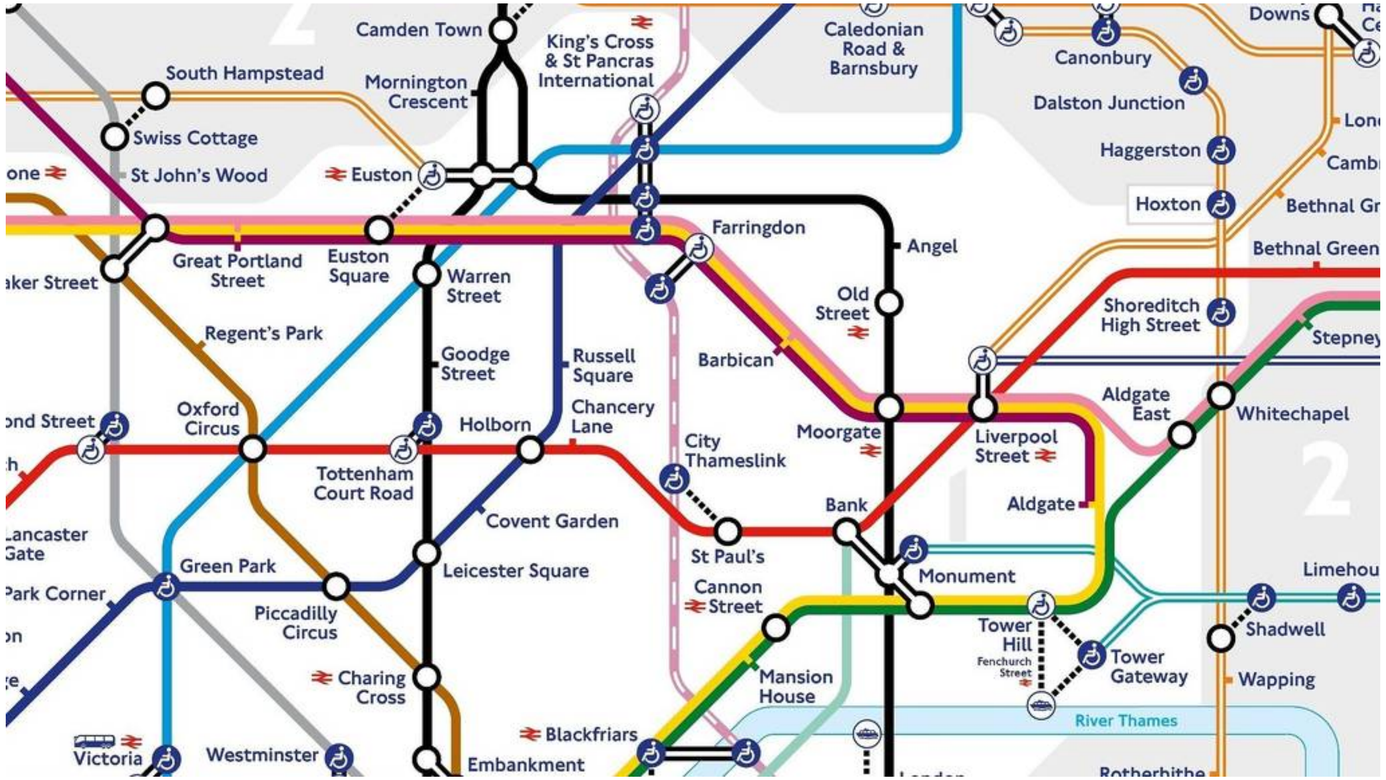
THE UNIVERSITY OF VERMONT
COLLEGE OF ENGINEERING &
MATHEMATICAL SCIENCES

Network Flows

Max flow / min cut: Ford-Fulkerson, part one

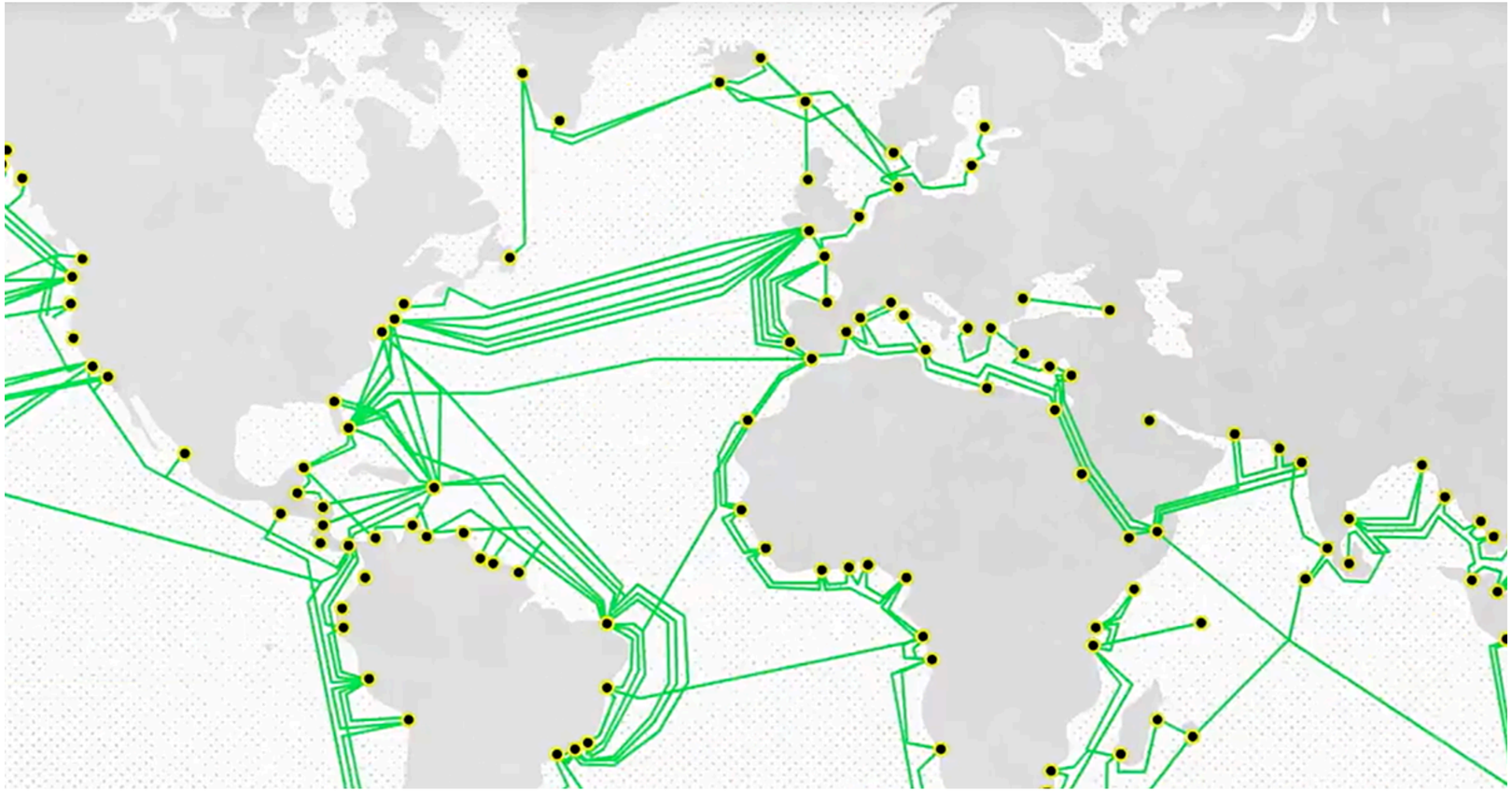
CS 124 / Department of Computer Science

Network flow



