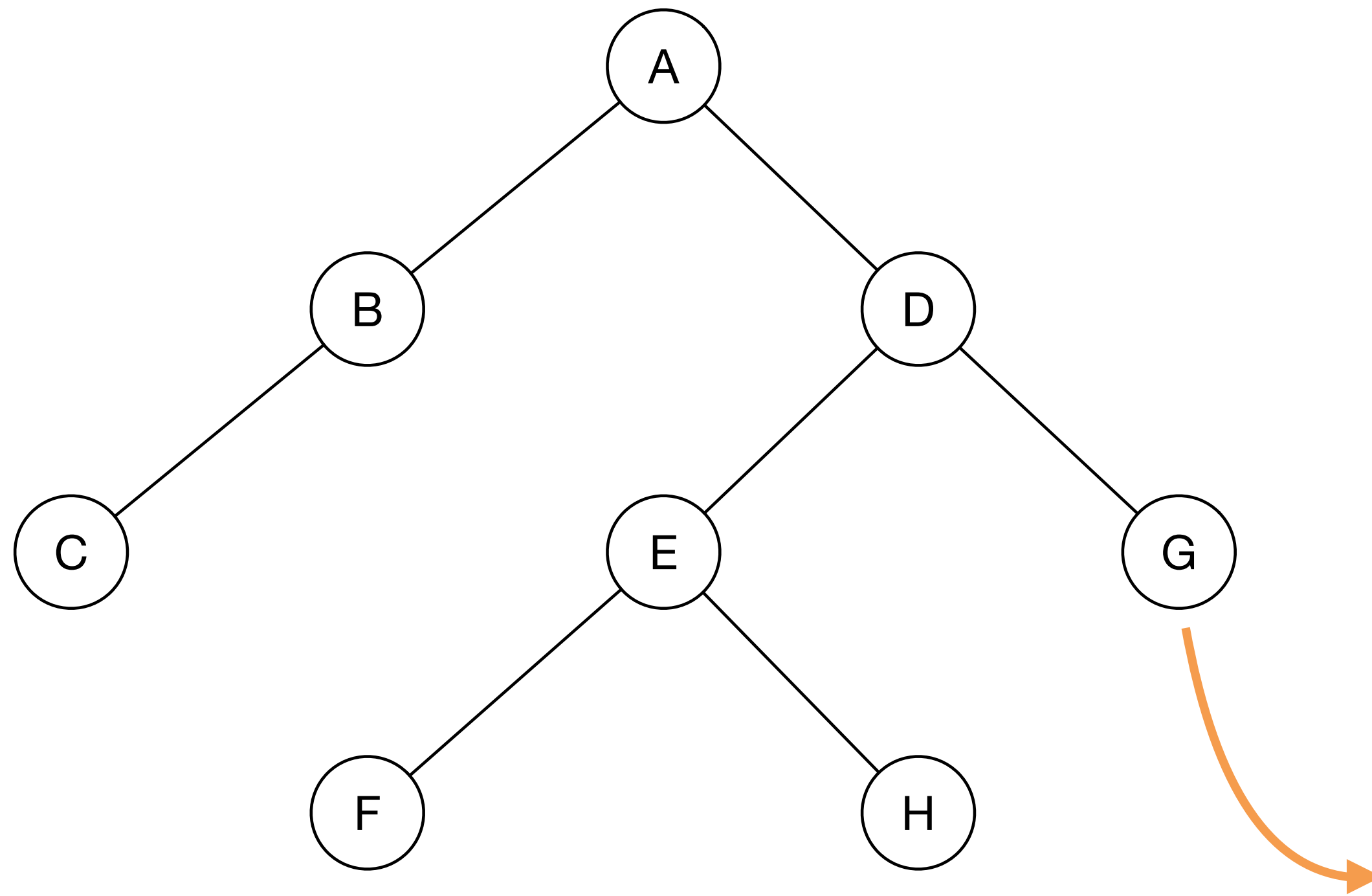
# Adjacency list



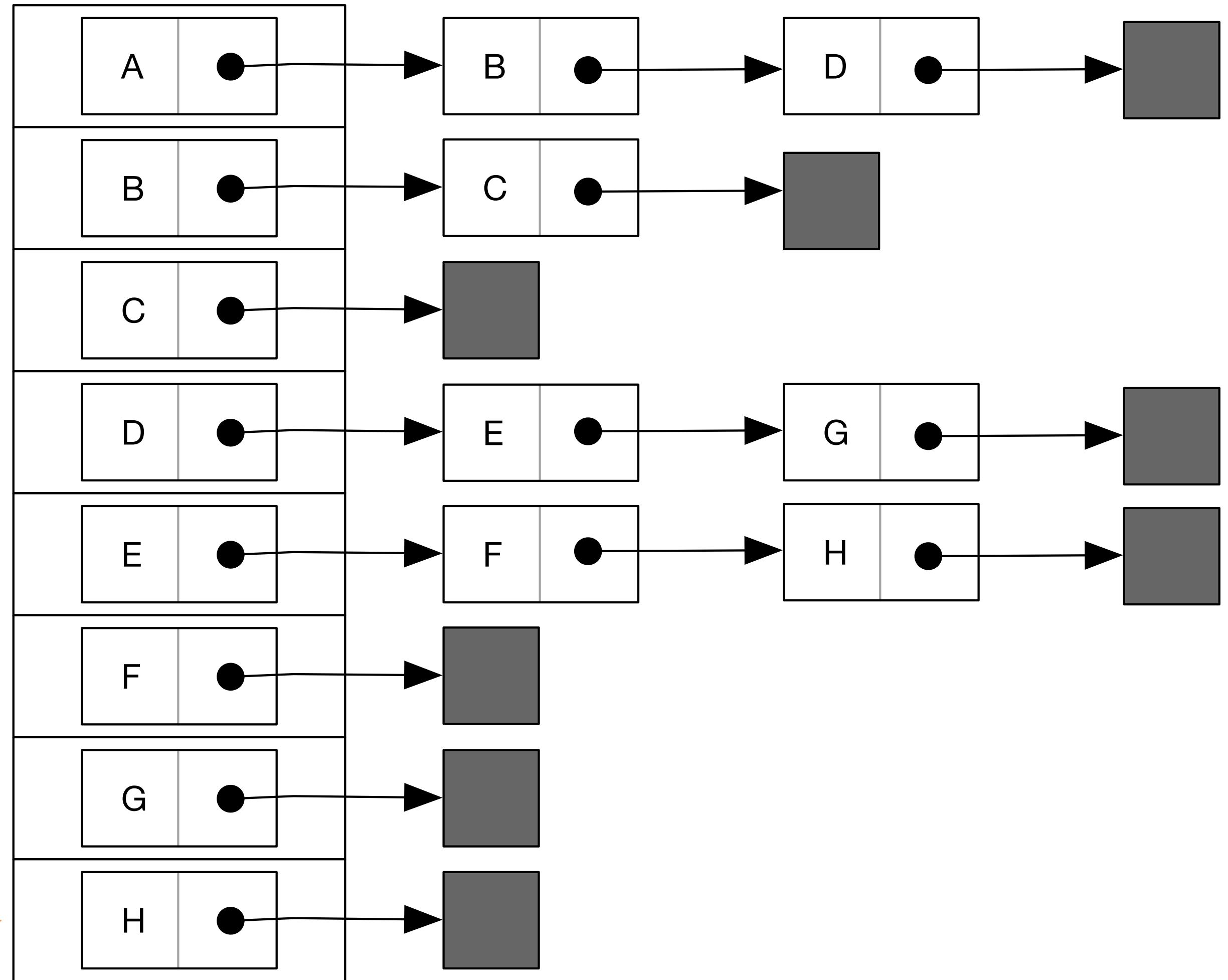
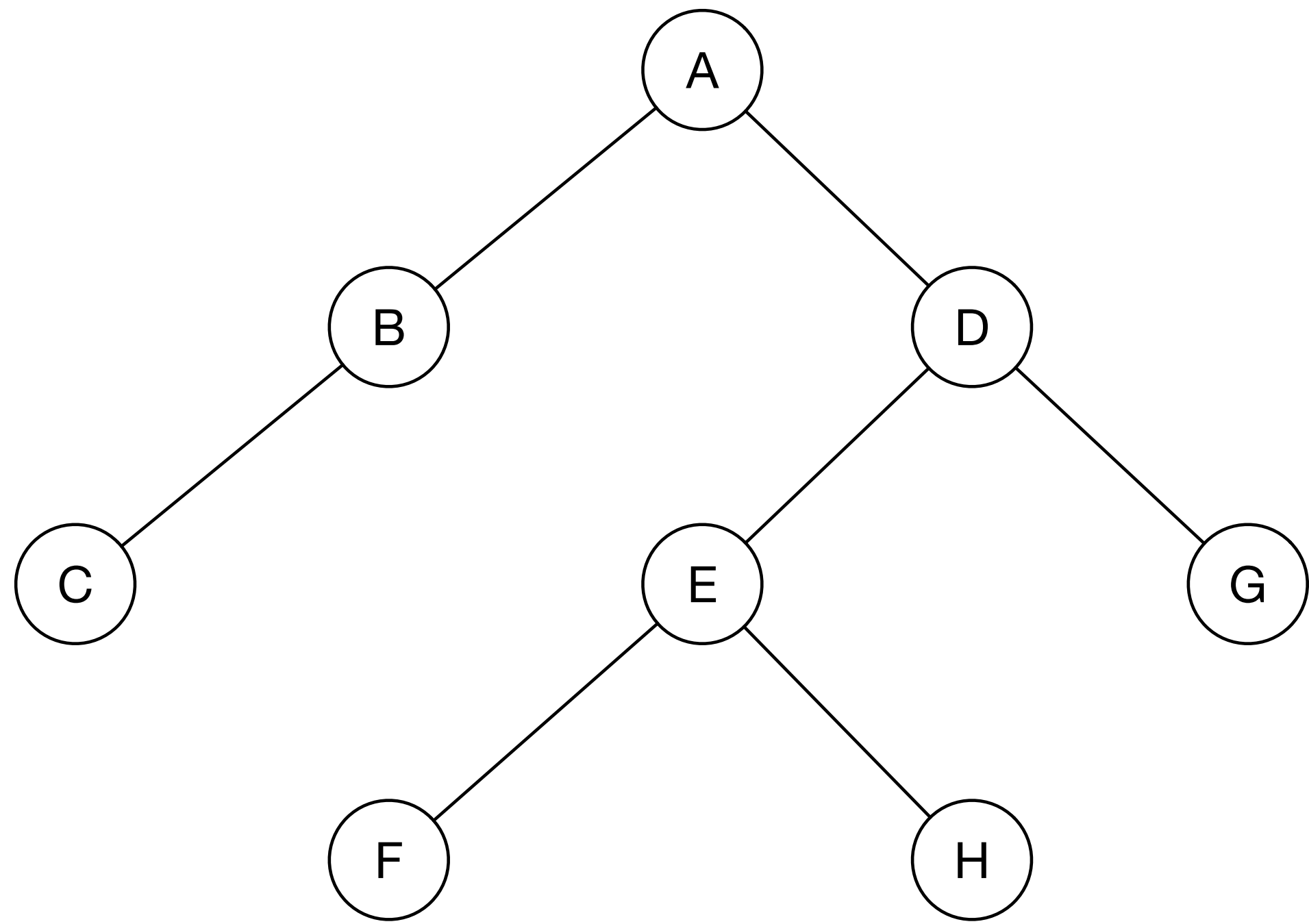
# Adjacency list



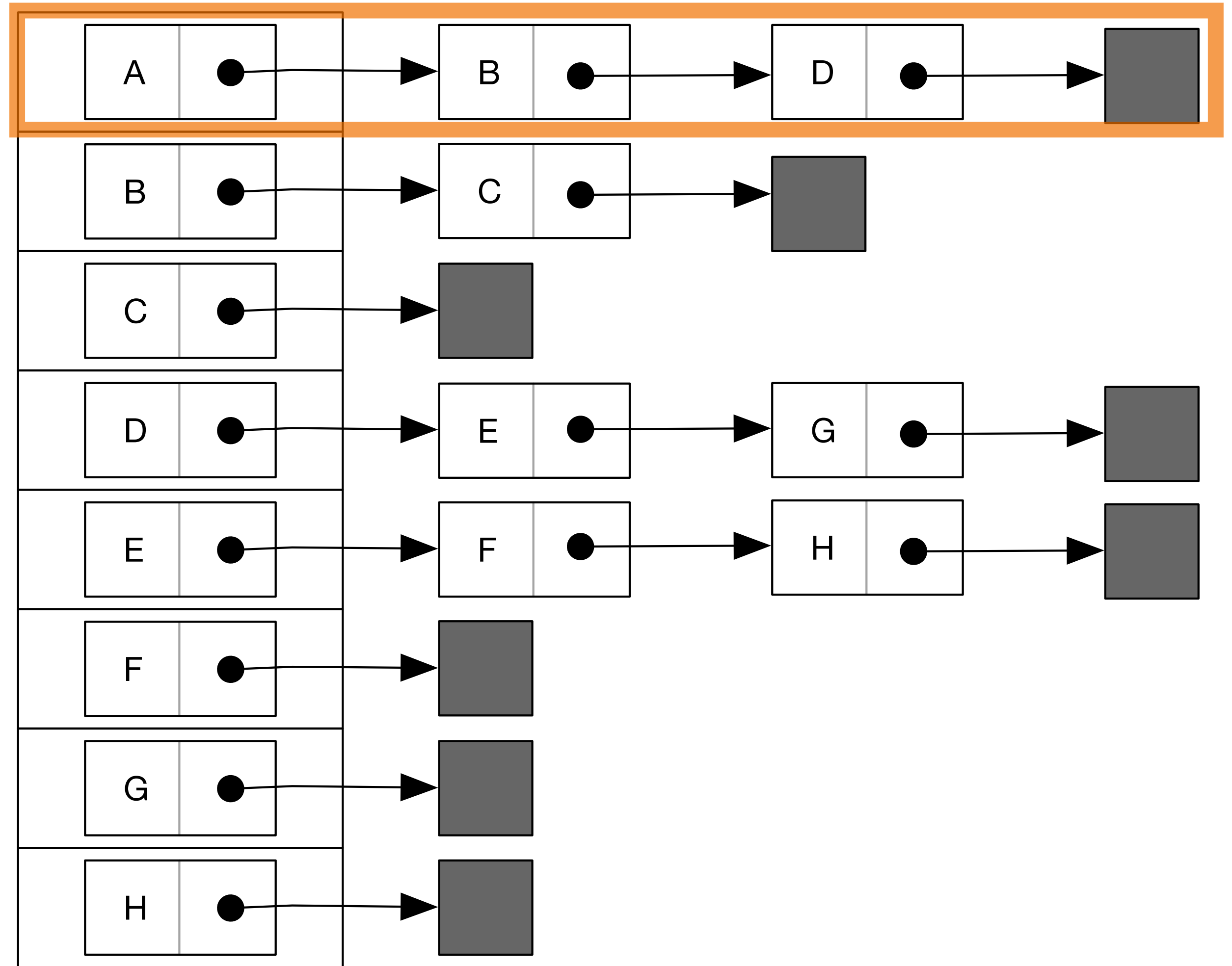
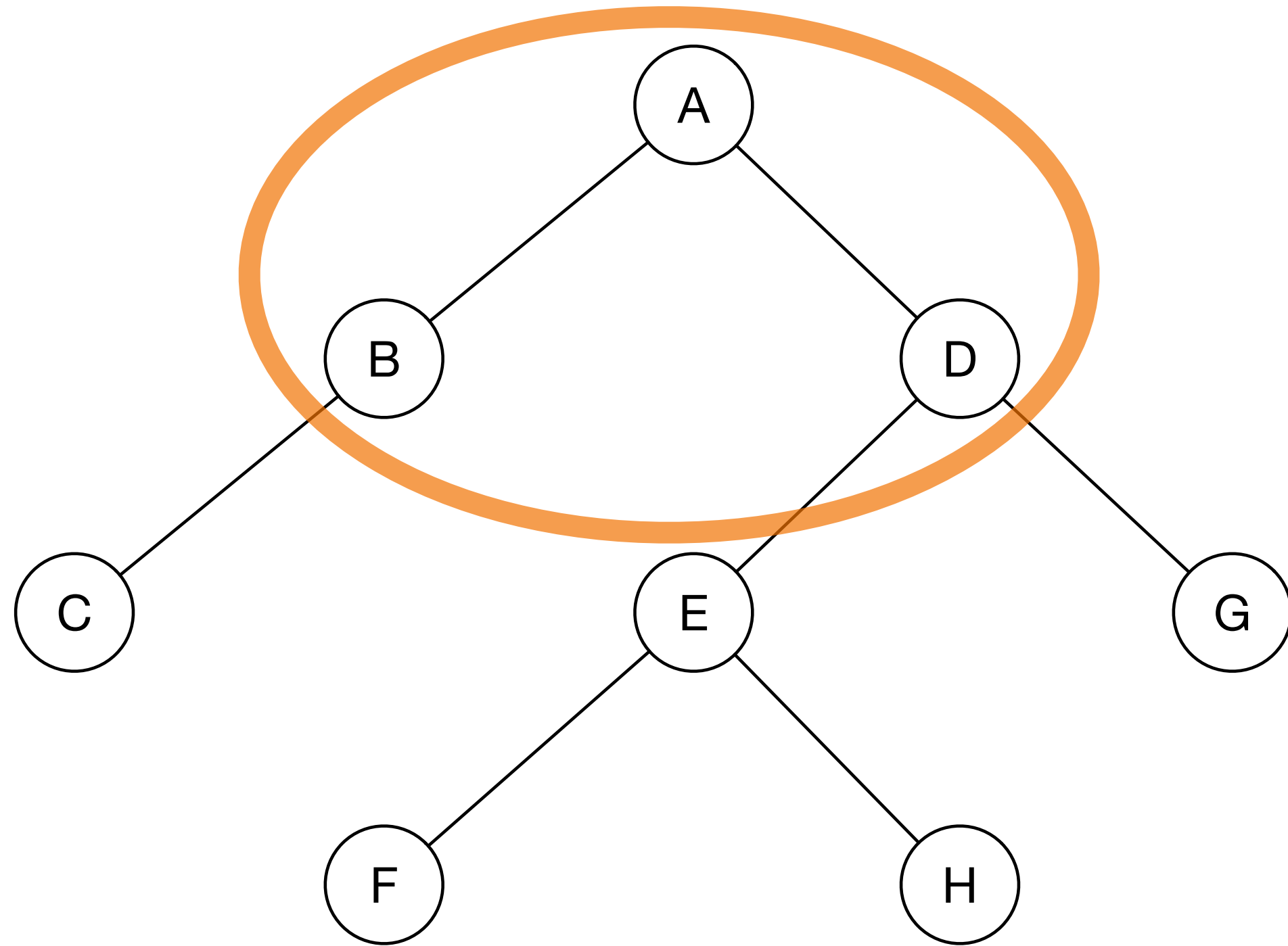
# Adjacency list