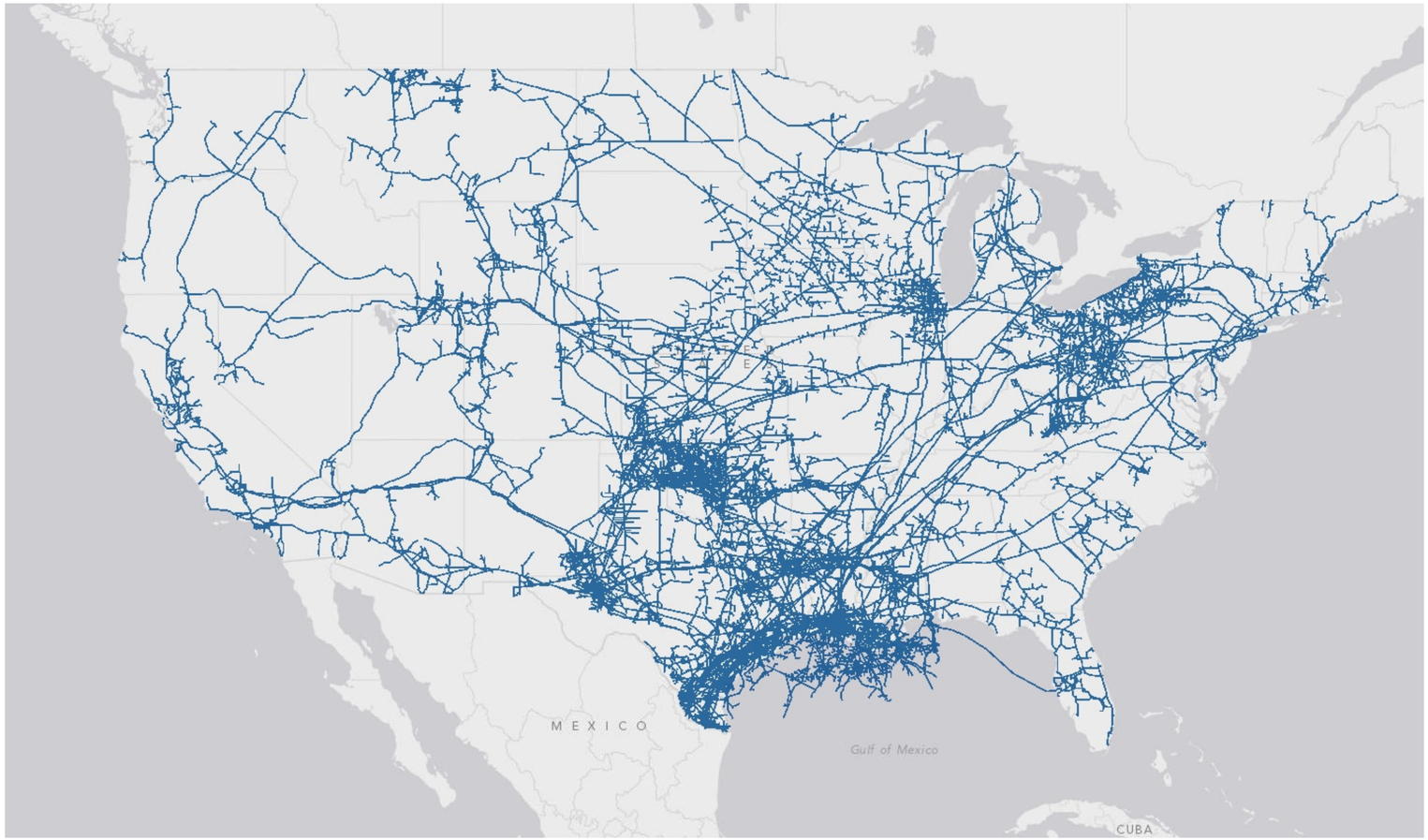
Source: Transport for London, tfl.gov.uk

Network flow



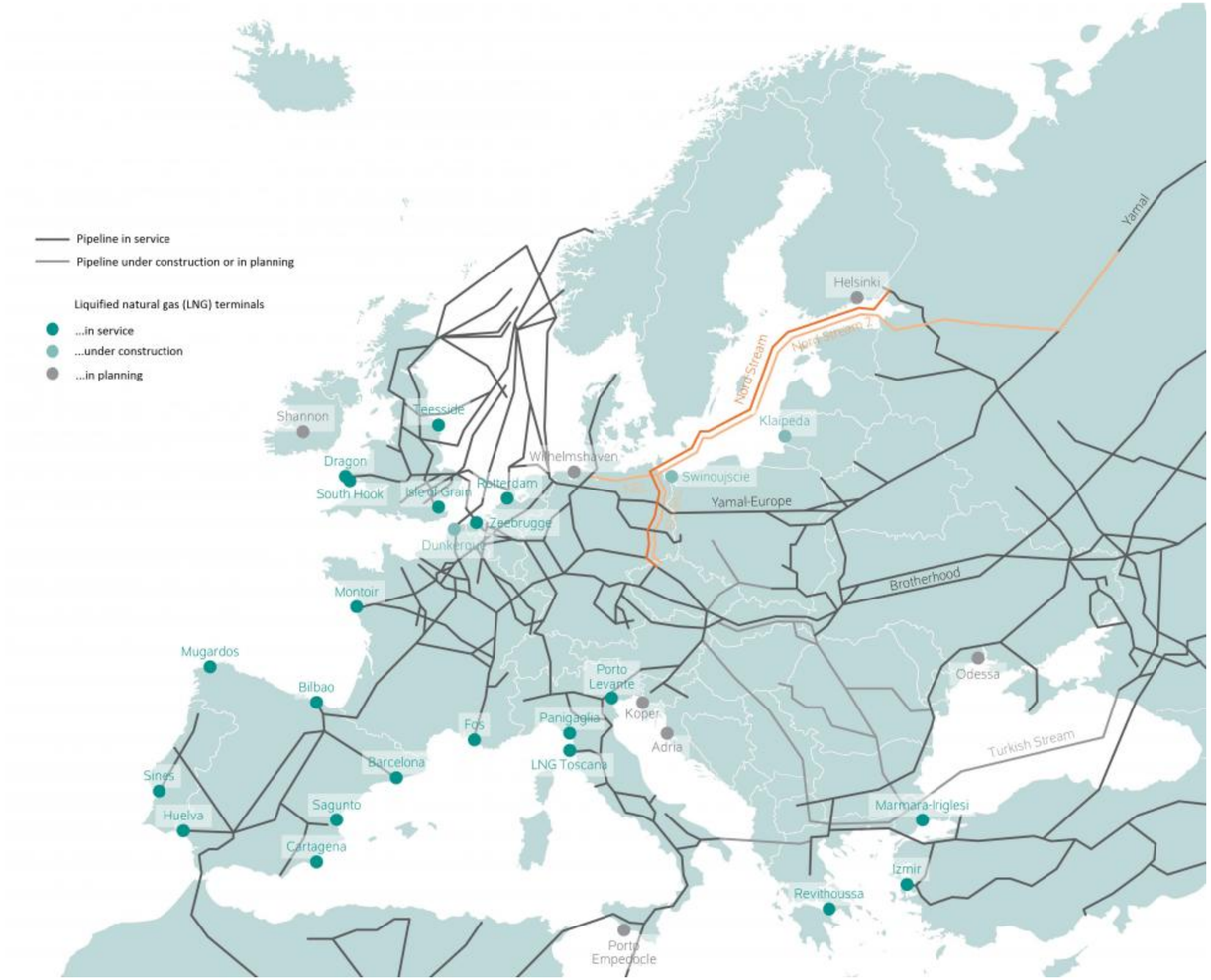
Source: The Visual Capitalist, https://www.visualcapitalist.com/wp-content/uploads/2019/09/global-fiber-optic-cable-network-map.png