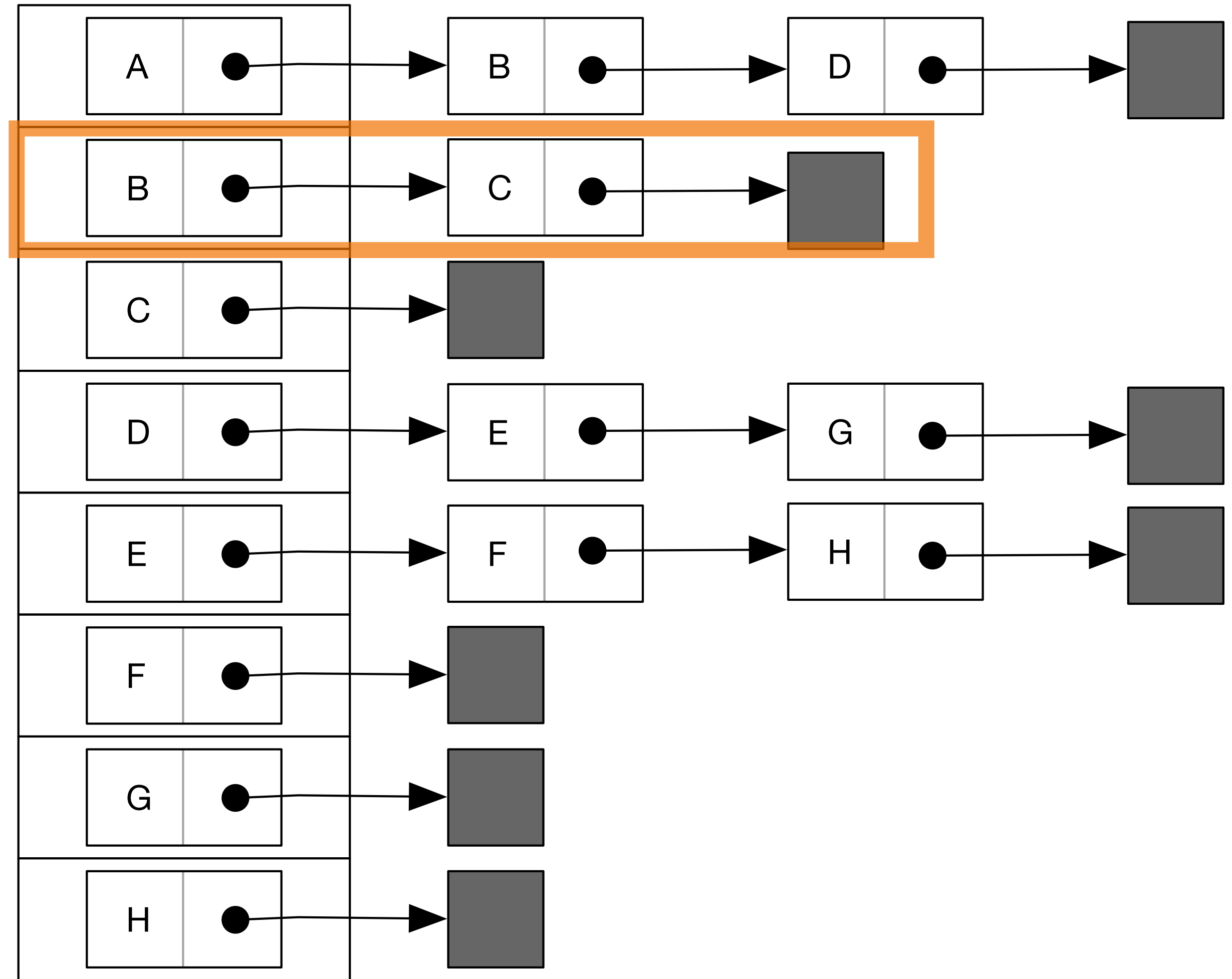
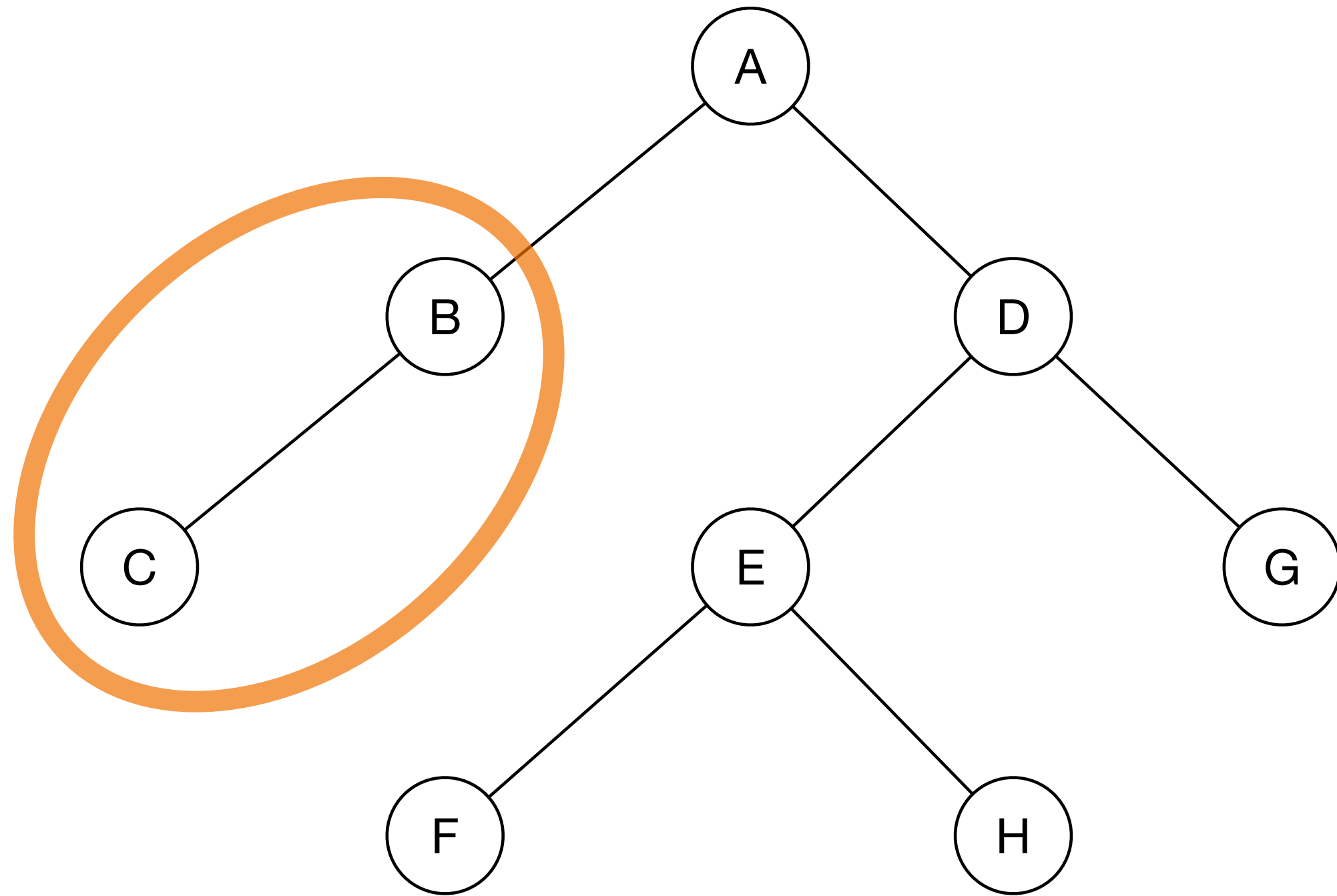
# Adjacency list



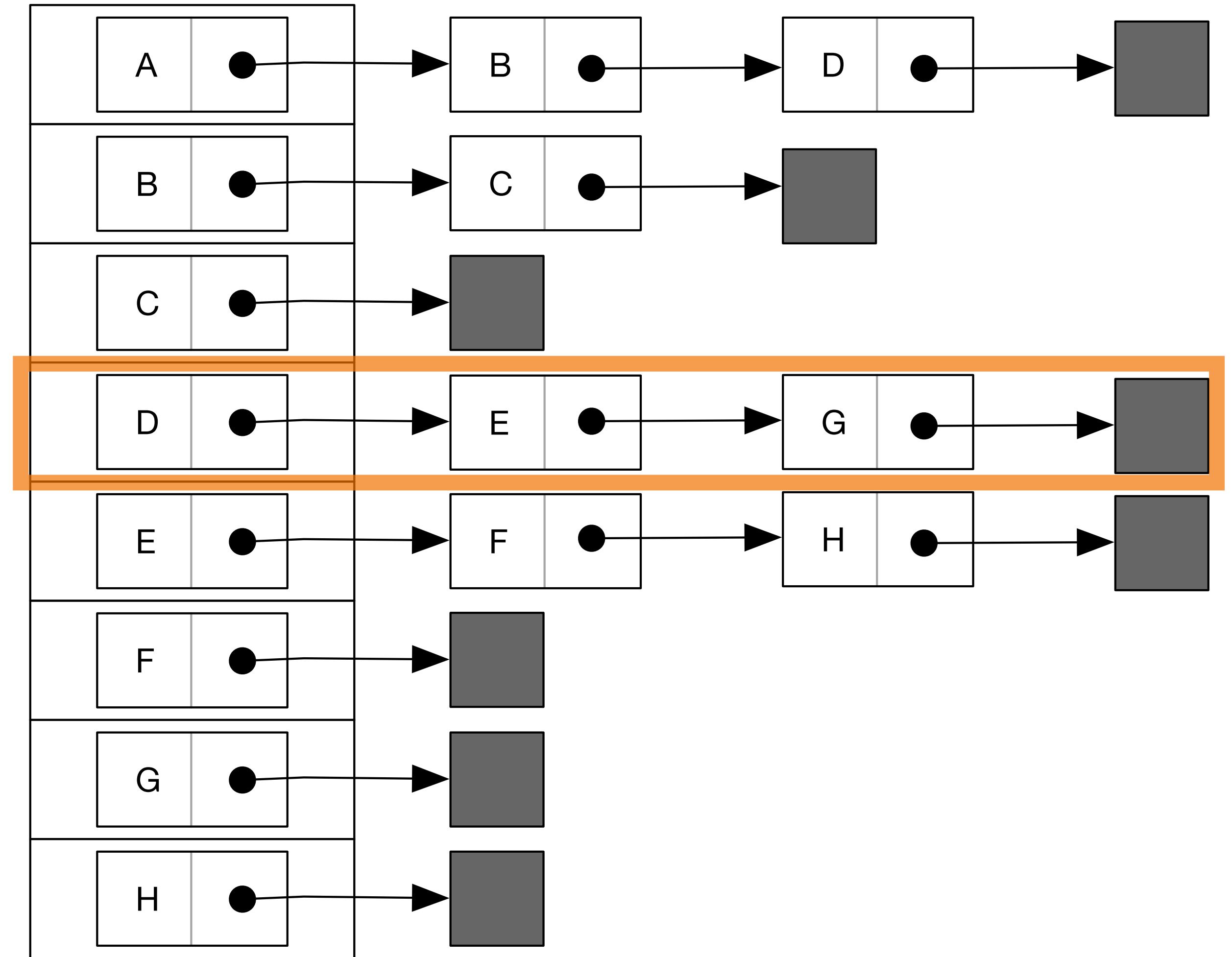
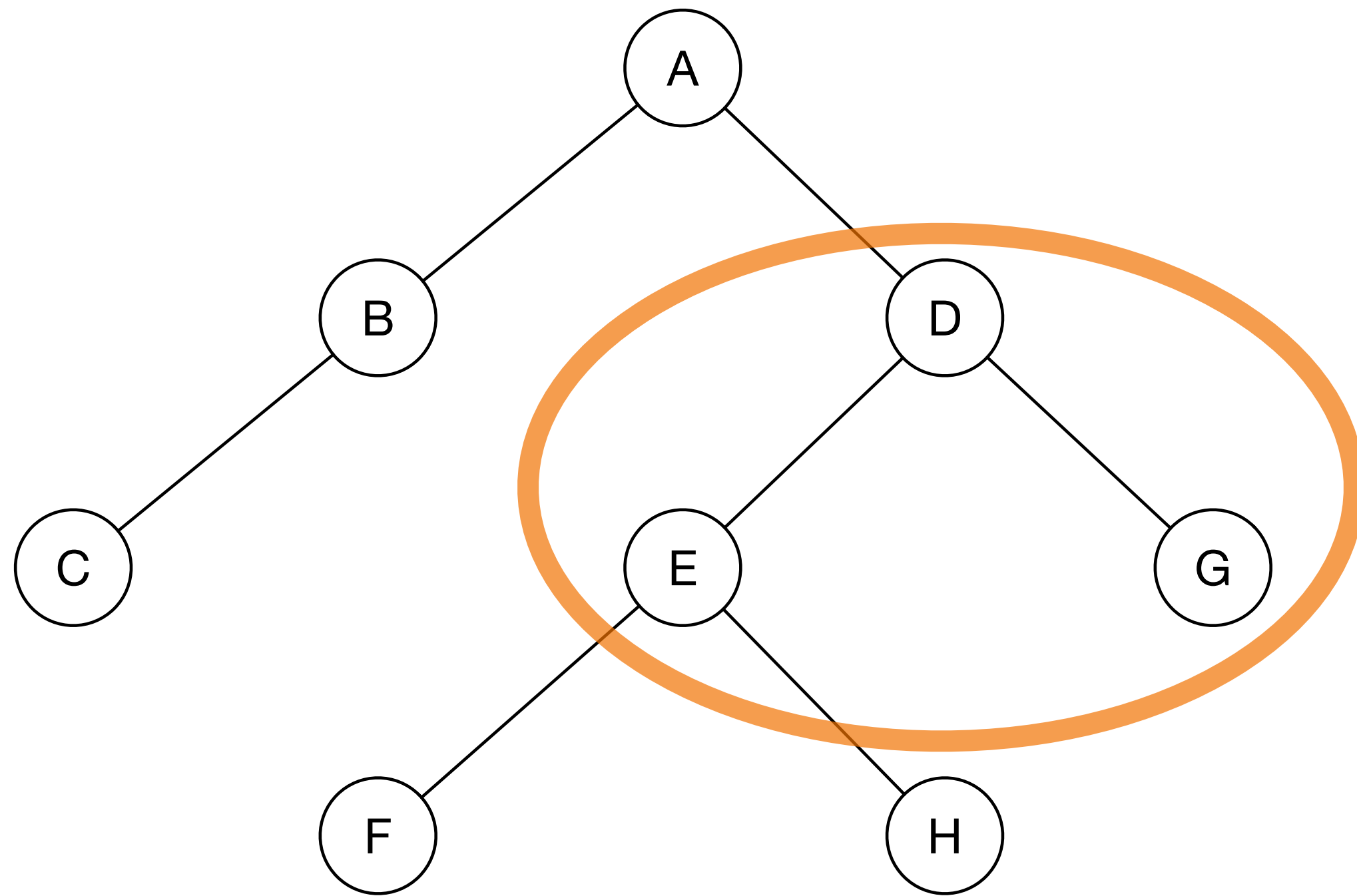
# Adjacency list



# Adjacency list

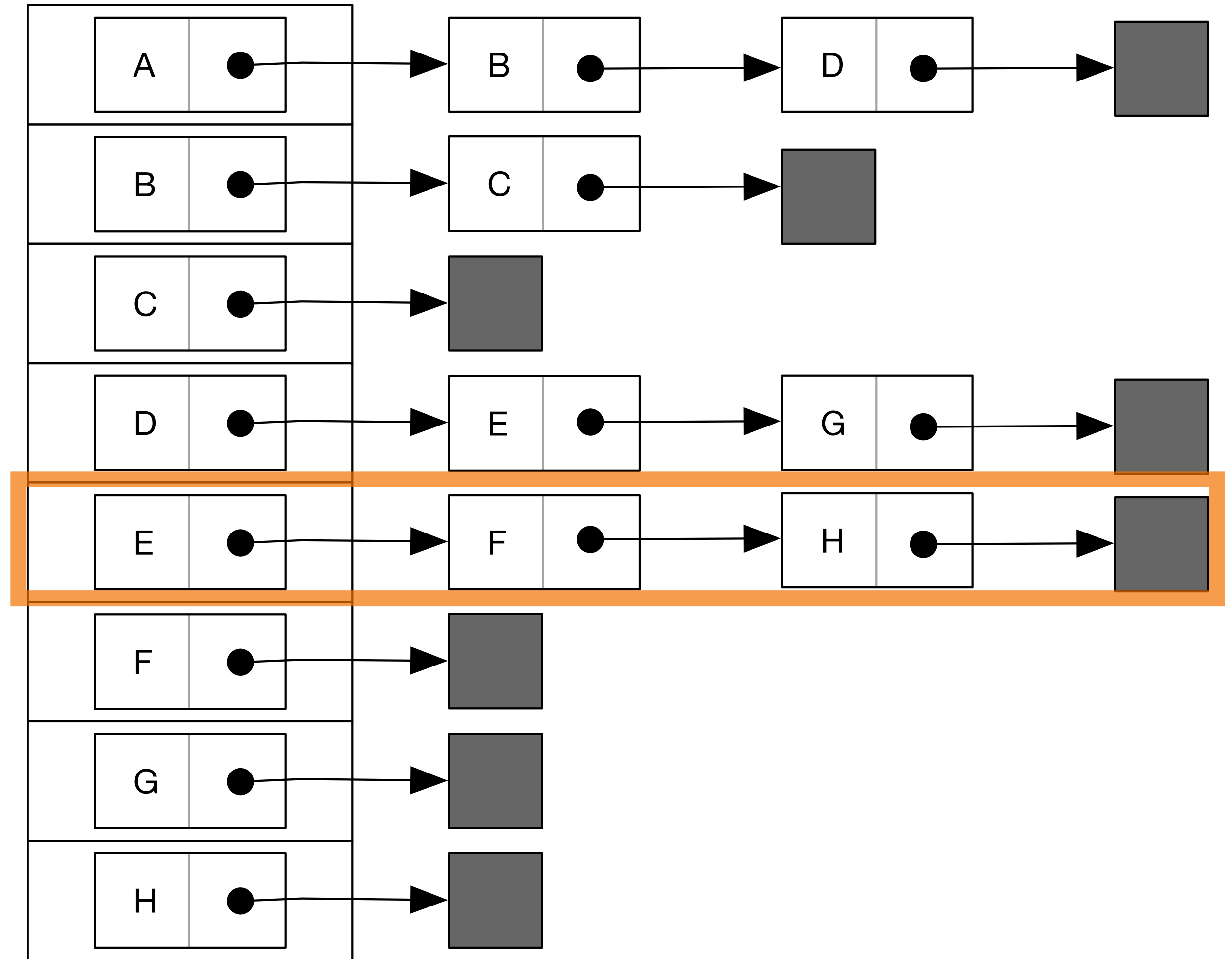
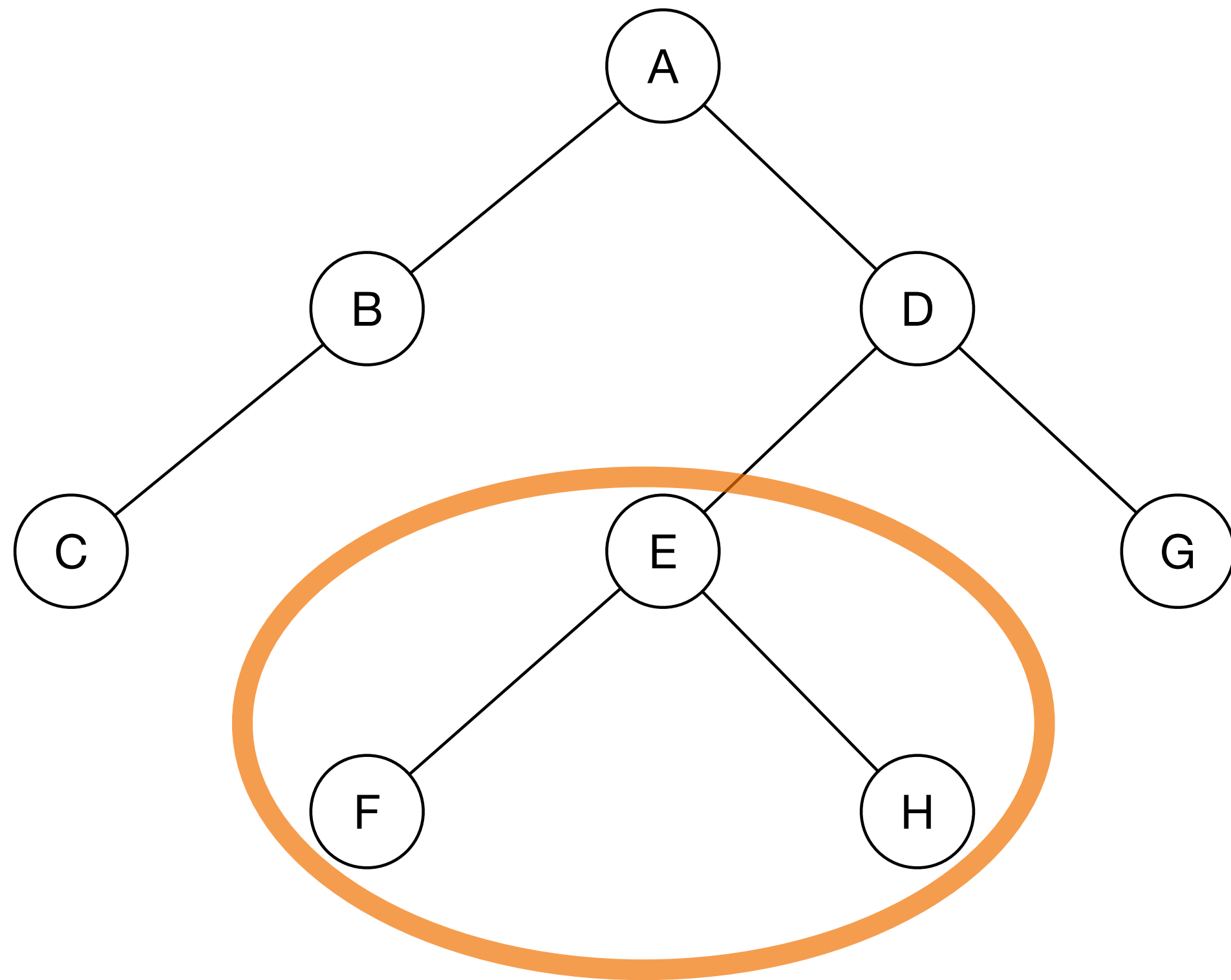