Network flow



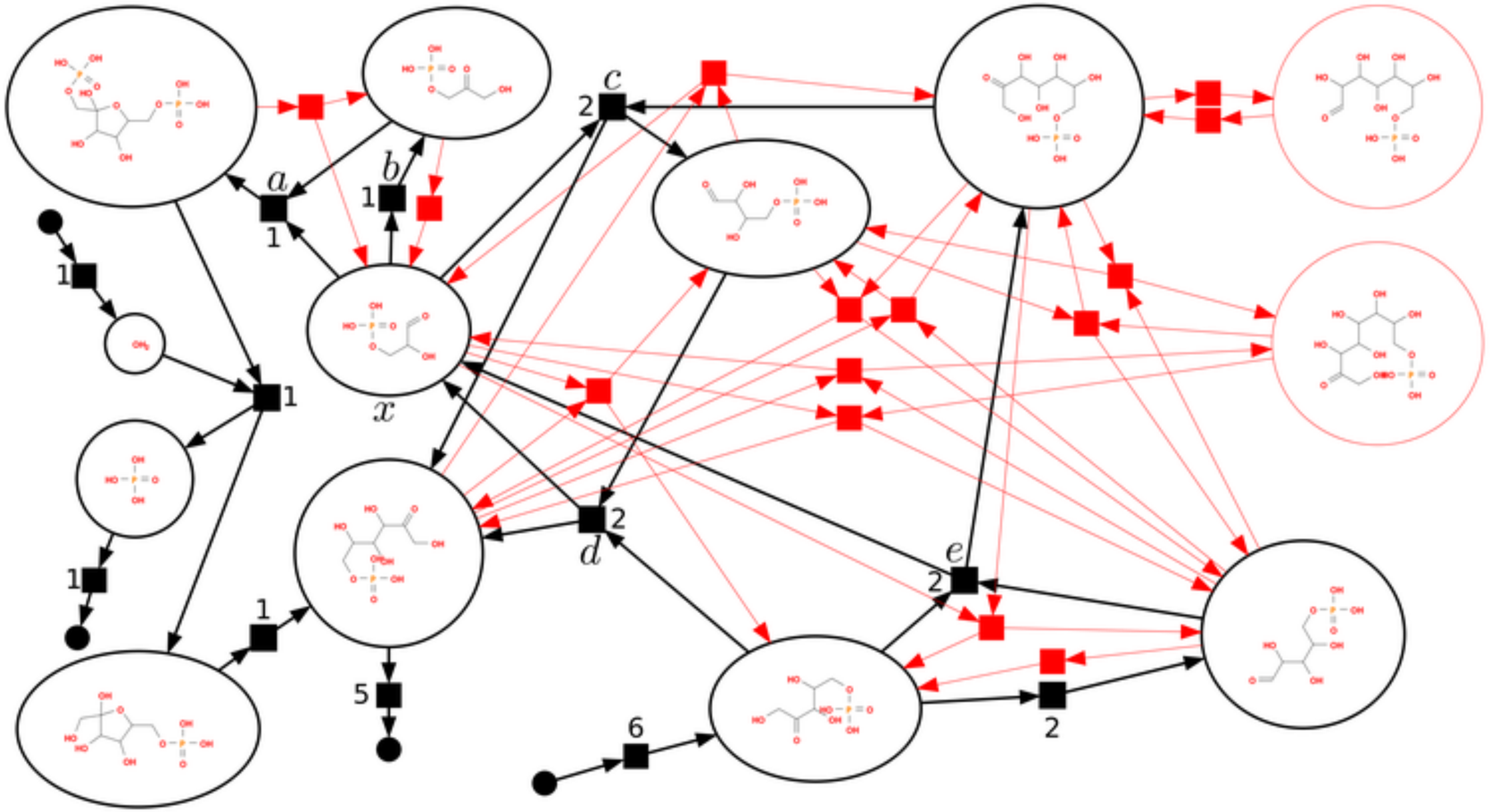
Source: Smithsonian Magazine,
https://www.smithsonianmag.com/science-nature/tour-the-countrys-energy-infrastructure-through-a-new-interactive-map-8844967/

Network flow



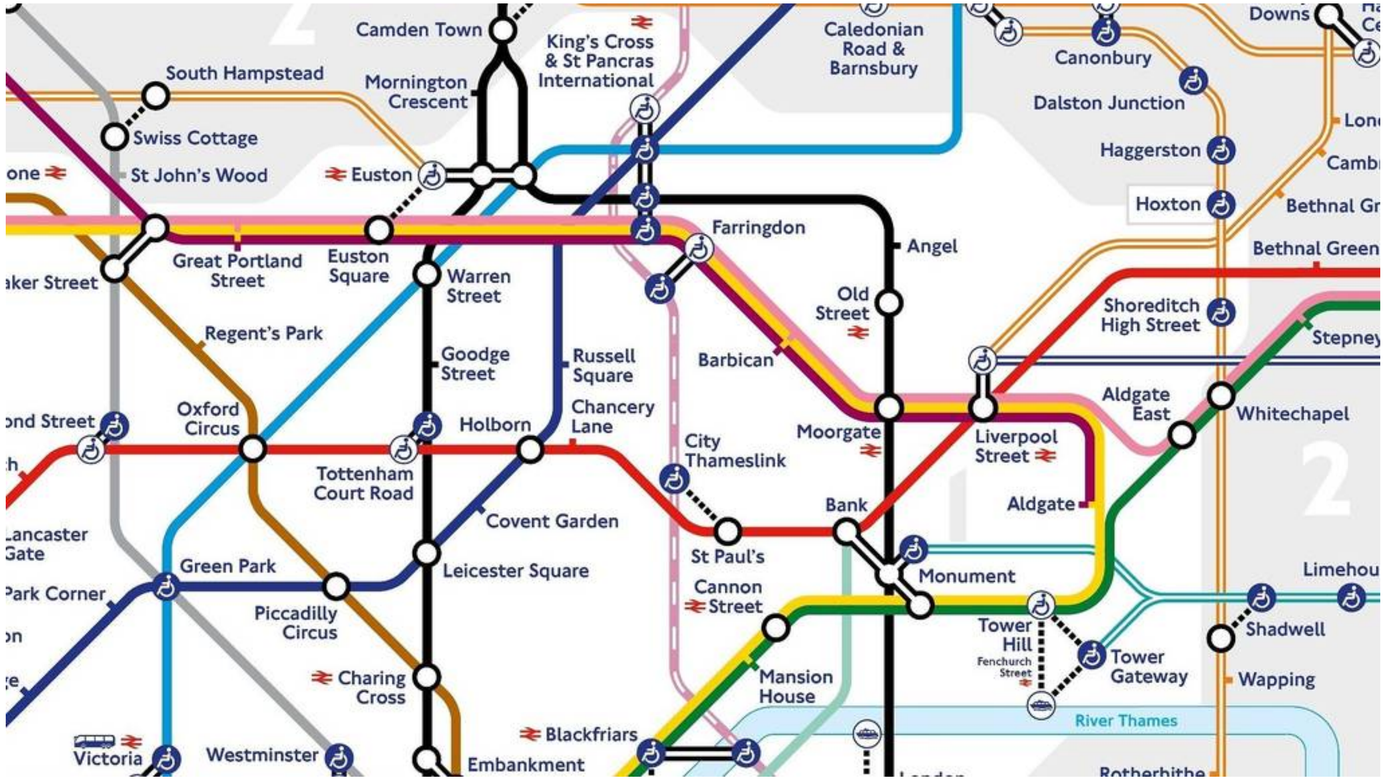
Source: DIW 2018, based on Kai-Olaf Lang and Kirsten Westphal, “Nord Stream 2 – Versuch einer politischen und wirtschaftlichen Einordnung,“ SWP Studie S21 (2016); ENTSO-G, Capacity Map (2017).

Network flow



Source: Max Plank Institute, https://www.mis.mpg.de/stadler/research/chemical-reaction-networks.html