# Adjacency list

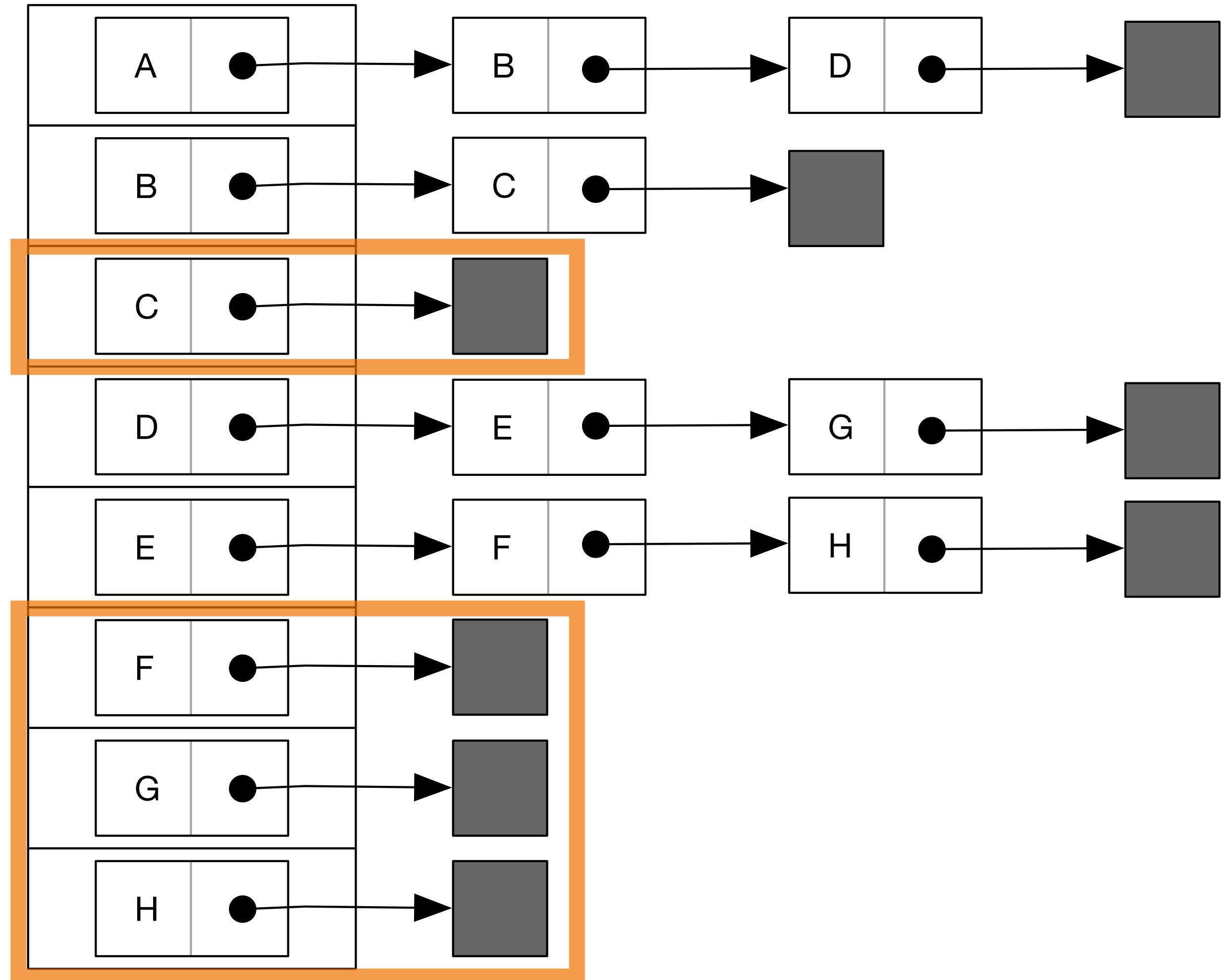
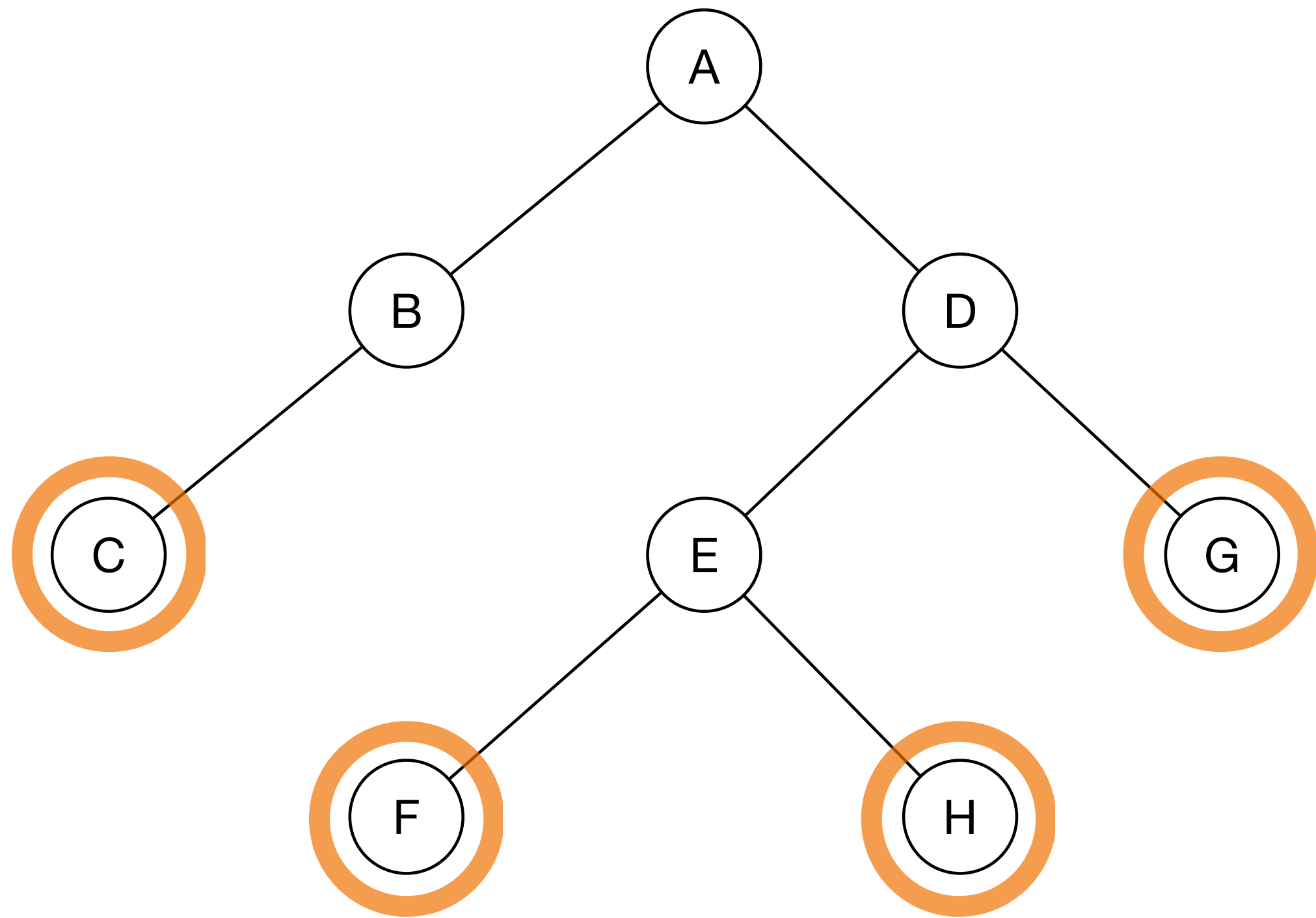




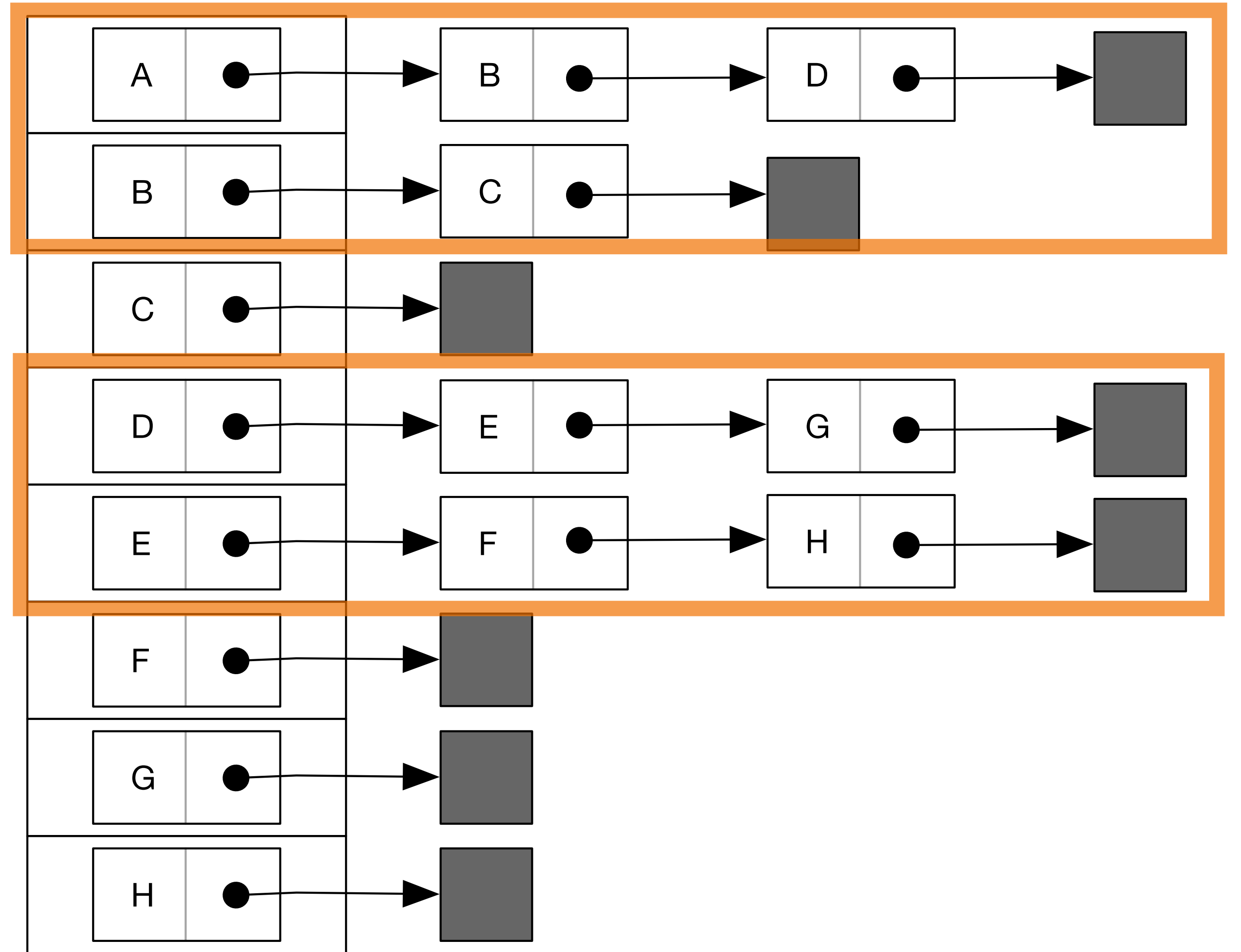
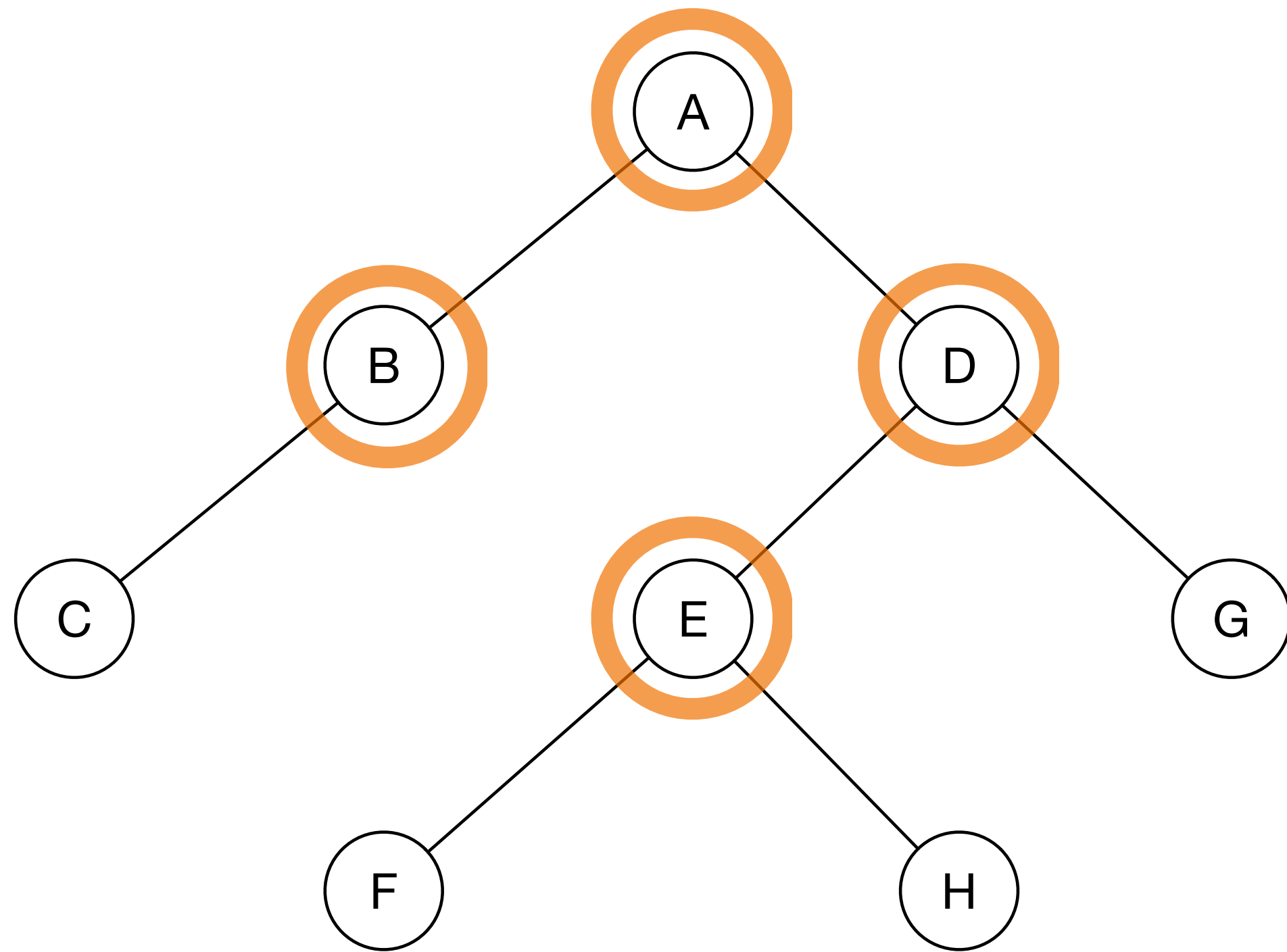
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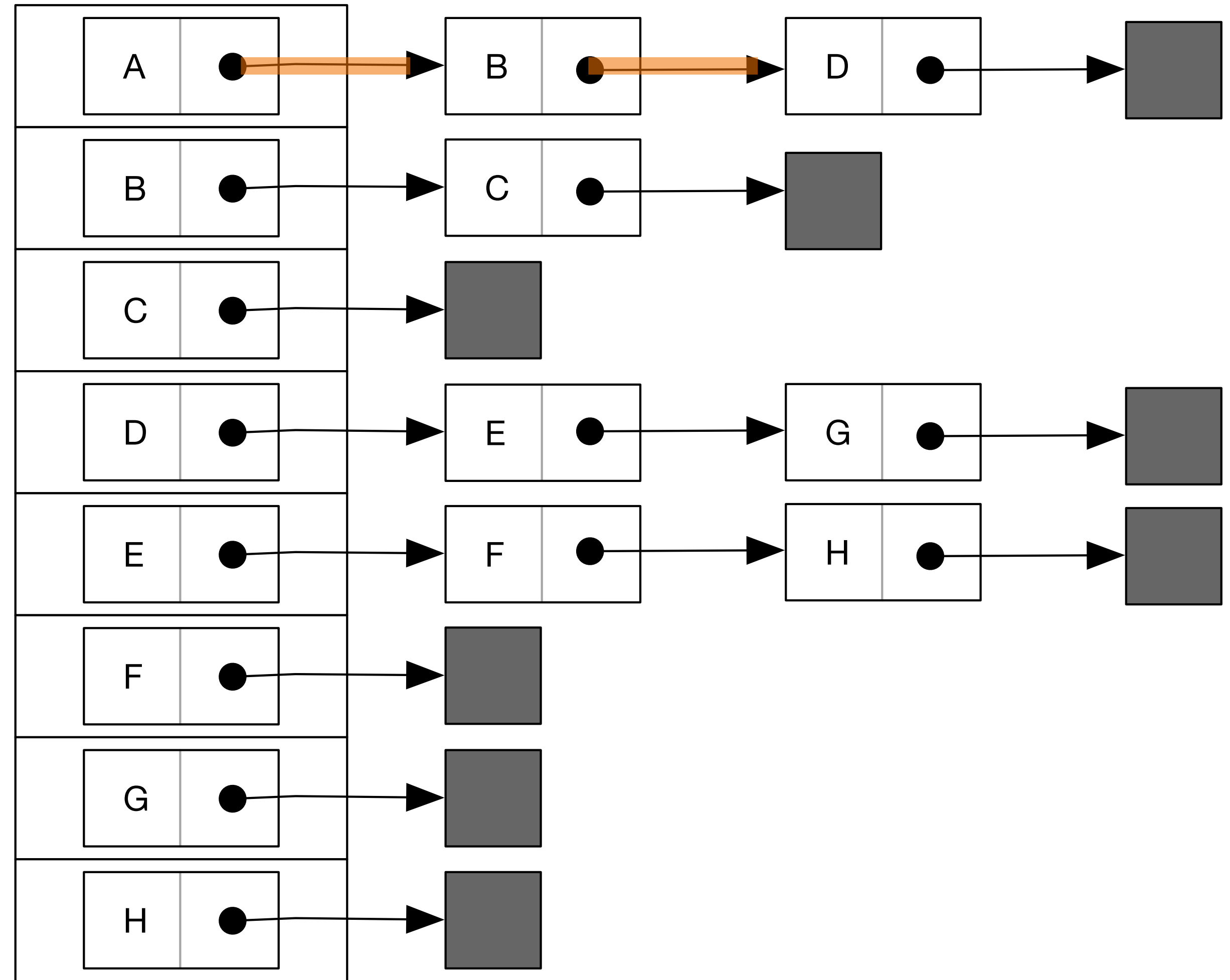
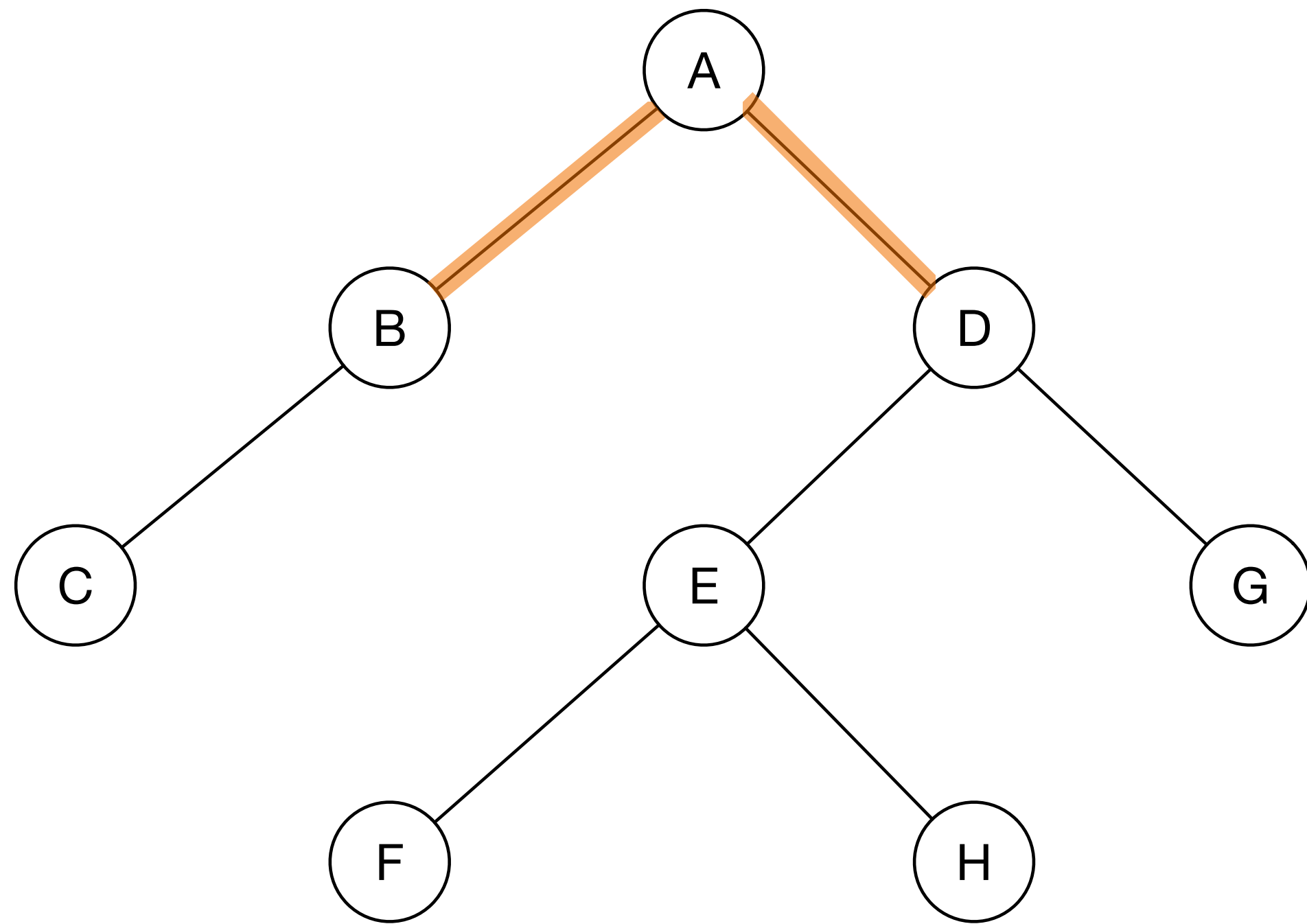
# Adjacency list