Network flow



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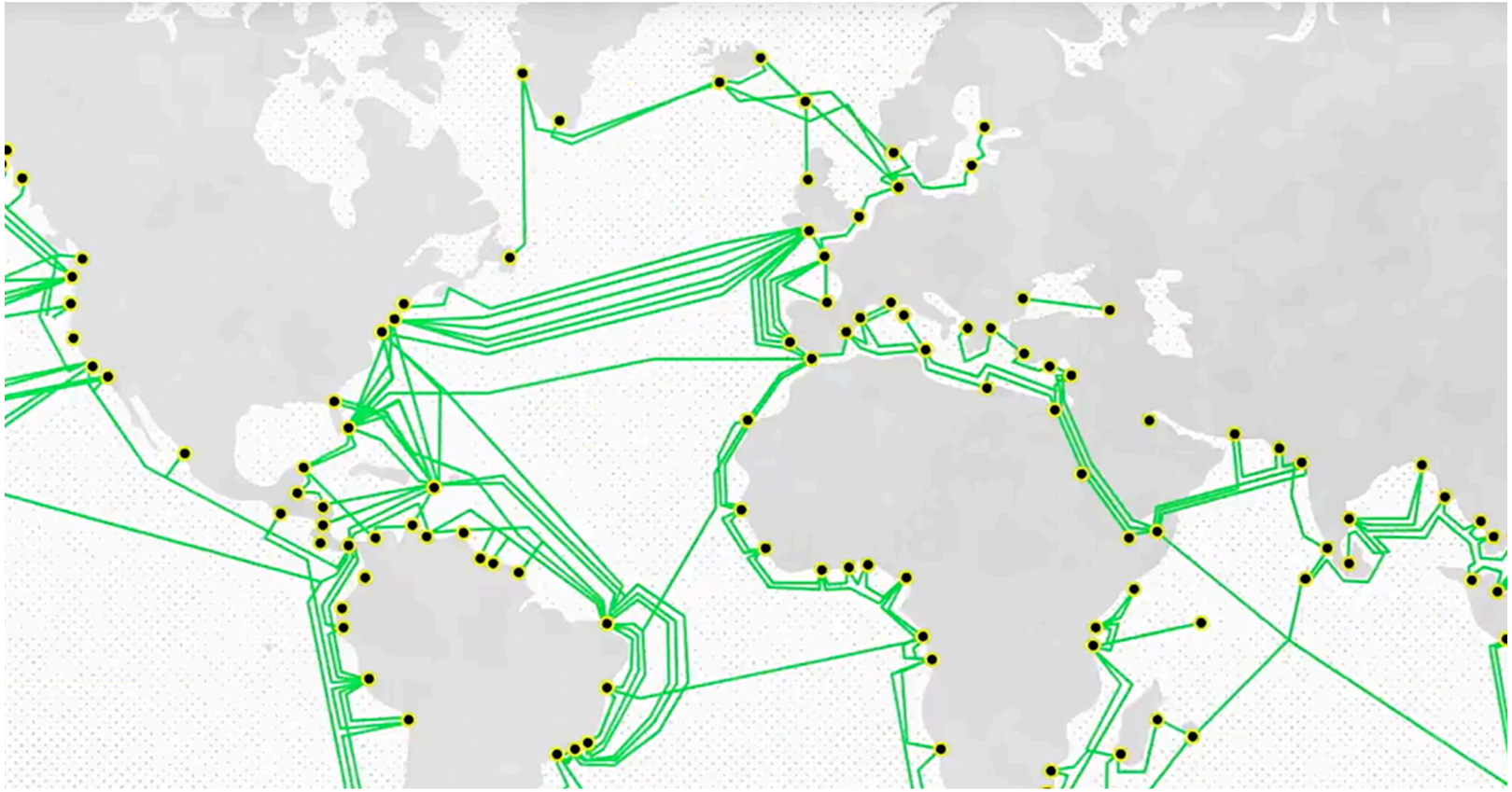

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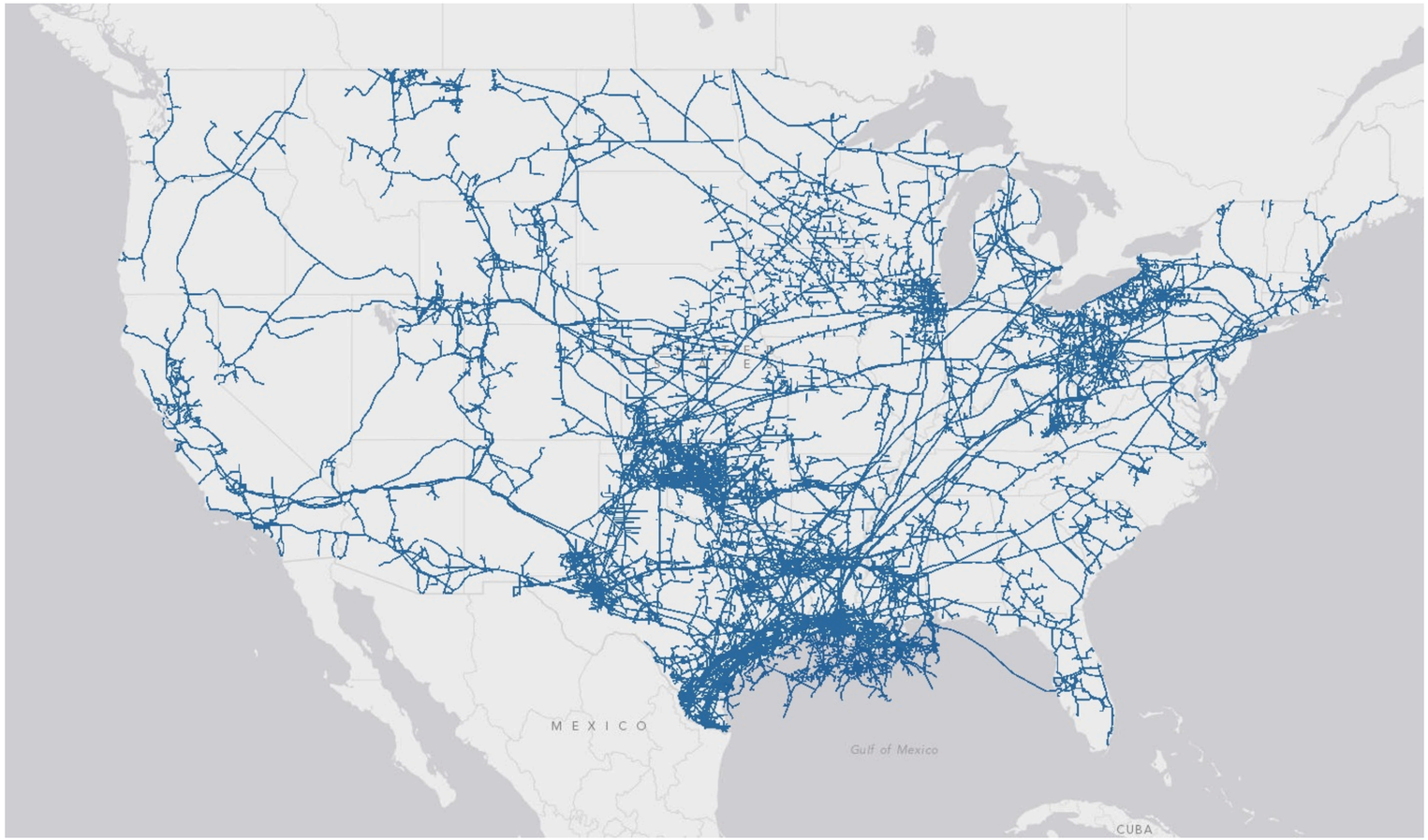

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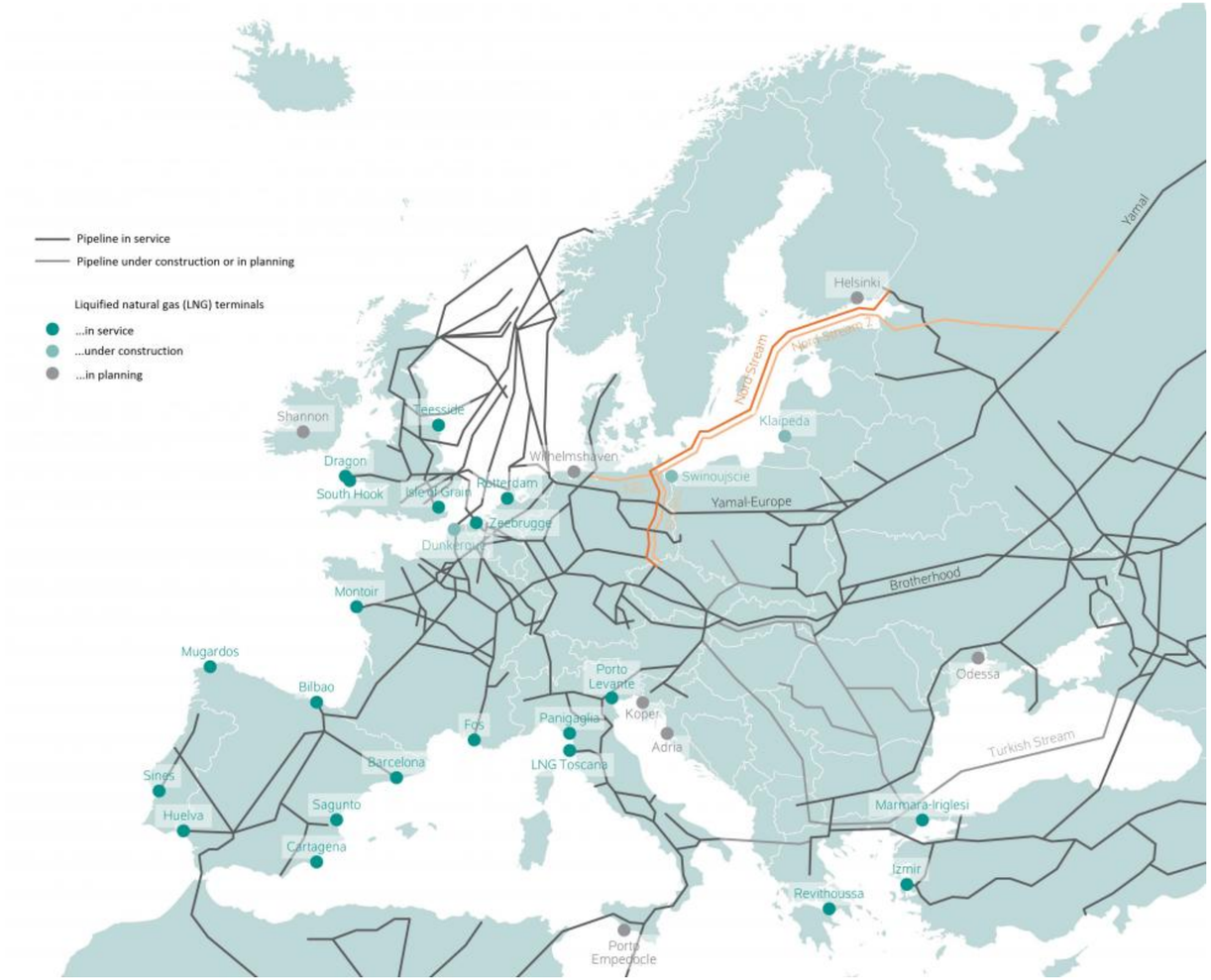

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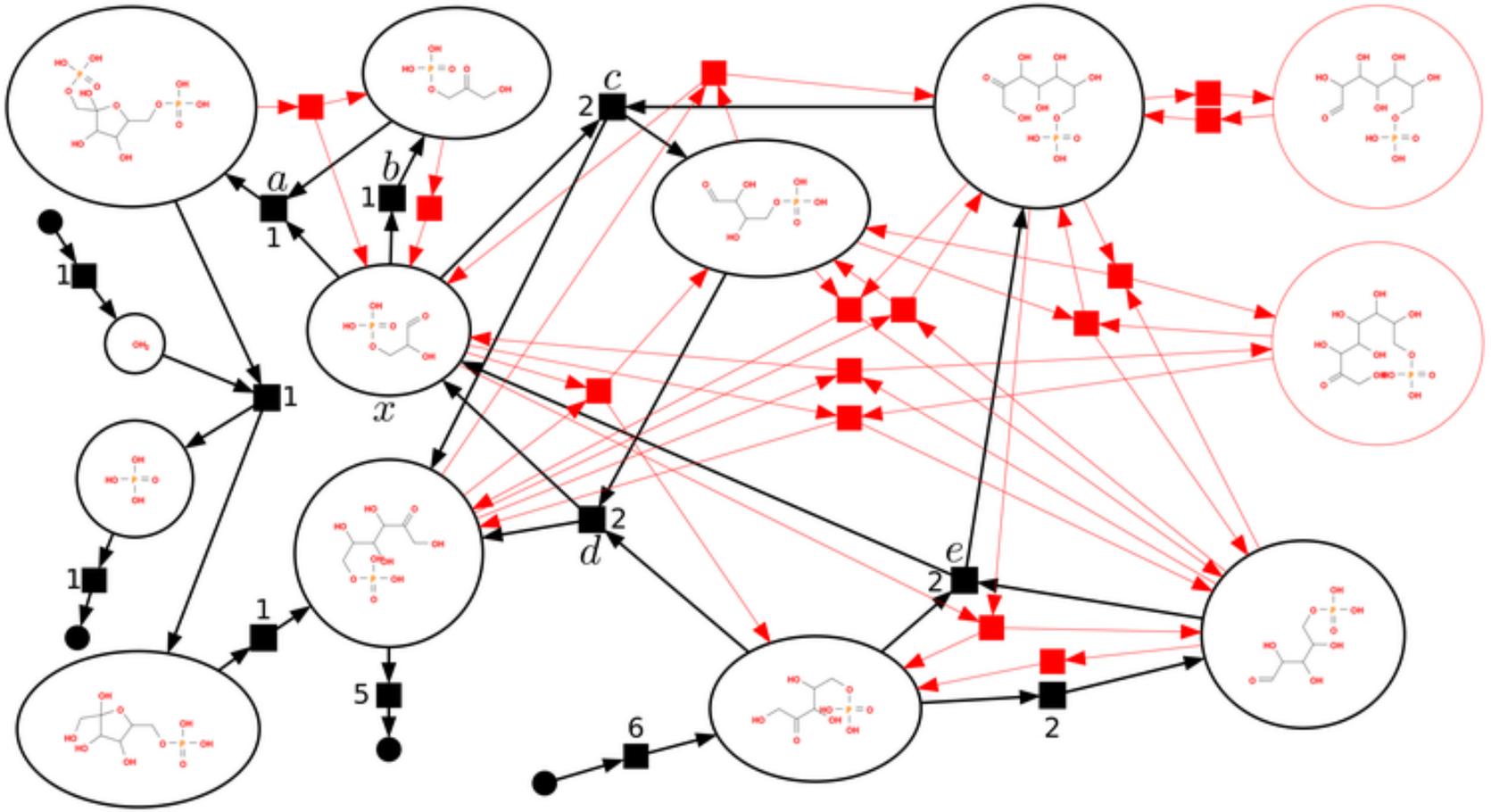

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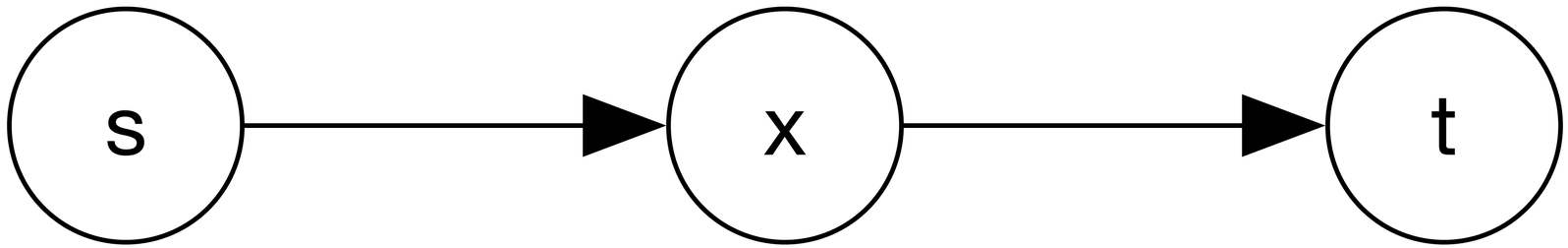
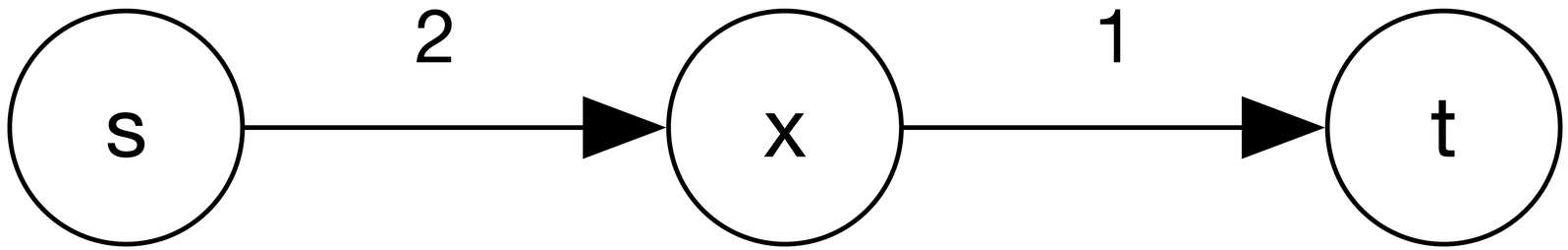


A very simple network!