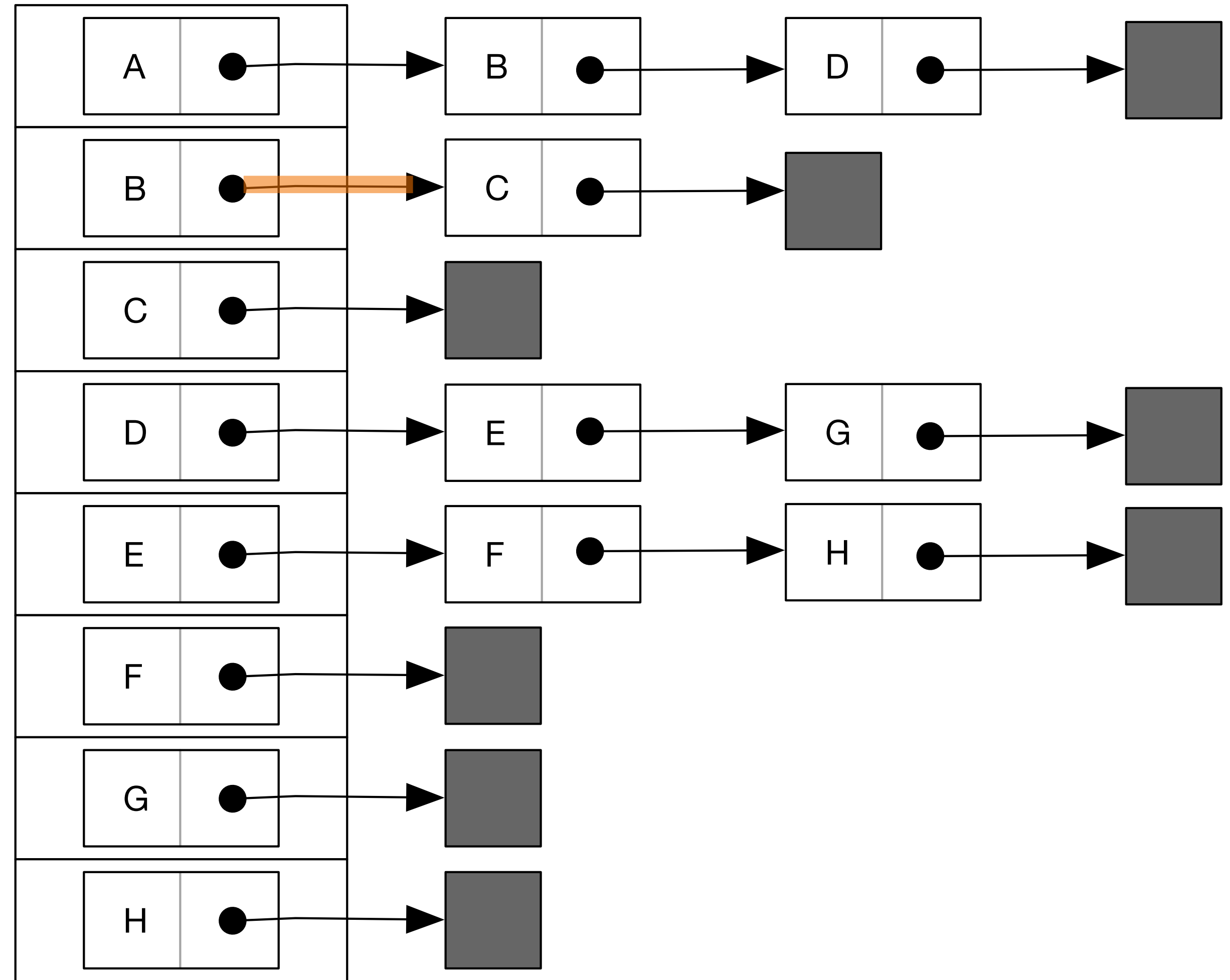
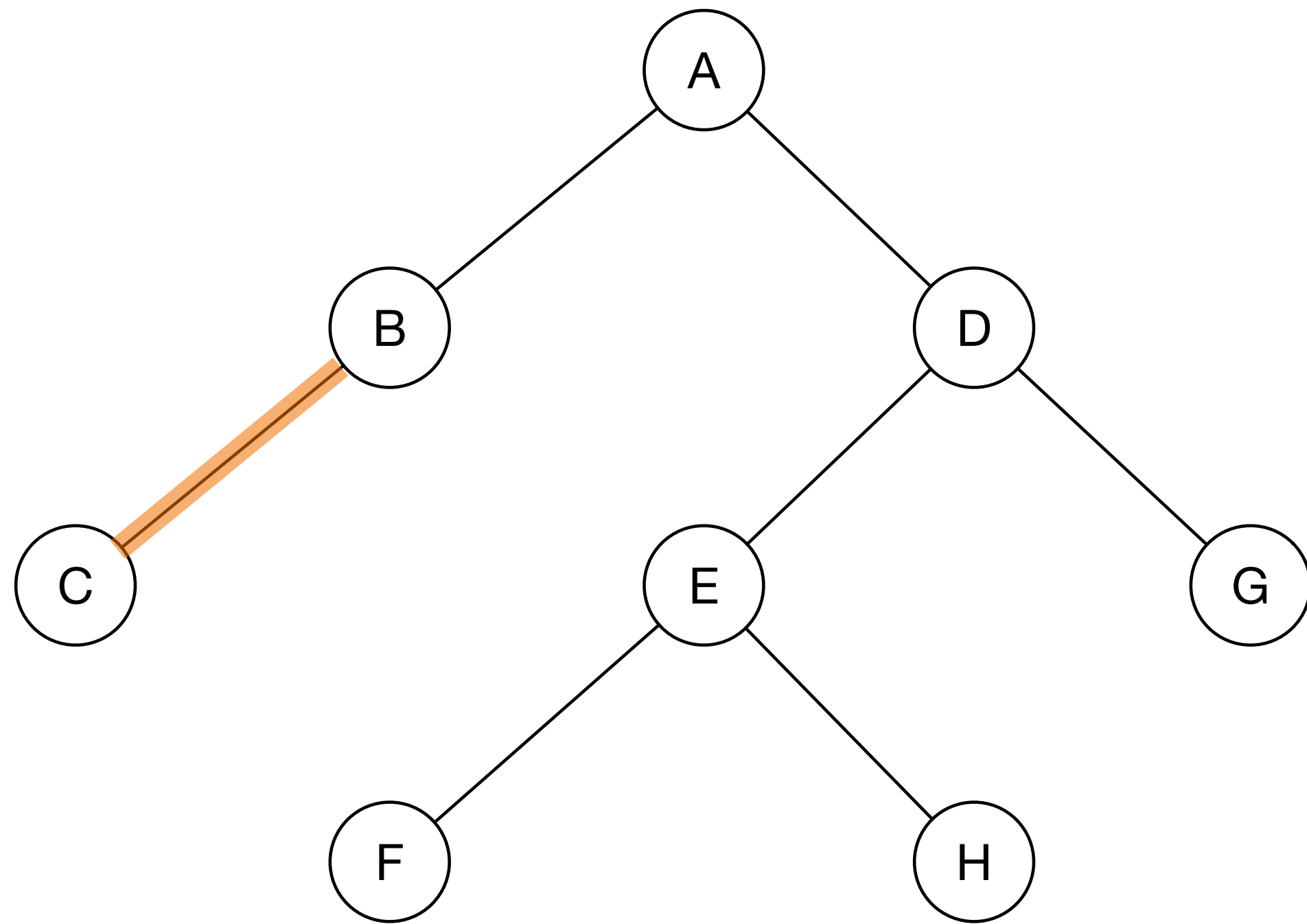
# Adjacency list



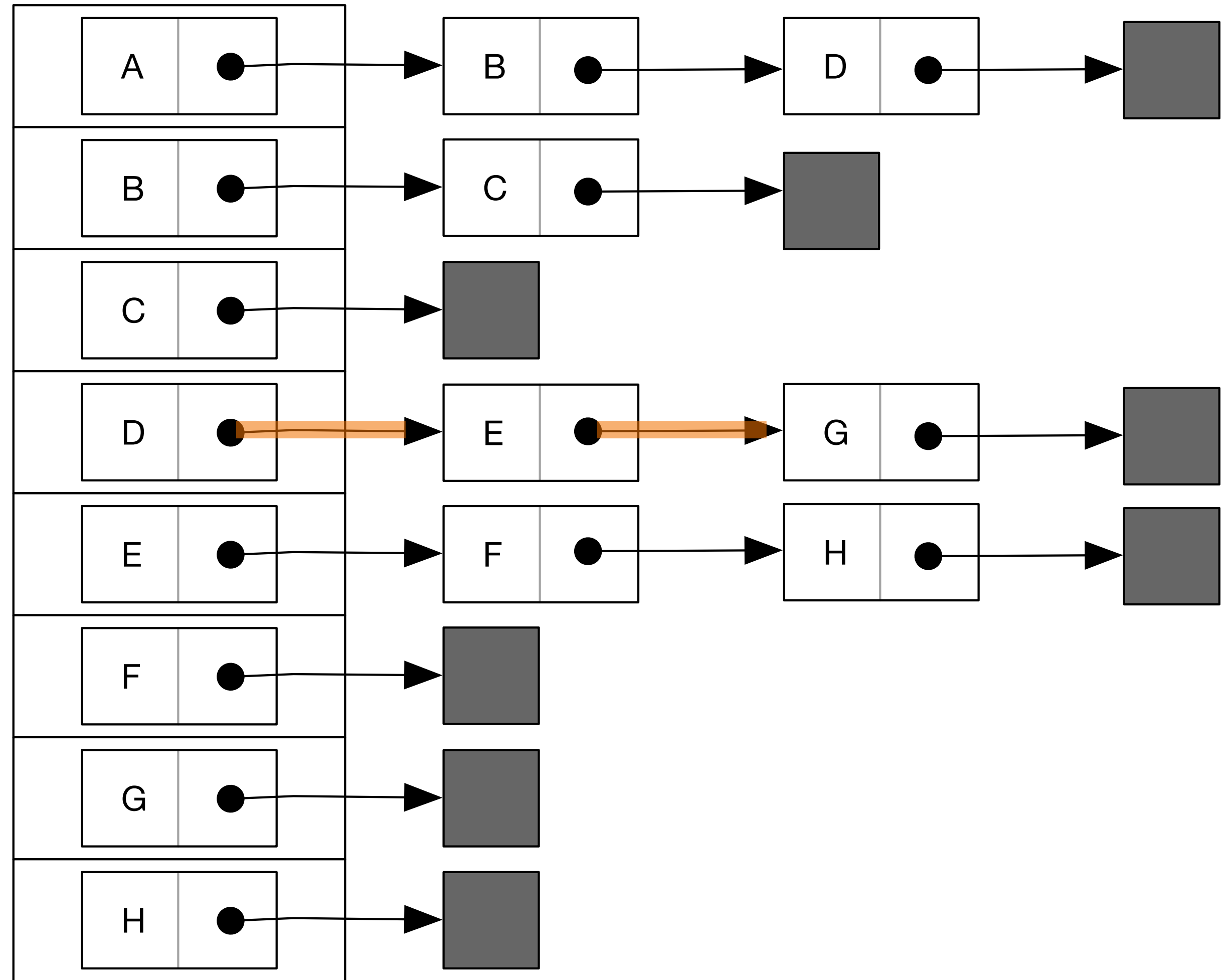
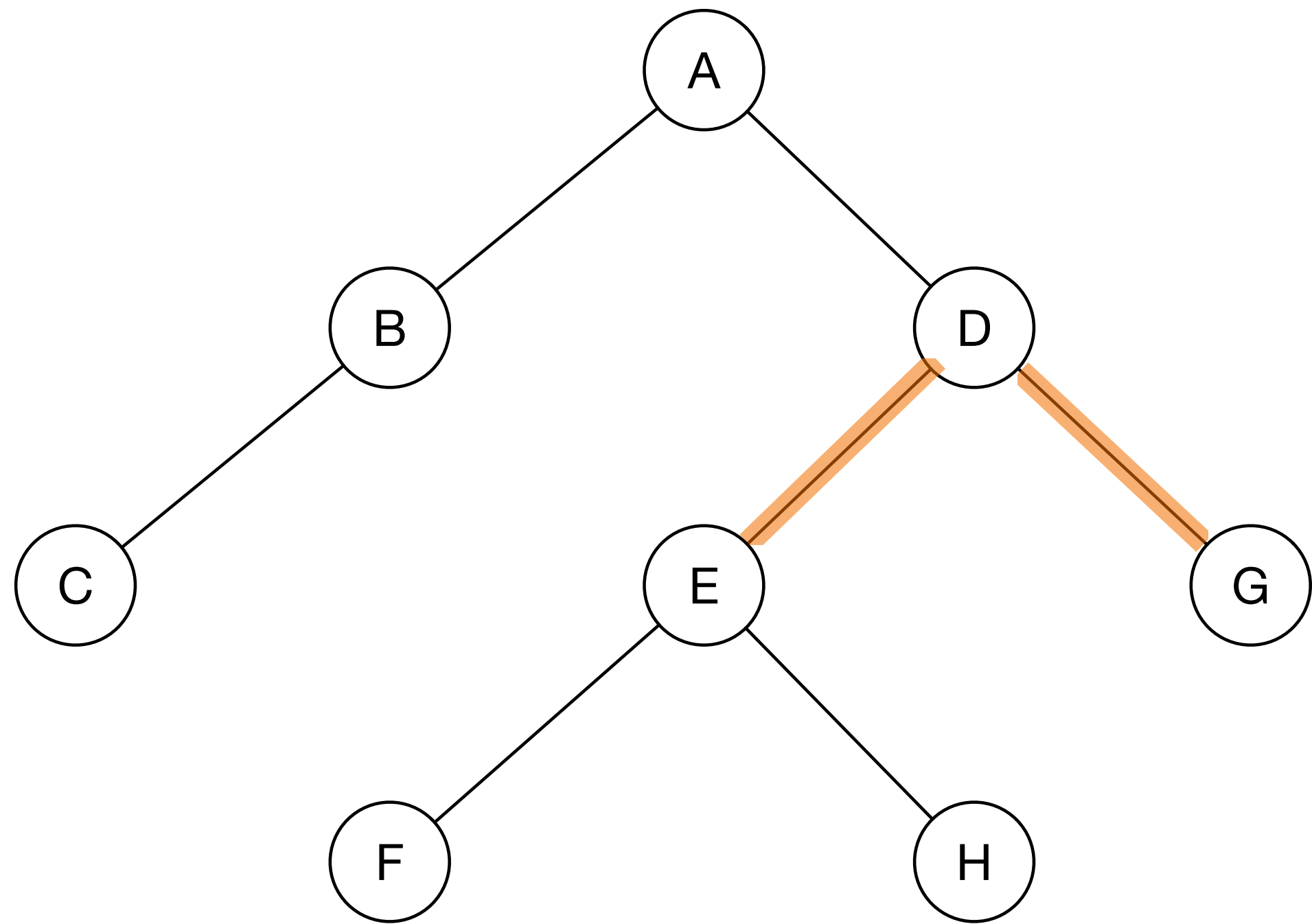
# Adjacency list



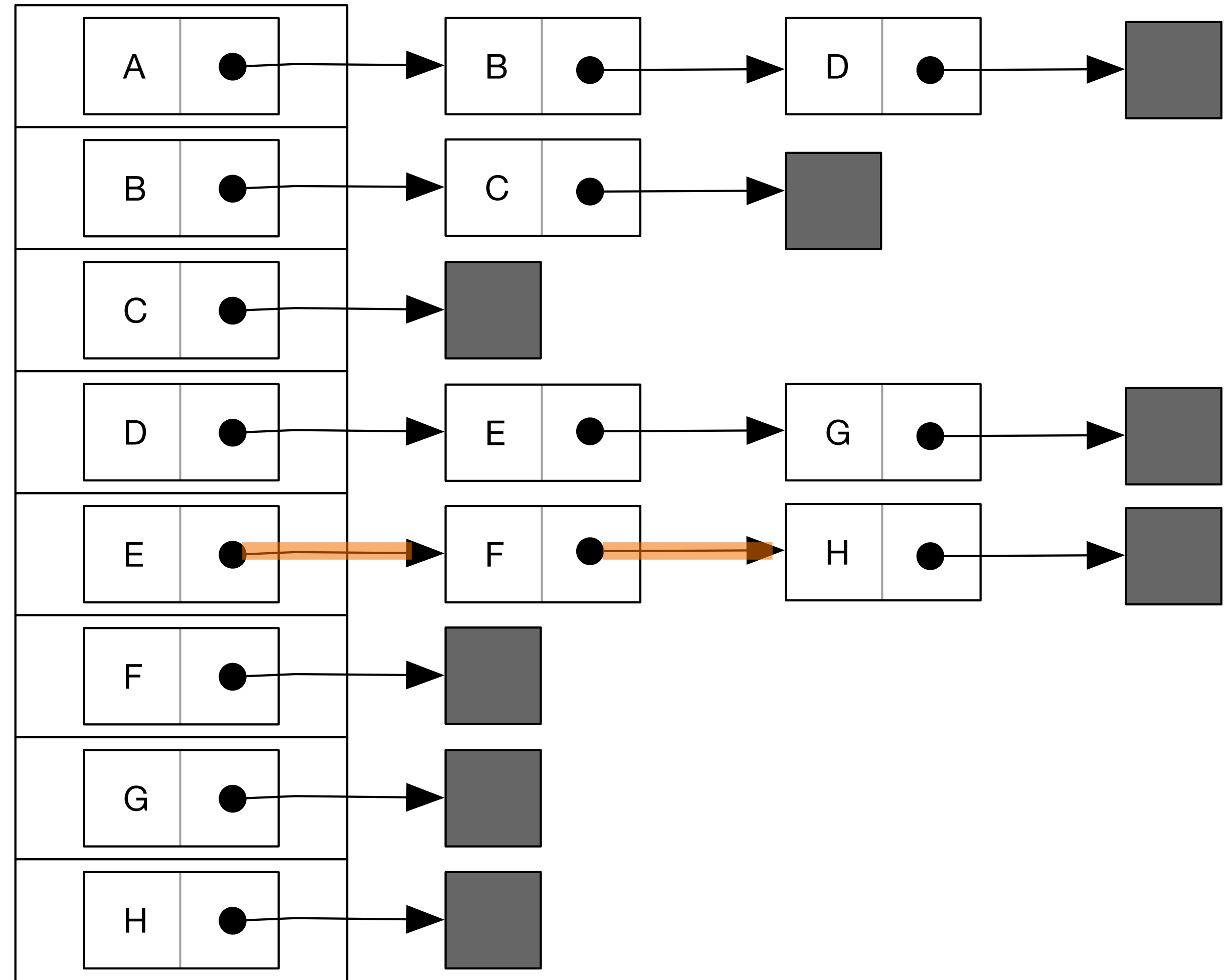
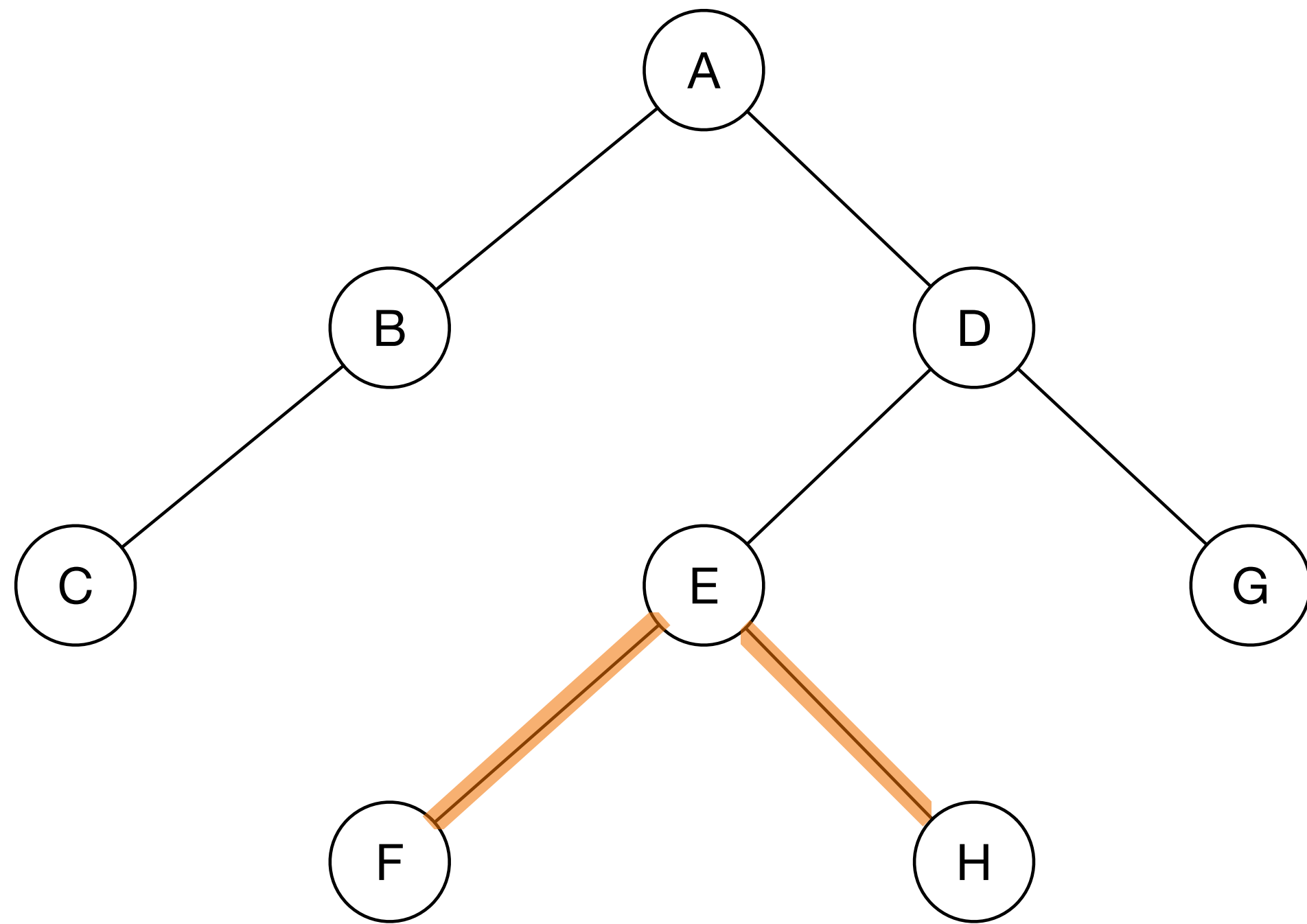
# Adjacency list