Network flow



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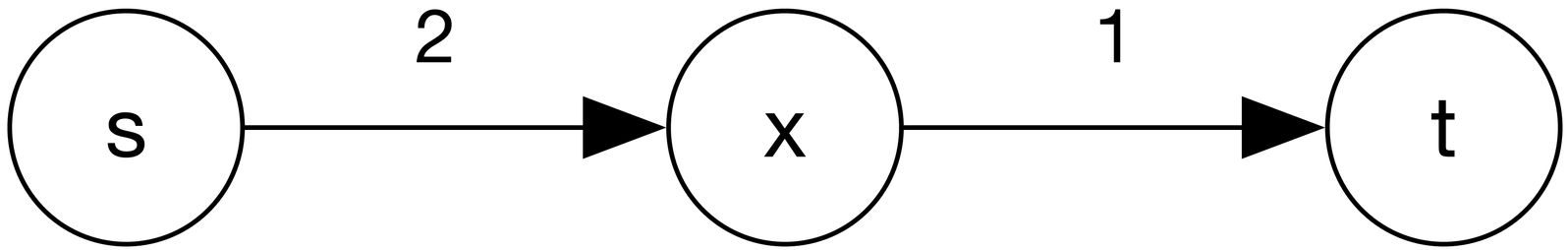

Network flow

$f_{u,v}$ is the flow from u to v

Network flow

$$f_{u,v} \text{ is the flow from } u \text{ to } v$$

$$f_{u,v} \leq c_{u,v}$$

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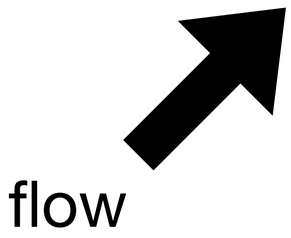

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flow across an edge cannot exceed capacity of the edge

Network flow

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$$\sum_{u:(u,v)\in E} f_{u,v} = \sum_{w:(v,w)\in E} f_{v,w}$$

Network flow

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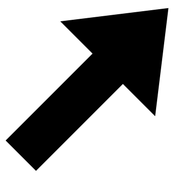

flow into node v

Network flow

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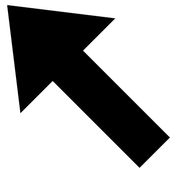
flow into node v



flow out of node v

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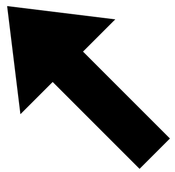
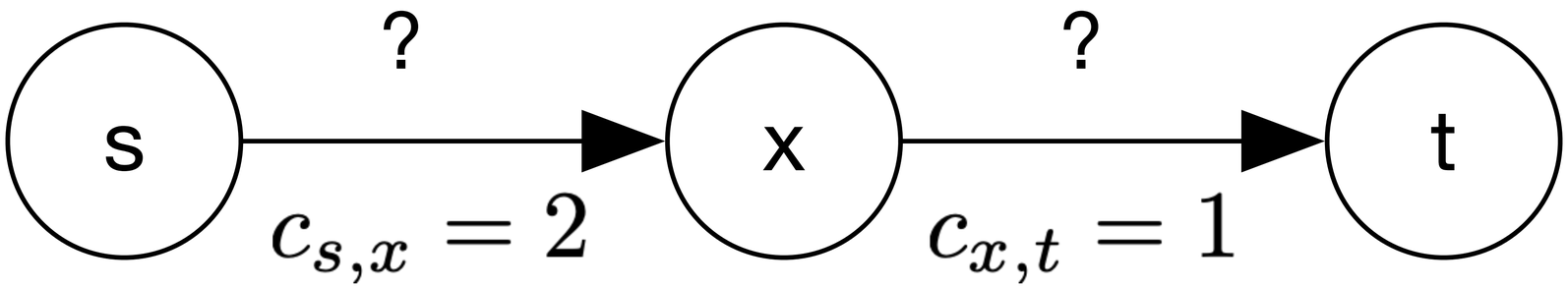
flow into node v



flow out of node v

for all nodes except source and sink, flow into a node = flow out from the node

Network flow



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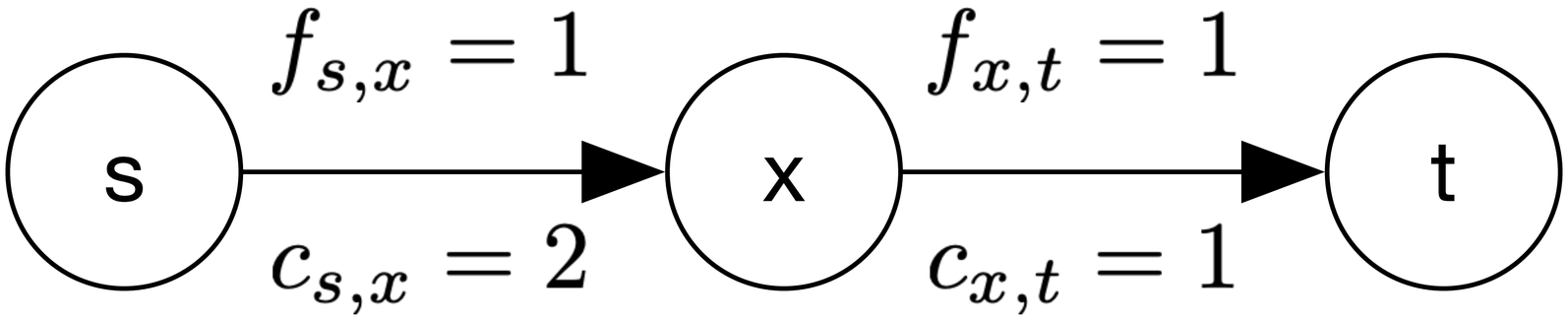
Network flow



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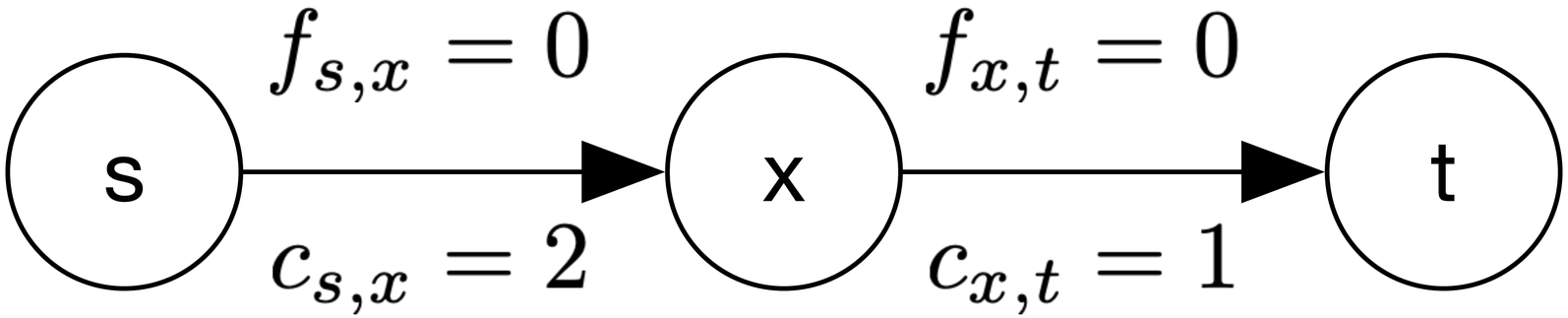
Network flow



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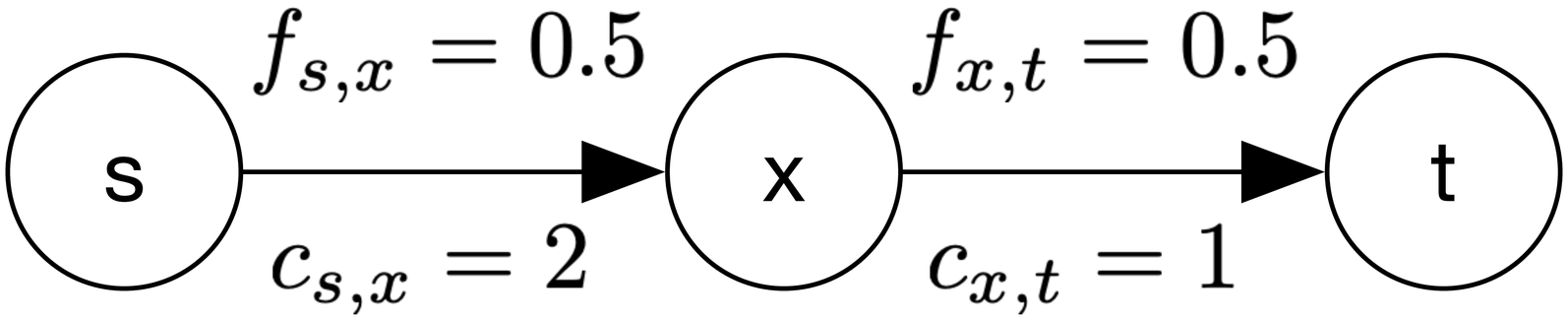

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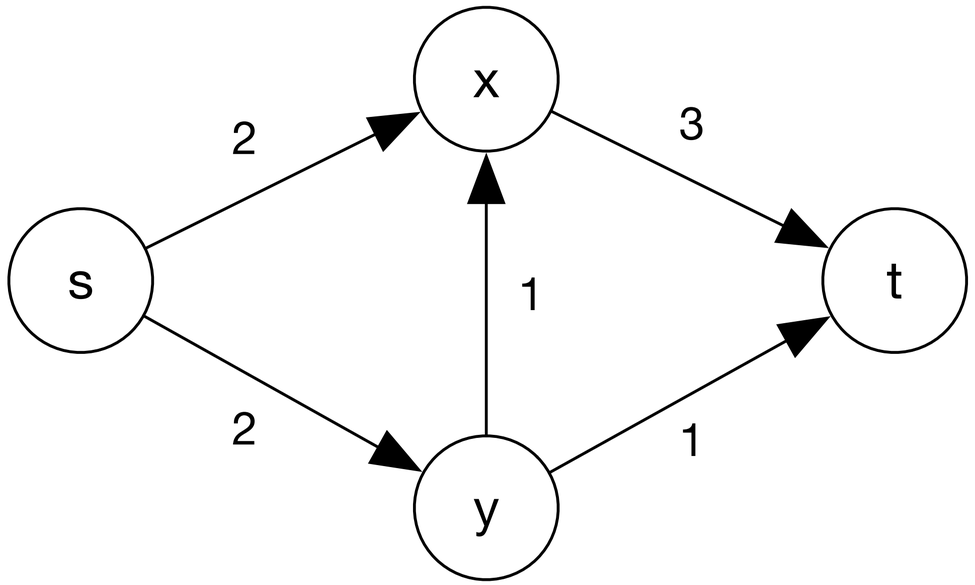


capacity = flow

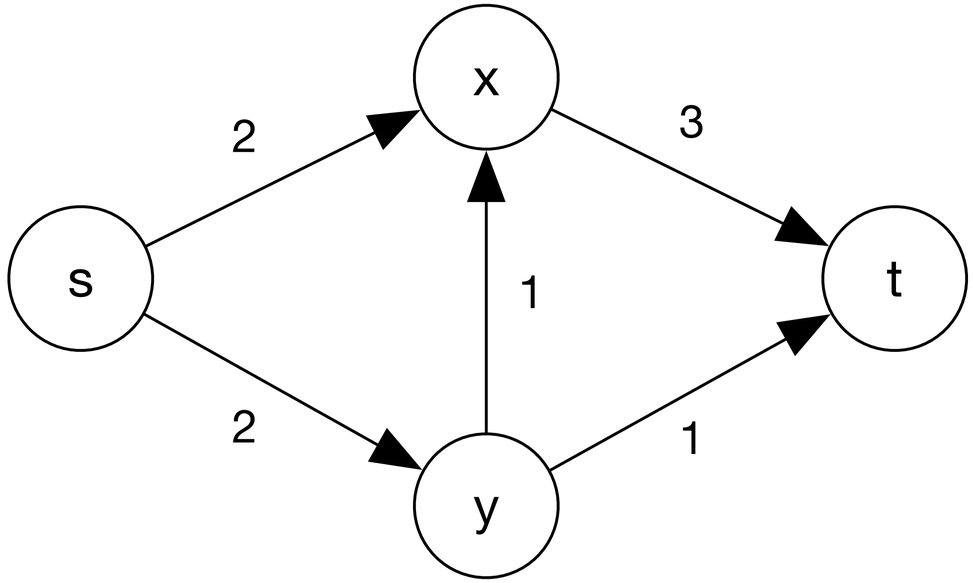
$$\sum_{u:(u,x)\in E} = \sum_{w:(x,w)\in E} = 3$$

flow into node *x* = flow out of node *x*

$$\sum_{u:(u,y)\in E} = \sum_{w:(y,w)\in E} = 2$$

flow into node *y* = flow out of node *y*

Network flow

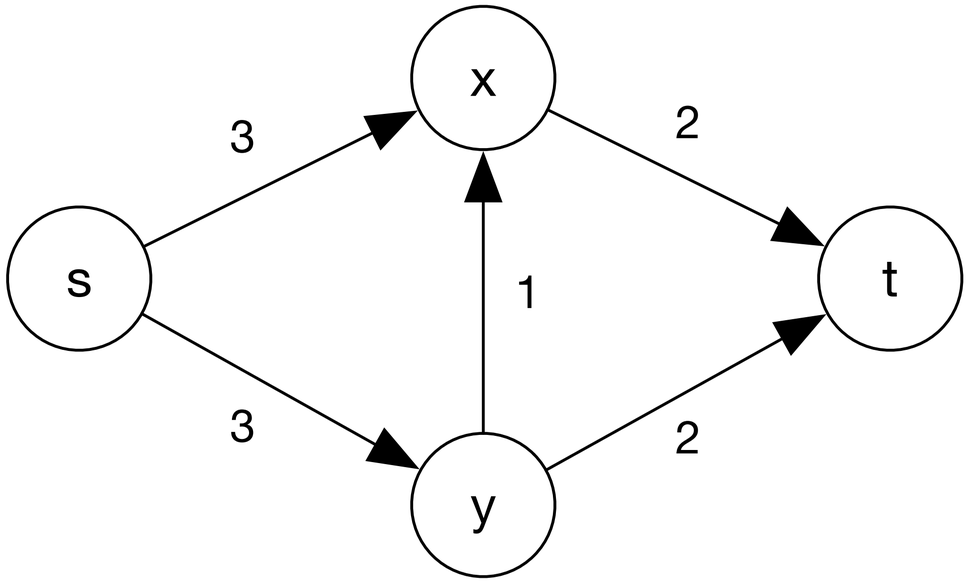


capacity = flow

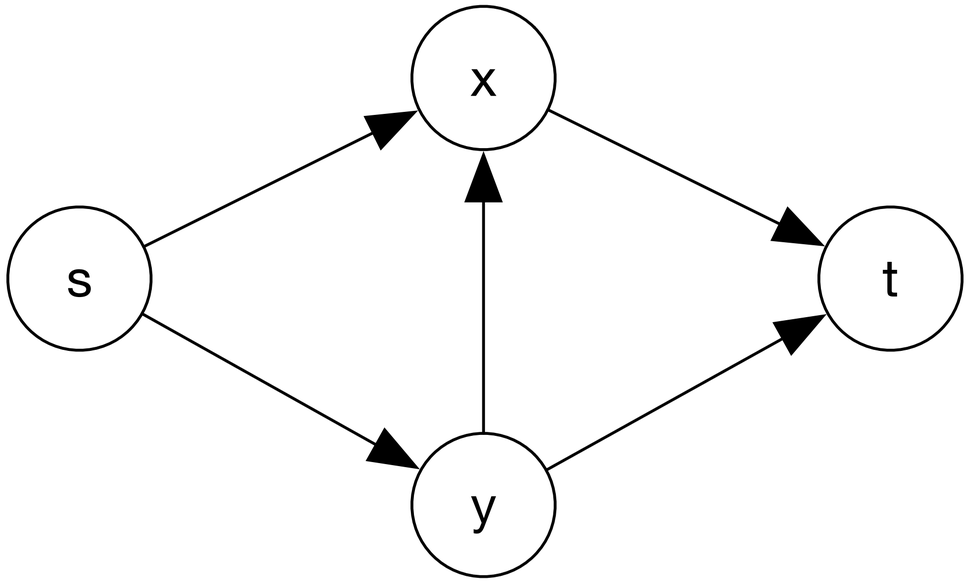
$$\sum_{u:(s,u)\in E} = \sum_{w:(w,t)\in E} = 4$$