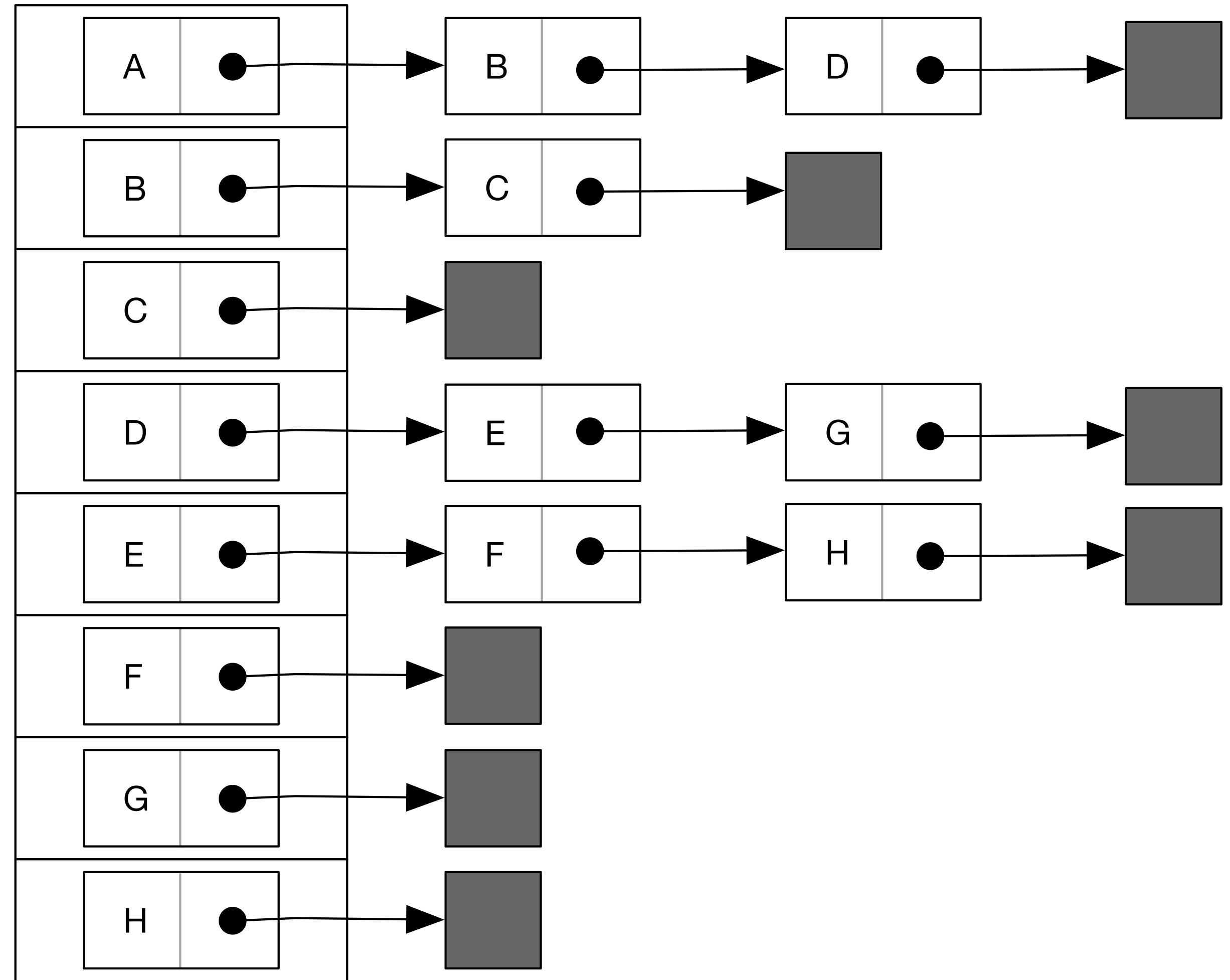
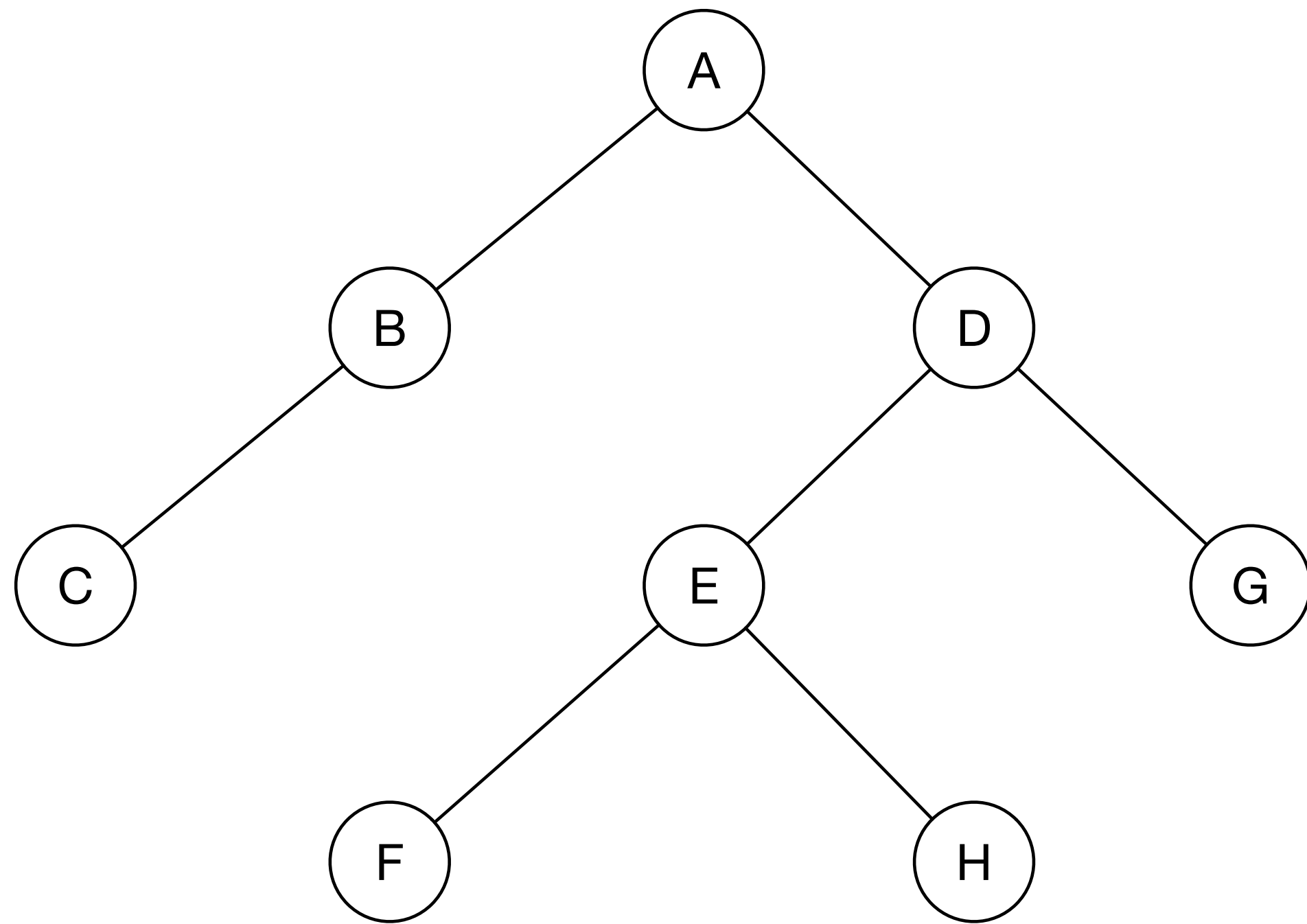
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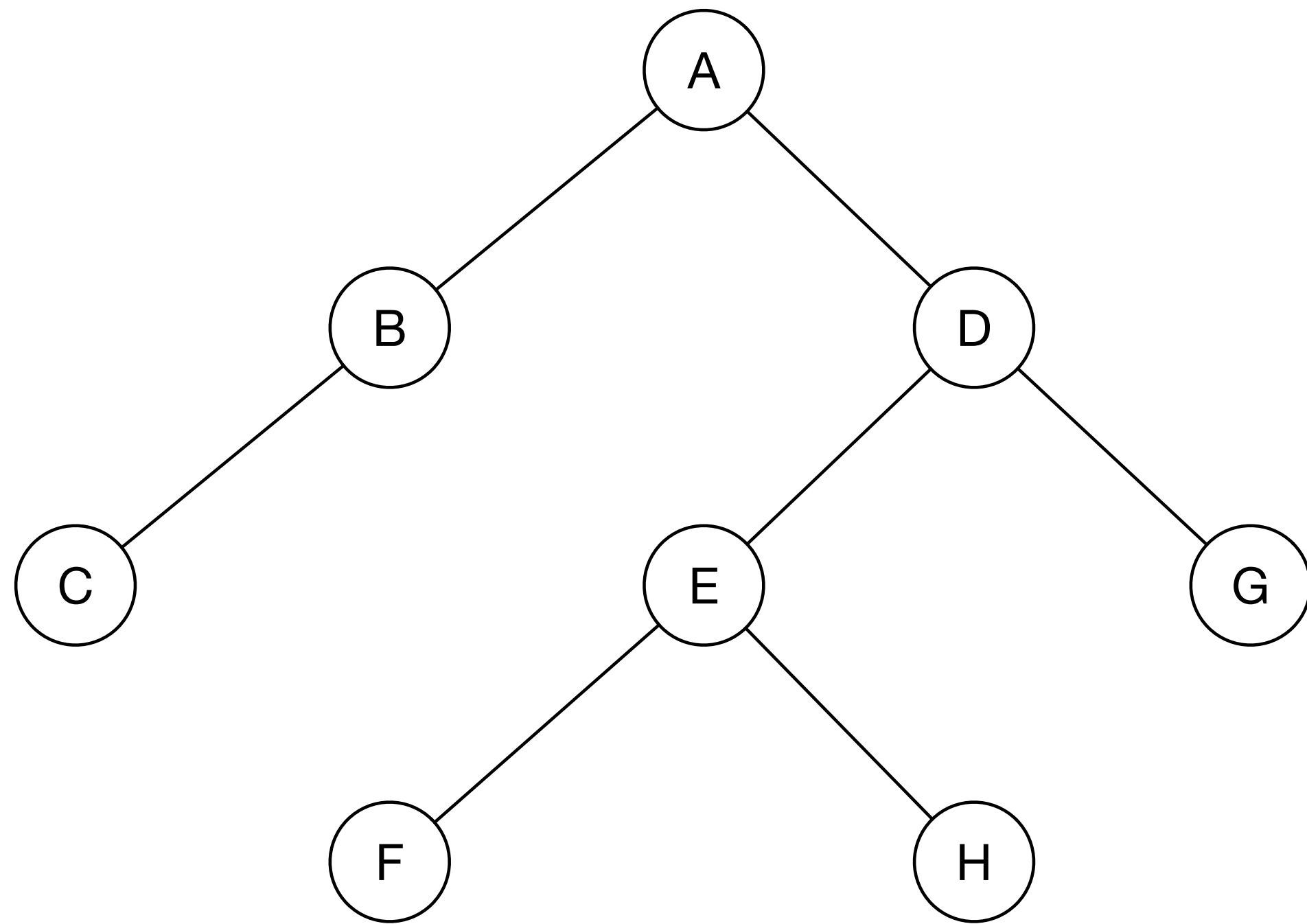


# Adjacency list



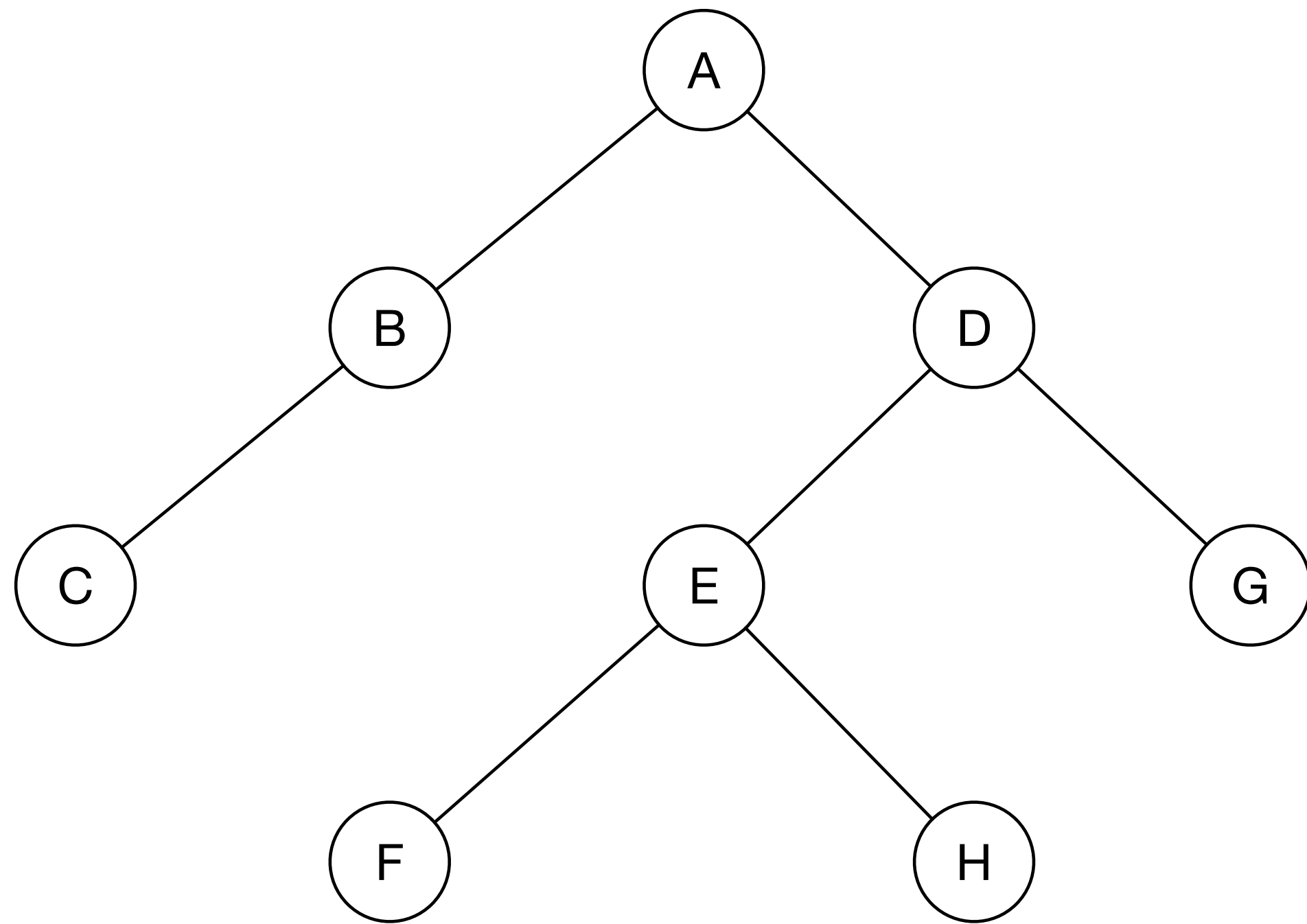


# Adjacency matrix



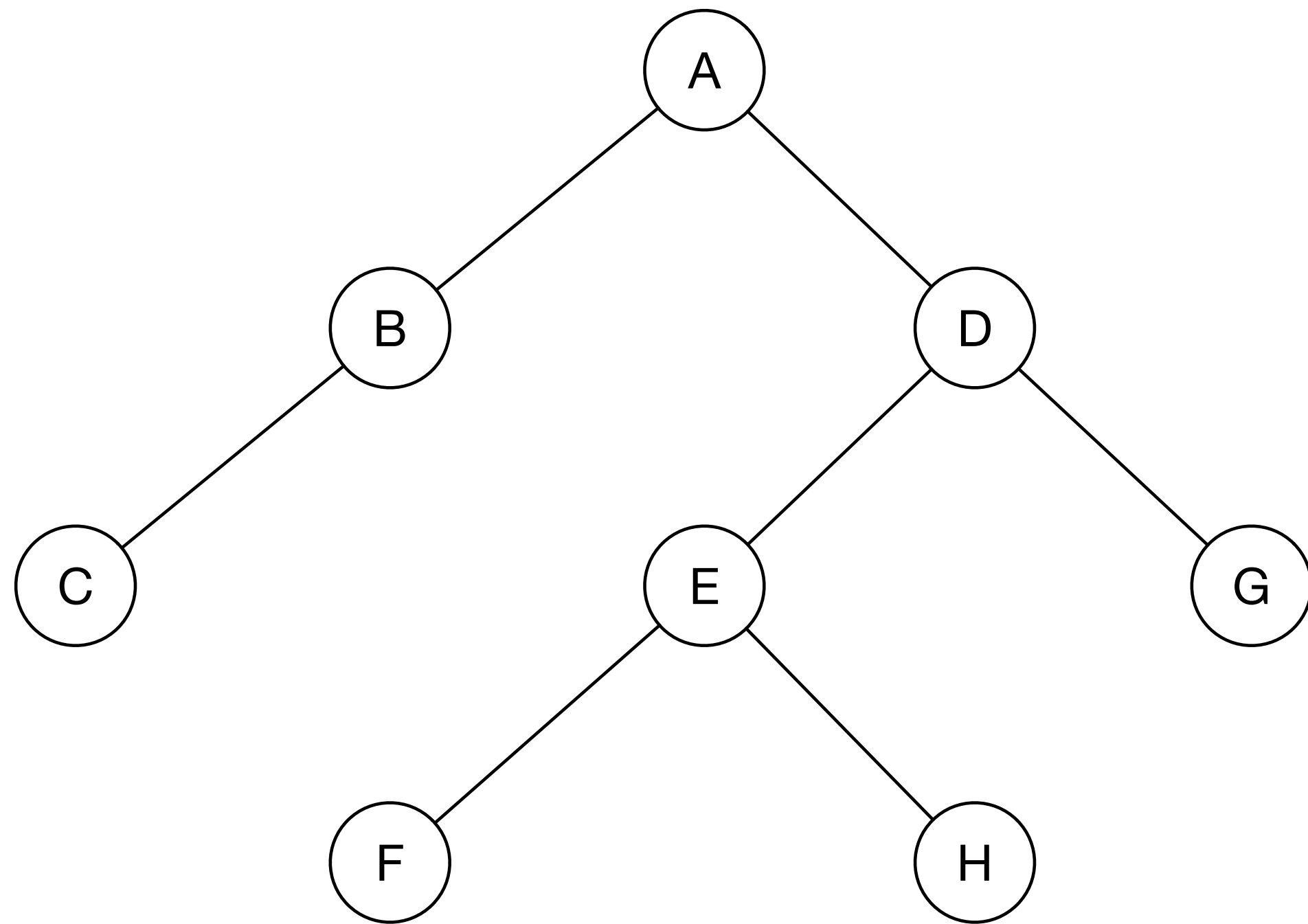
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B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency matrix



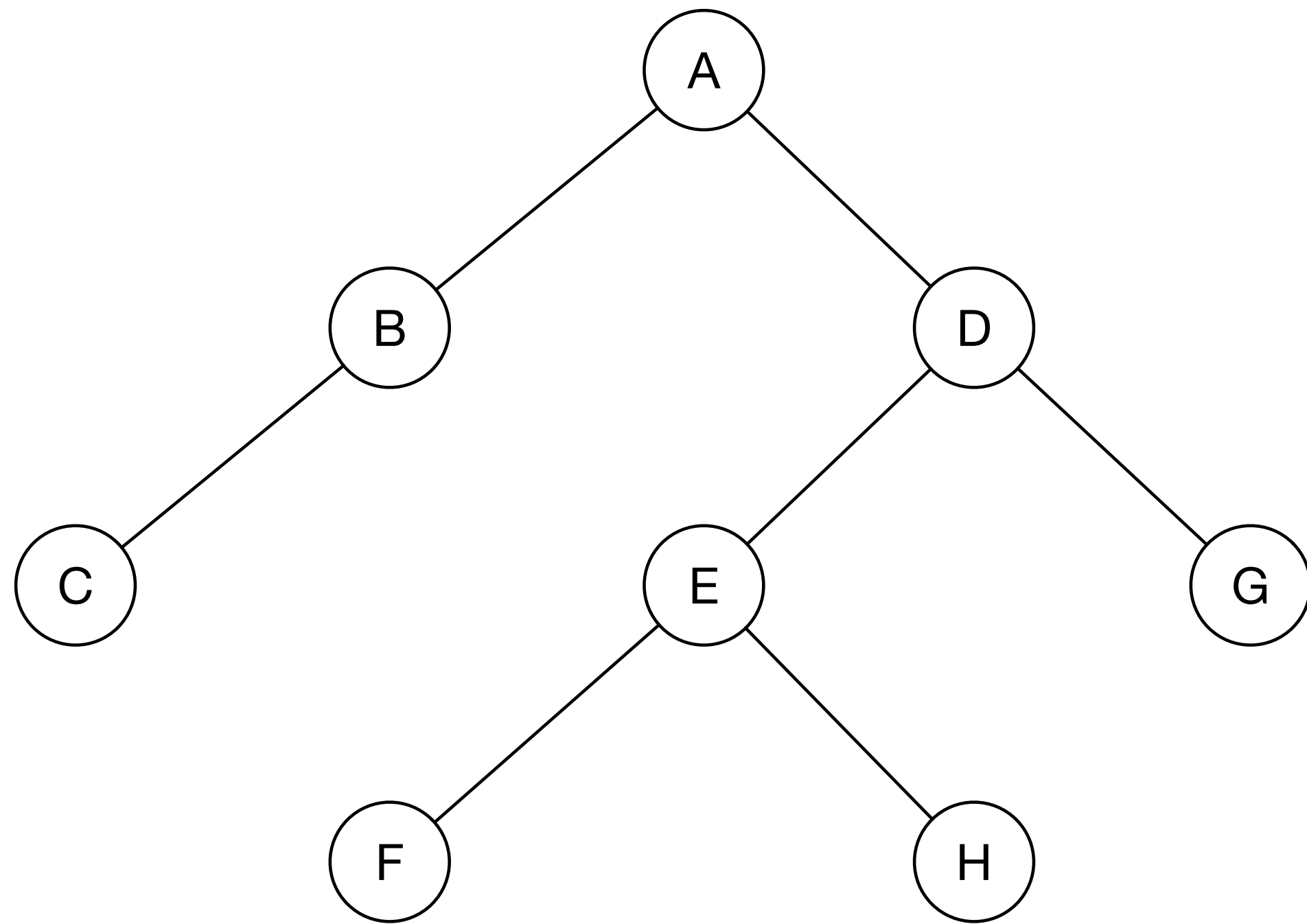
	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency matrix



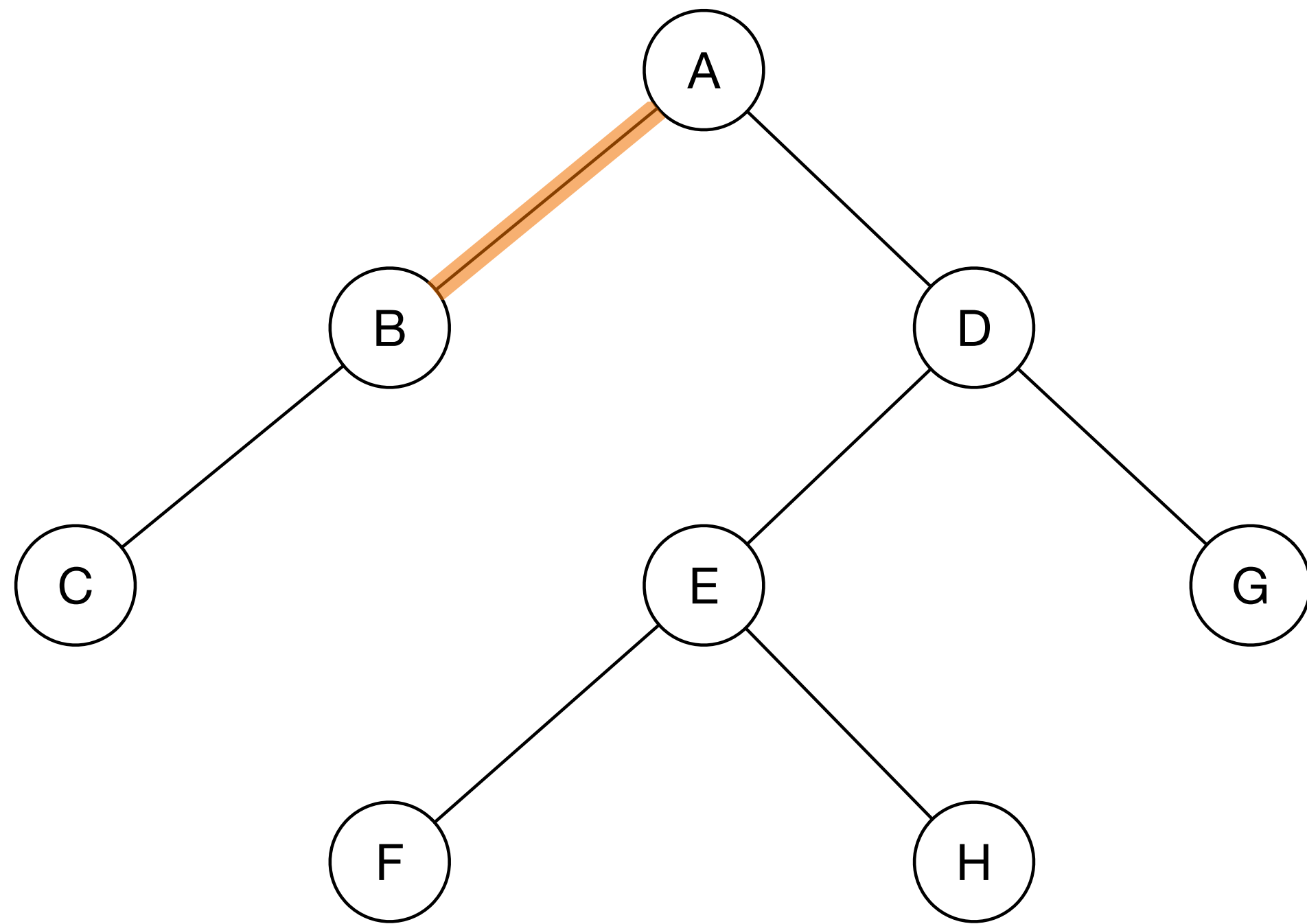
	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency matrix



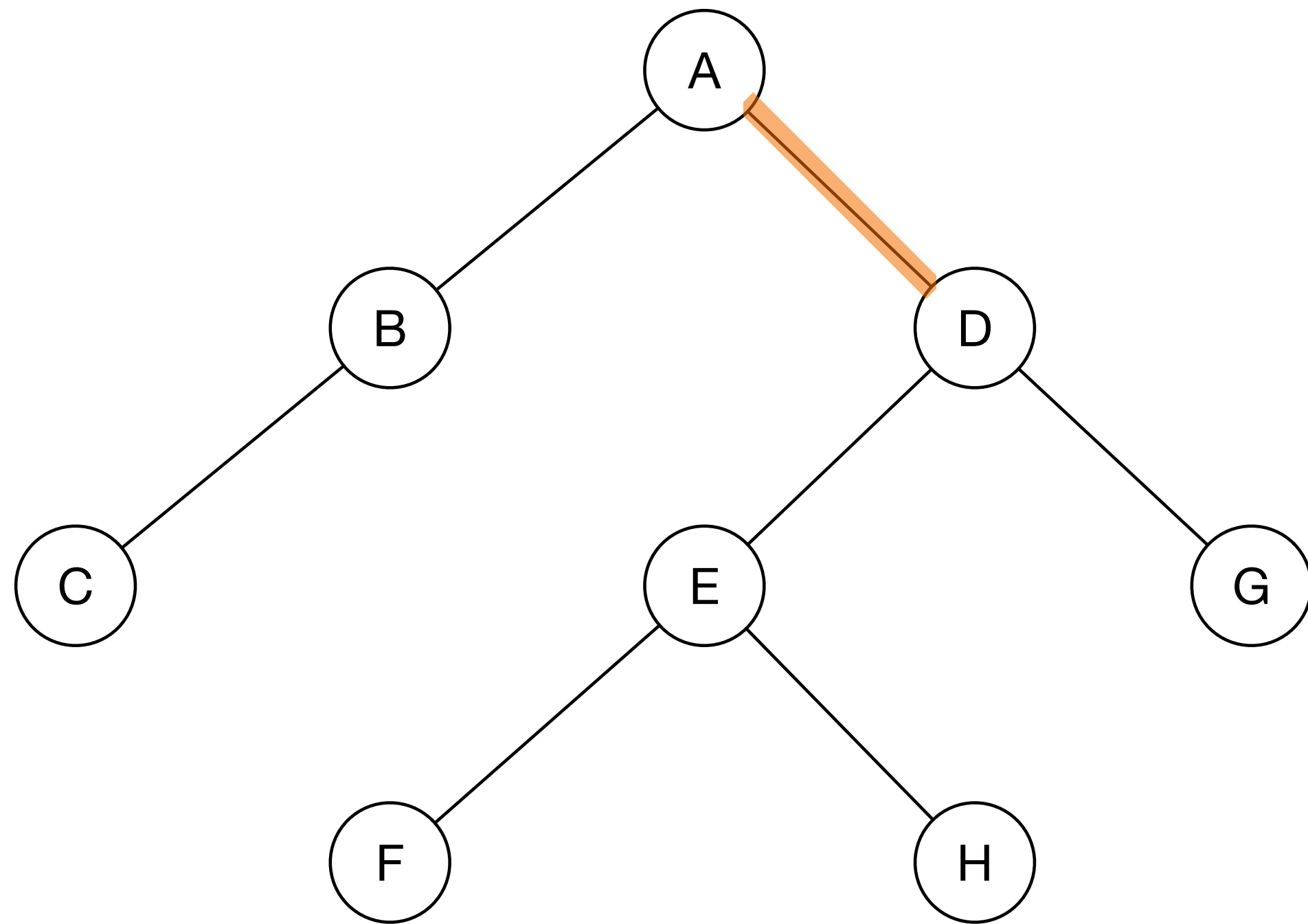
	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency matrix



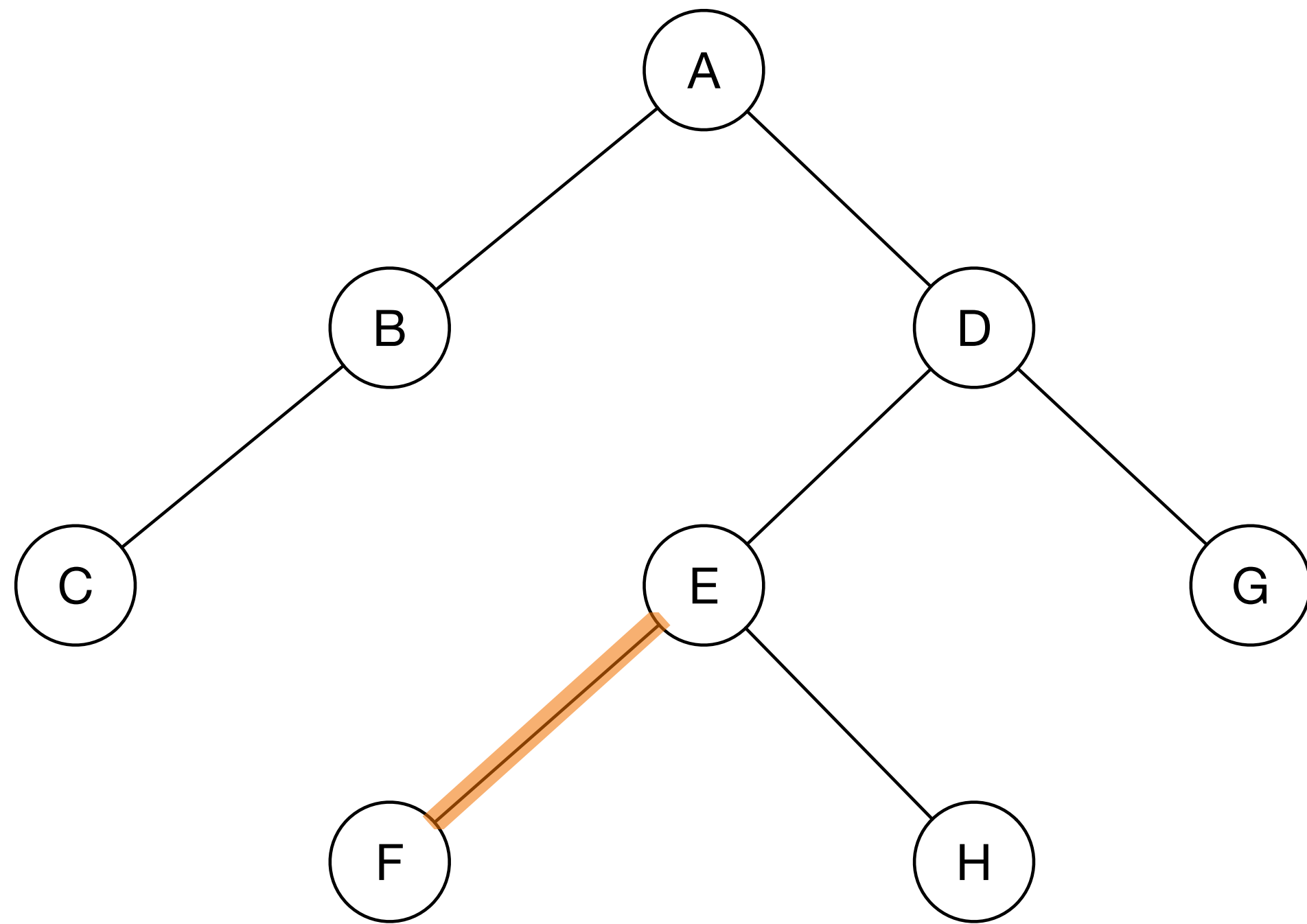
	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency matrix



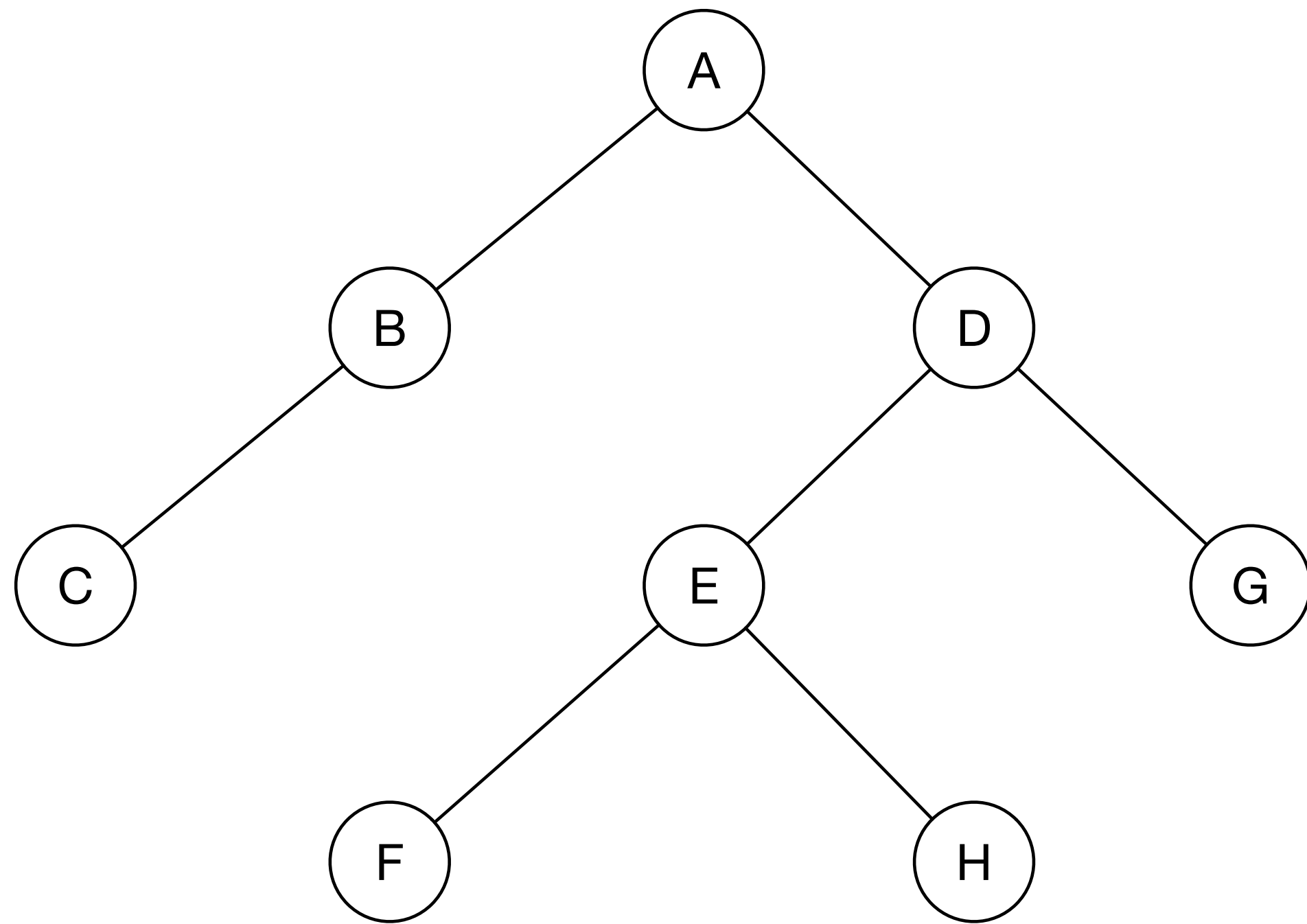
	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency matrix



	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

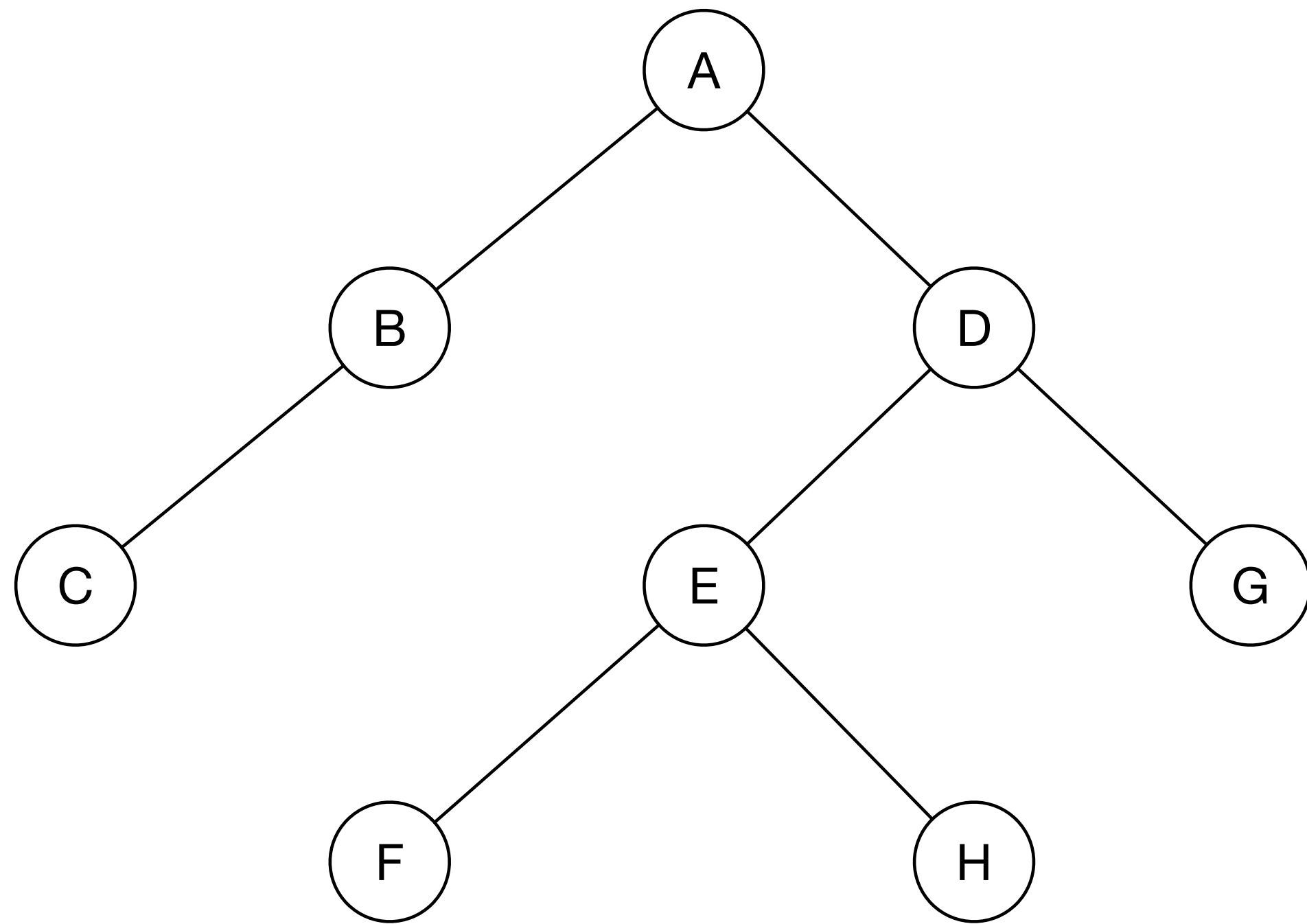
# Adjacency matrix



	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

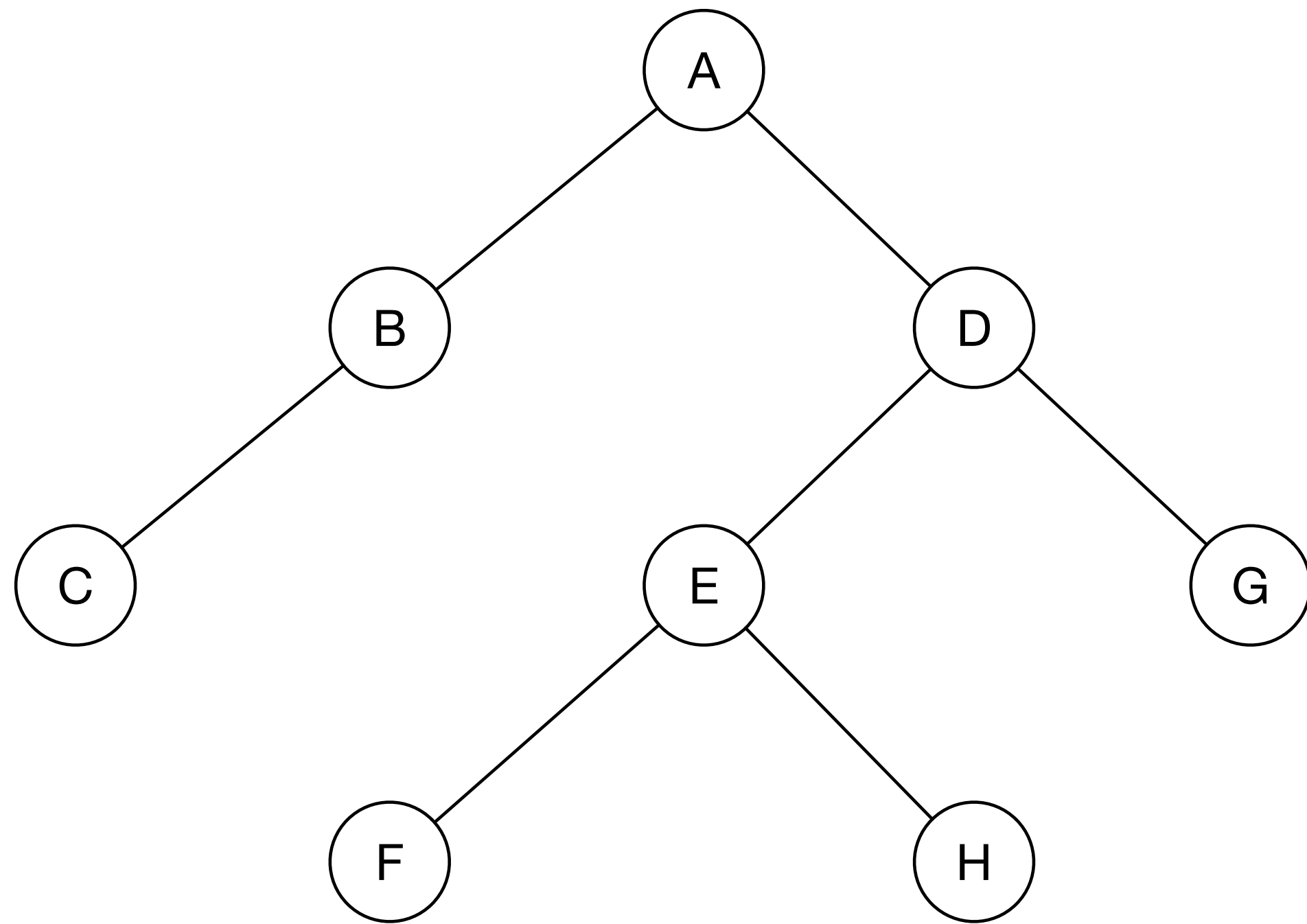


# Adjacency matrix



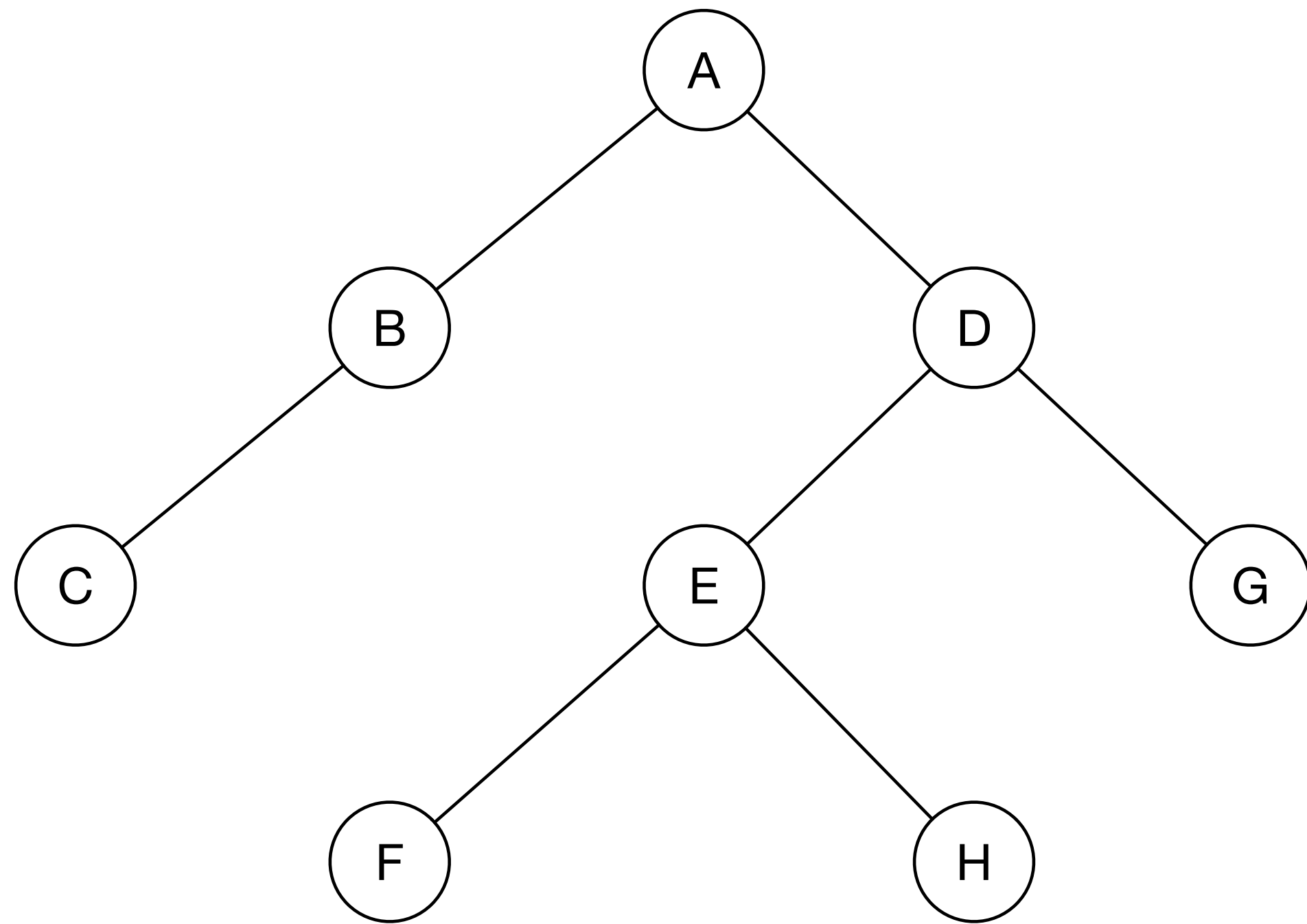
	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency matrix



	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency matrix



	A	B	C	D	E	F	G	H
A	0	1	0	1	0	0	0	0
B	1	0	1	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	1	0	0	0	1	0	1	0
E	0	0	0	1	0	1	0	1
F	0	0	0	0	1	0	0	0
G	0	0	0	1	0	0	0	0
H	0	0	0	0	1	0	0	0

# Adjacency List vs Adjacency Matrix

## Adjacency list

Good for sparse graphs

Only encodes edges present

Takes time to traverse list. If  $d$  is the degree of a node, this will be  $O(d_{\max})$

Easier to find all adjacent nodes

## Adjacency matrix

Good for dense graphs

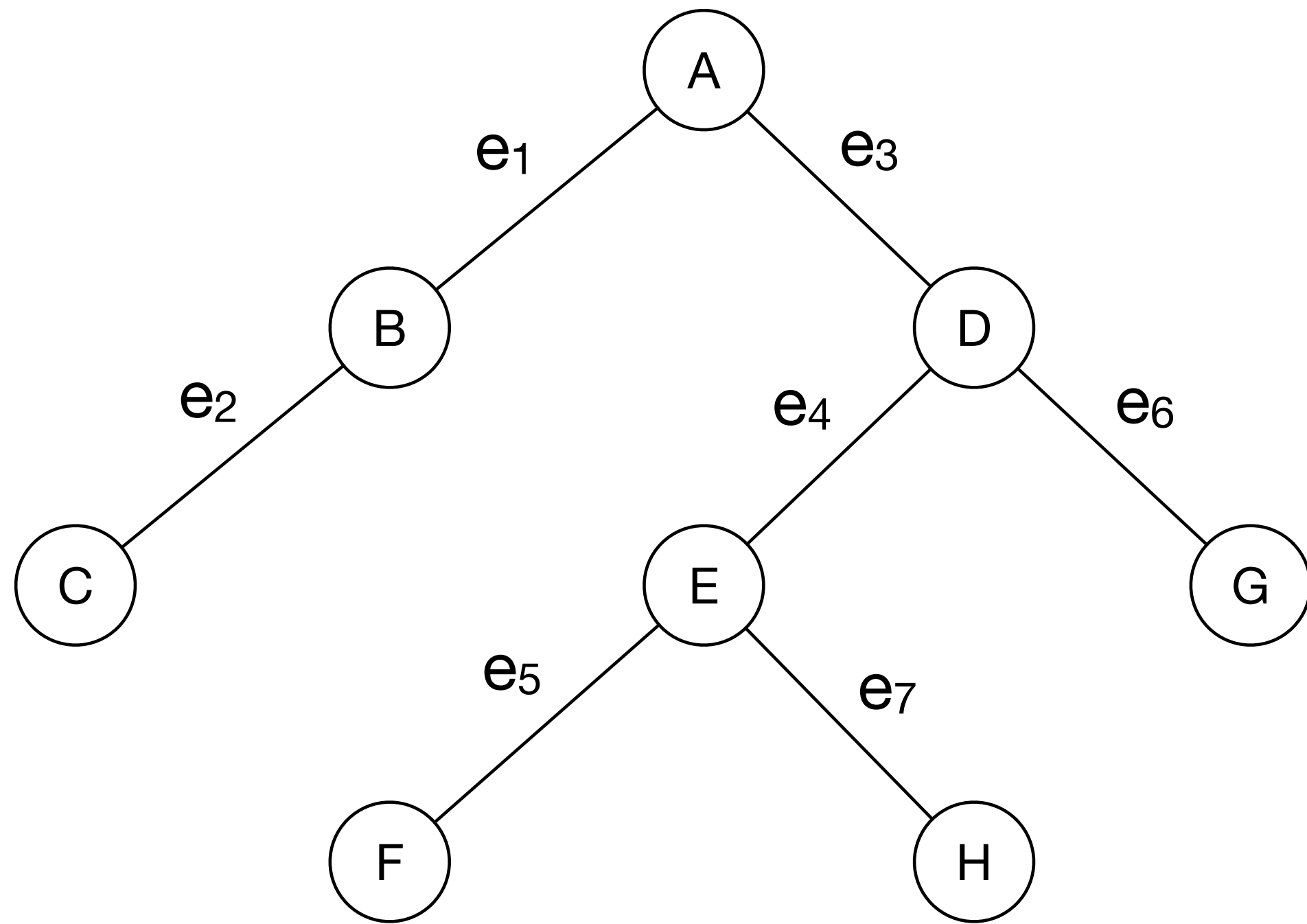
Encodes present *and* absent edges

Can check if two nodes are adjacent in constant time  $O(1)$

Must scan entire row or column to find all adjacent nodes,  $O(N)$

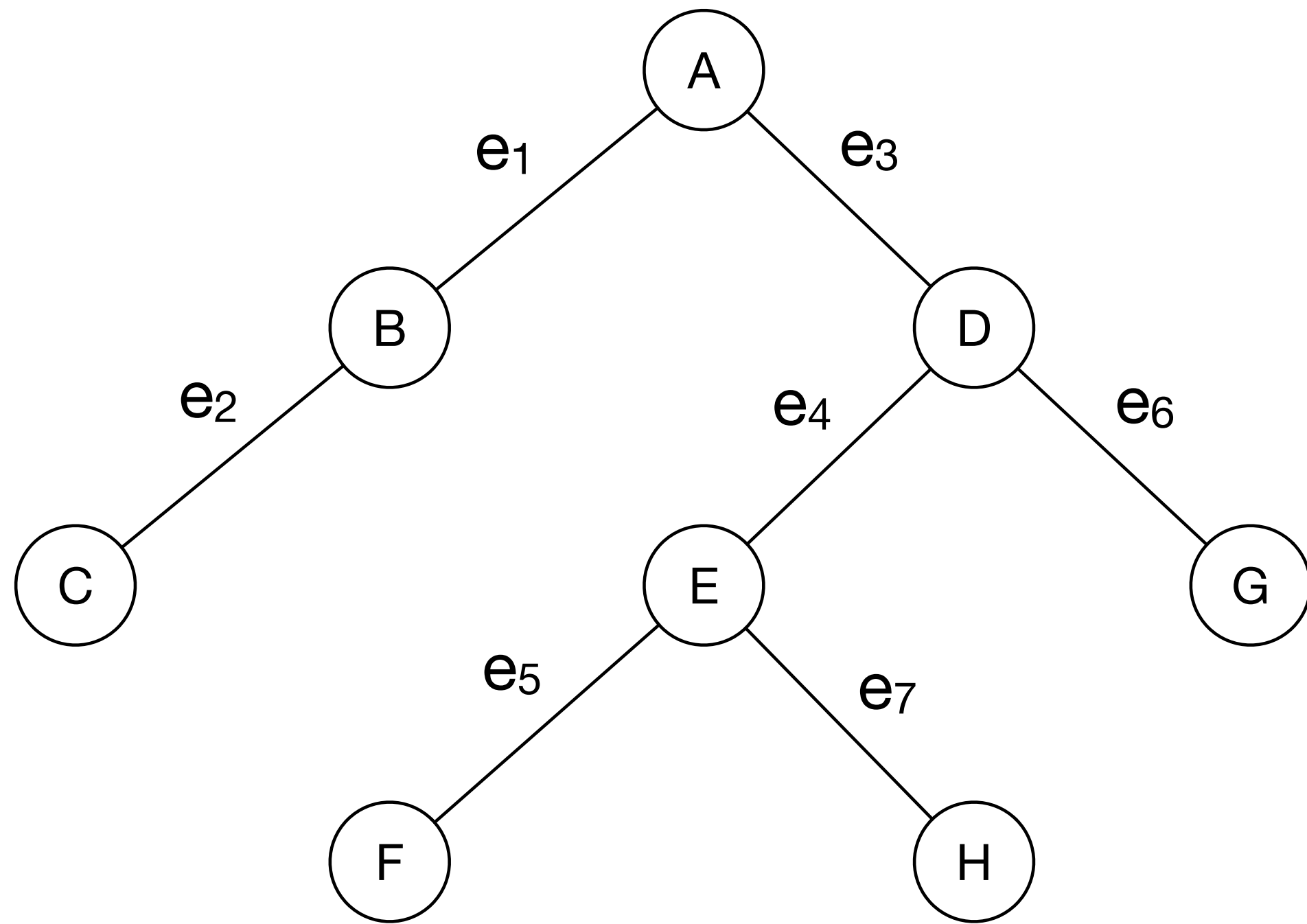
Can do cool stuff with matrix calculations (spectral graph theory, an advanced topic)

# Incidence matrix



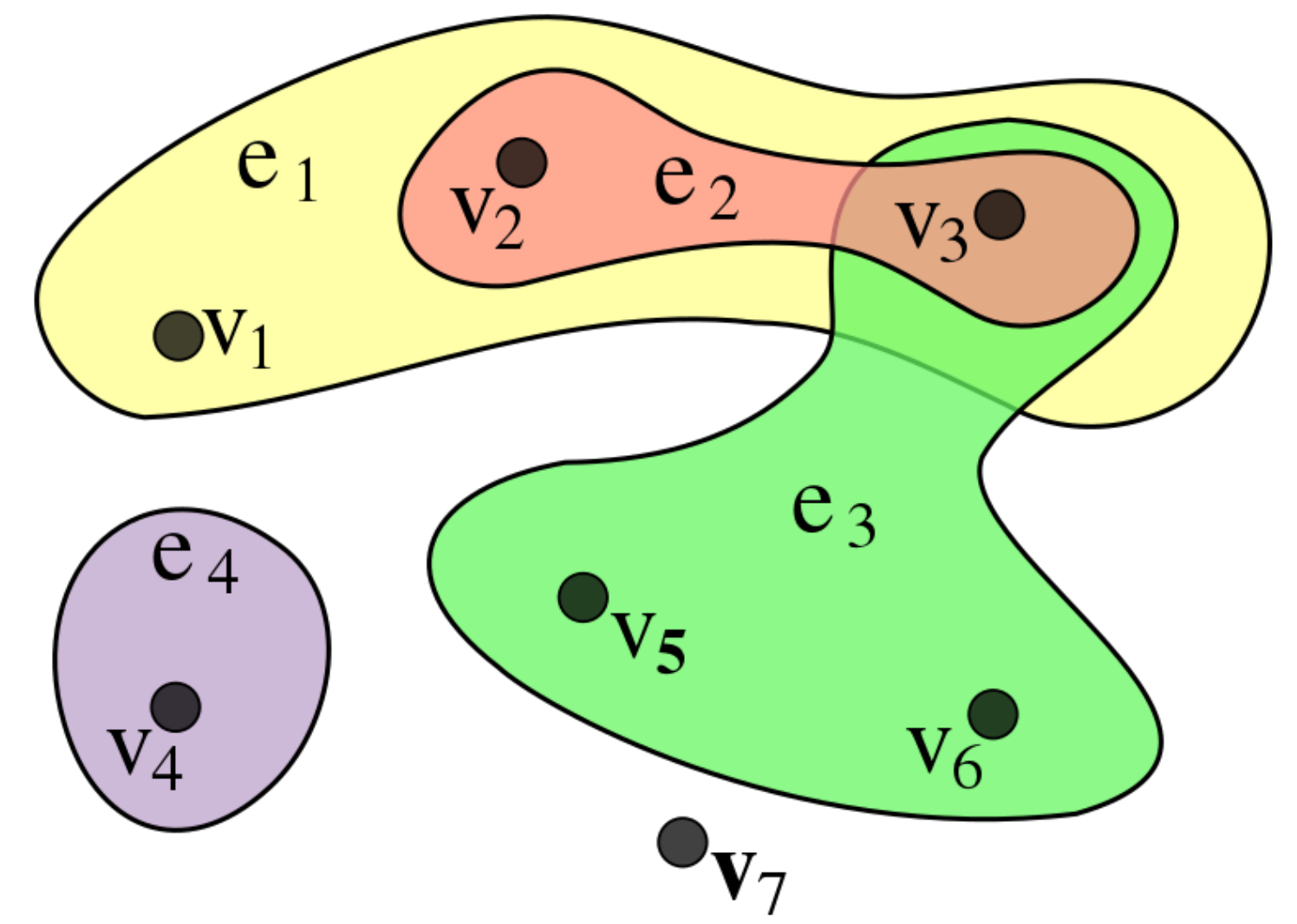
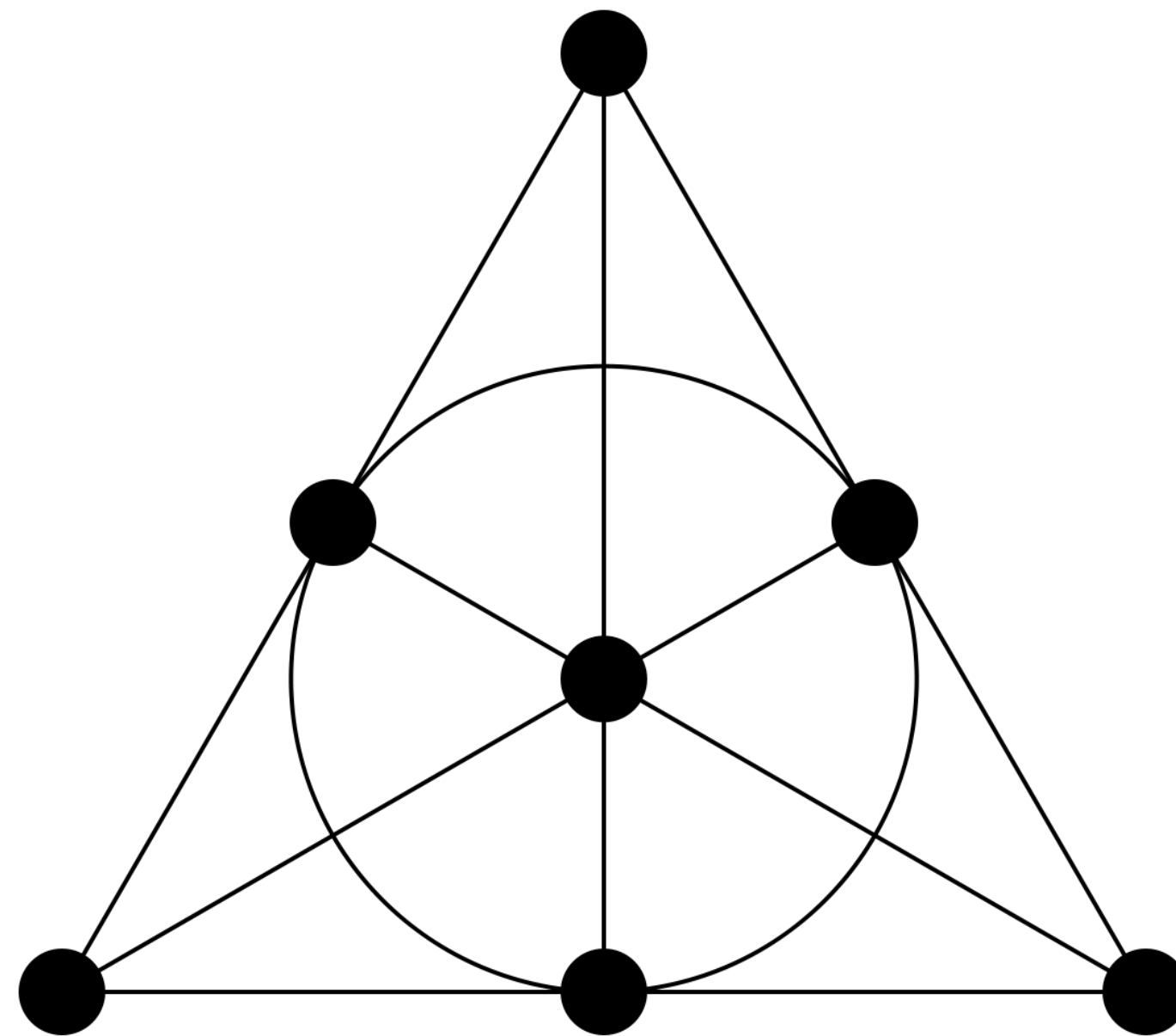
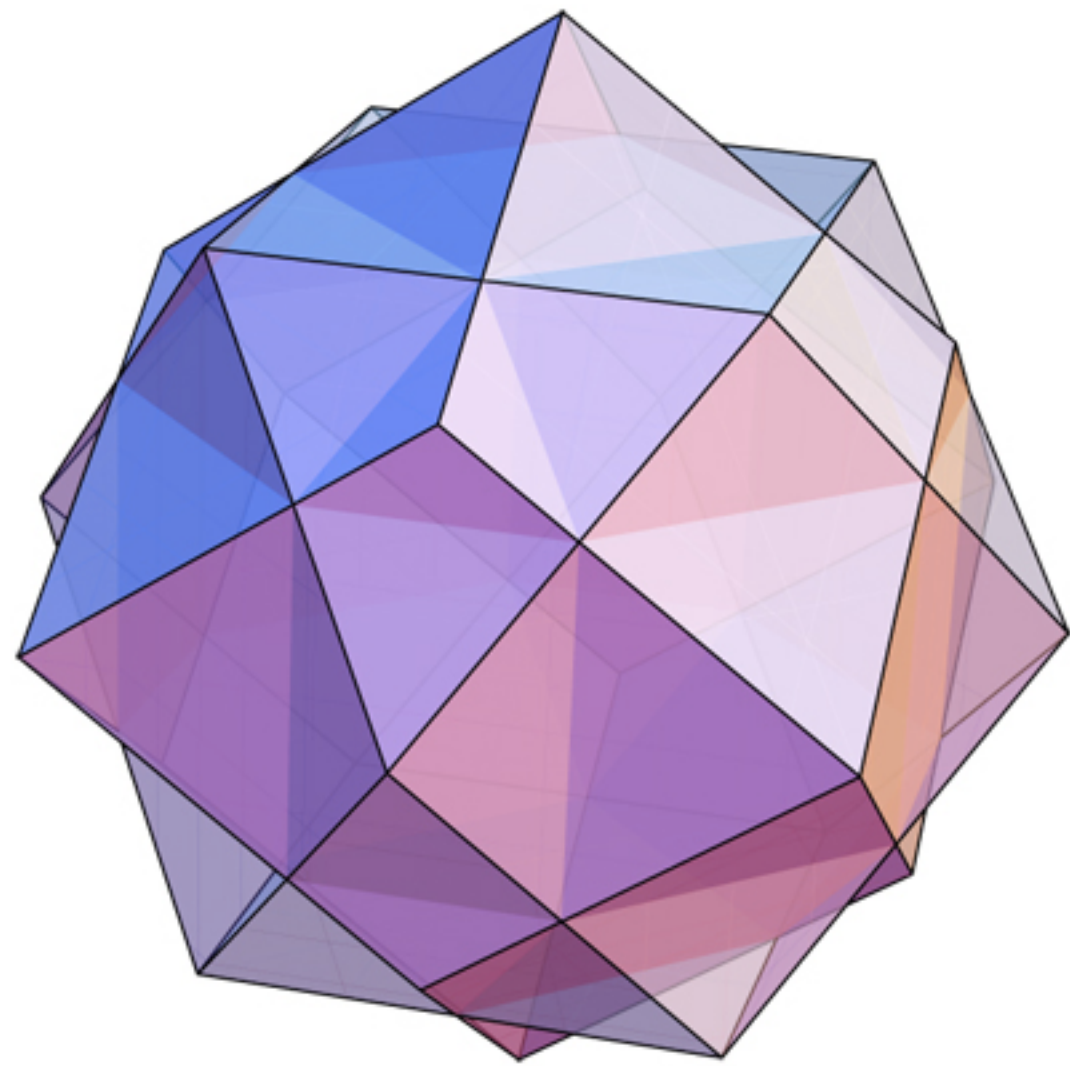
	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$
A	1	0	1	0	0	0	0
B	1	1	0	0	0	0	0
C	0	1	0	0	0	0	0
D	0	0	1	1	0	1	0
E	0	0	0	1	1	0	1
F	0	0	0	0	1	0	0
G	0	0	0	0	0	1	0
H	0	0	0	0	1	0	1

# Incidence matrix

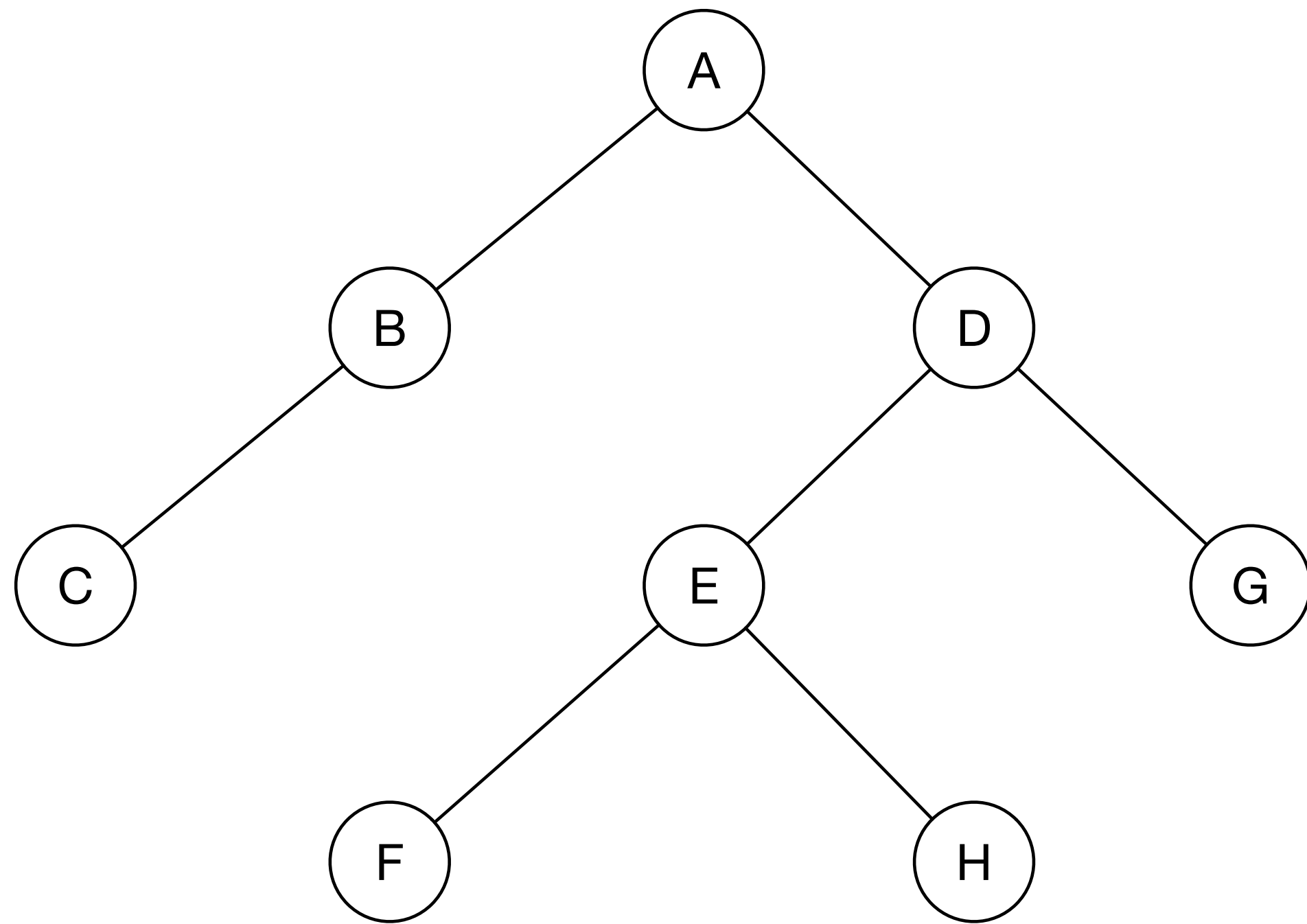


	$e_1$	$e_2$	$e_3$	$e_4$	$e_5$	$e_6$	$e_7$
A	1	0	1	0	0	0	0
B	1	1	0	0	0	0	0
C	0	1	0	0	0	0	0
D	0	0	1	1	0	1	0
E	0	0	0	1	1	0	1
F	0	0	0	0	1	0	0
G	0	0	0	0	0	1	0
H	0	0	0	0	1	0	1

# Incidence matrix



# Simple L/R Object-oriented Approach

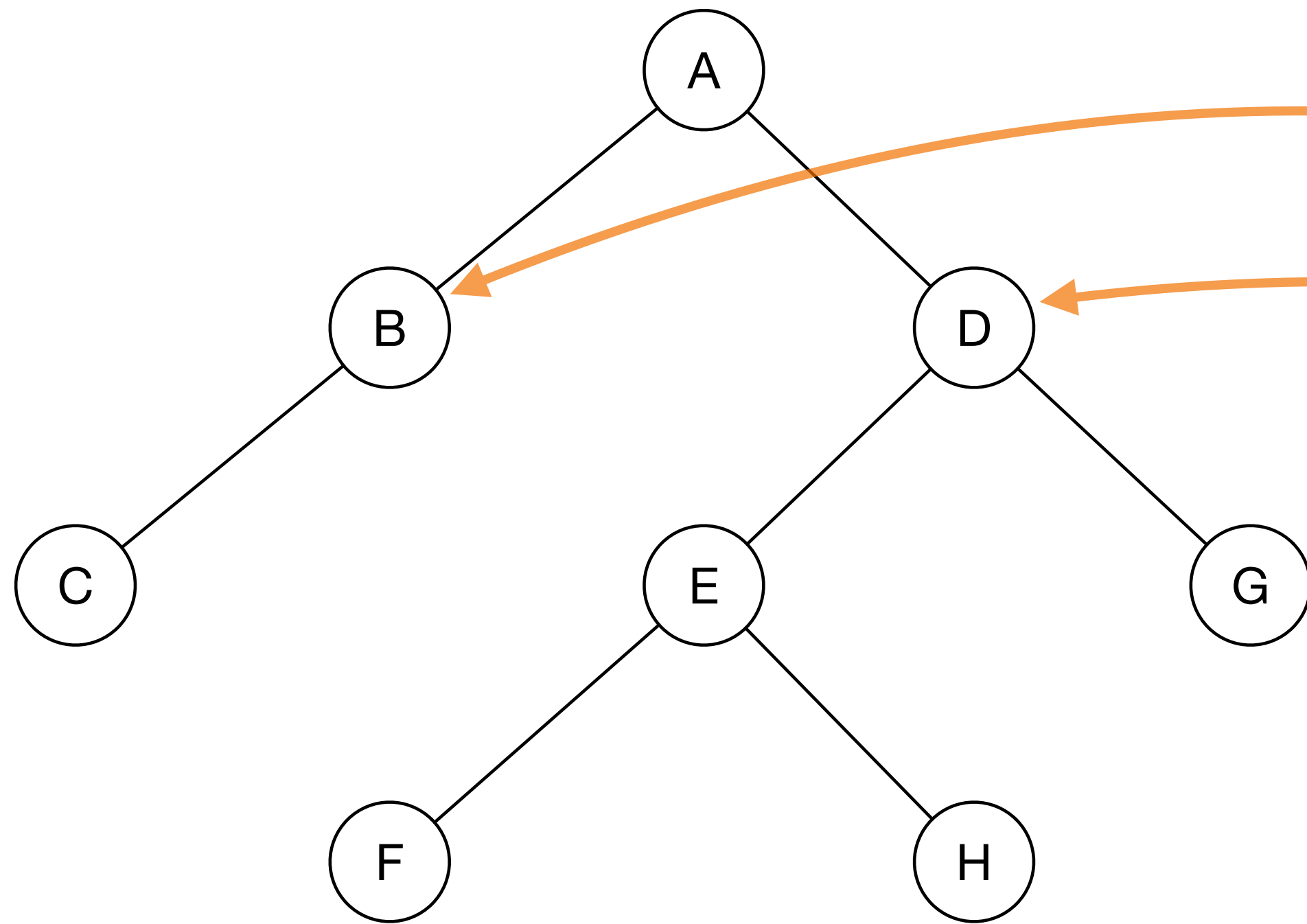


```
class Node {  
private:  
    Node *leftChild;  
    Node *rightChild;  
  
    ...  
}
```



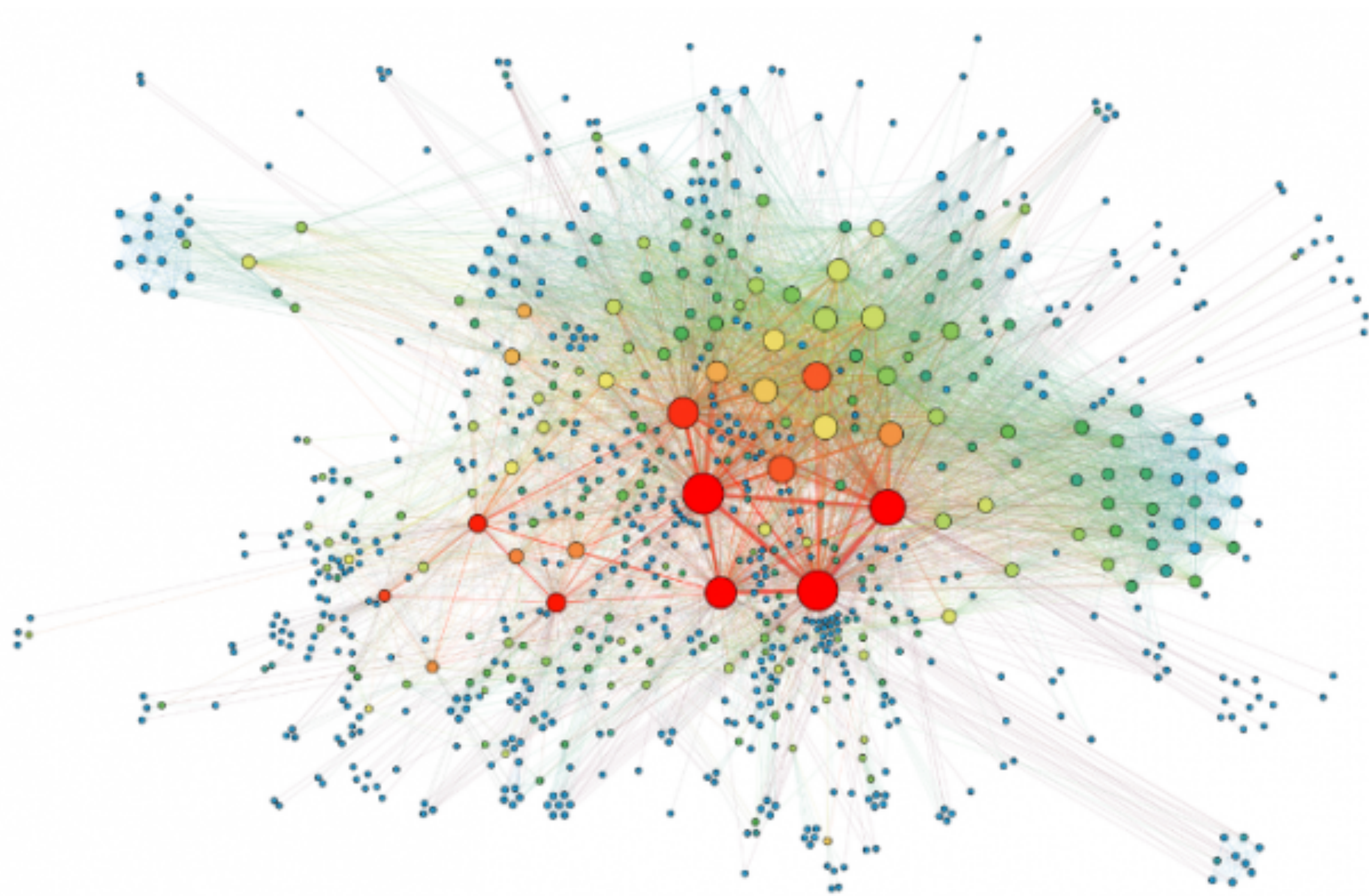
# Simple L/R Object-oriented Approach

A



```
class Node {  
private:  
    Node *leftChild;  
    Node *rightChild;  
    ...  
}
```

# Simple L/R Object-oriented Approach



```
class Node {  
private  
    Node leftChild;  
    Node rightChild;  
    ...  
}
```

A large red 'X' is drawn over the code, indicating that this approach is incorrect or discouraged.

# How do we represent a tree?

## Choosing a data structure

- Adjacency list - sparse objects
- Adjacency matrix - dense objects
- Simple L/R object-oriented approach - binary trees