flow out of source = flow into sink

Network flow

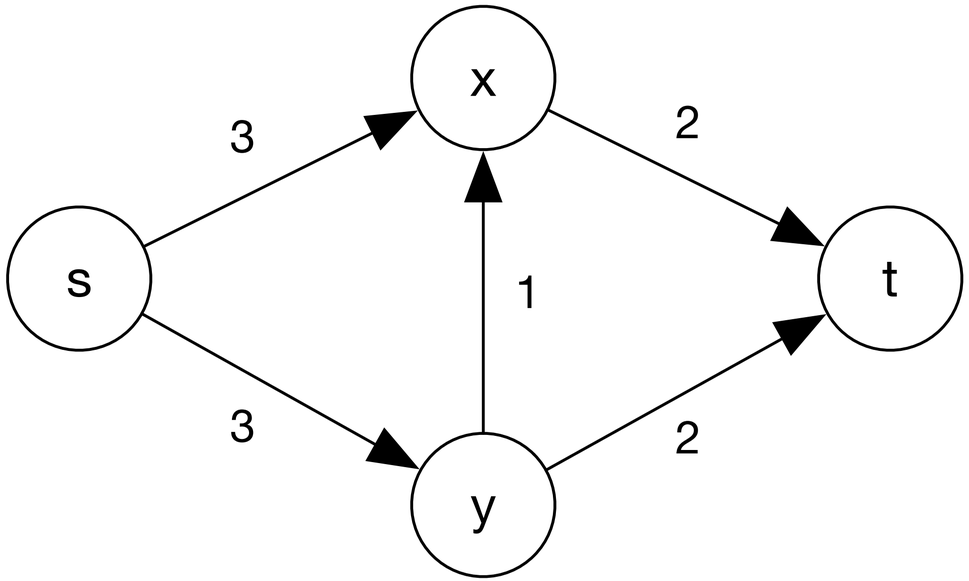


capacity

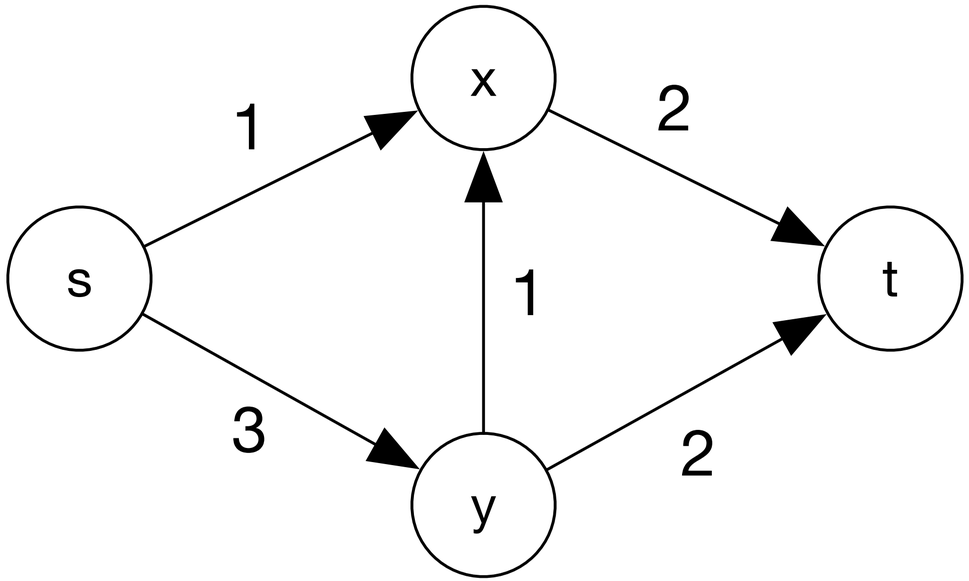


flow

Network flow

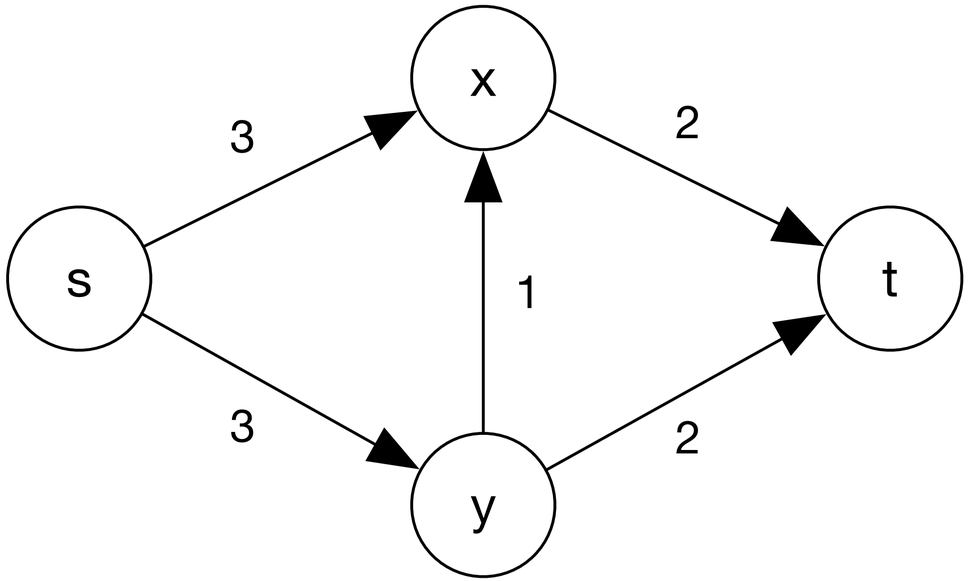


capacity

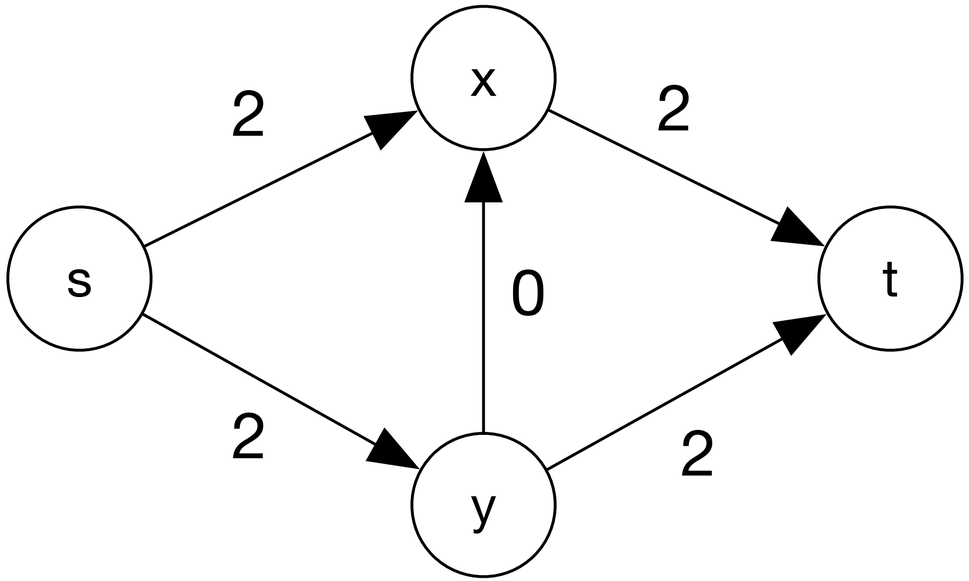


flow

Network flow



capacity



flow

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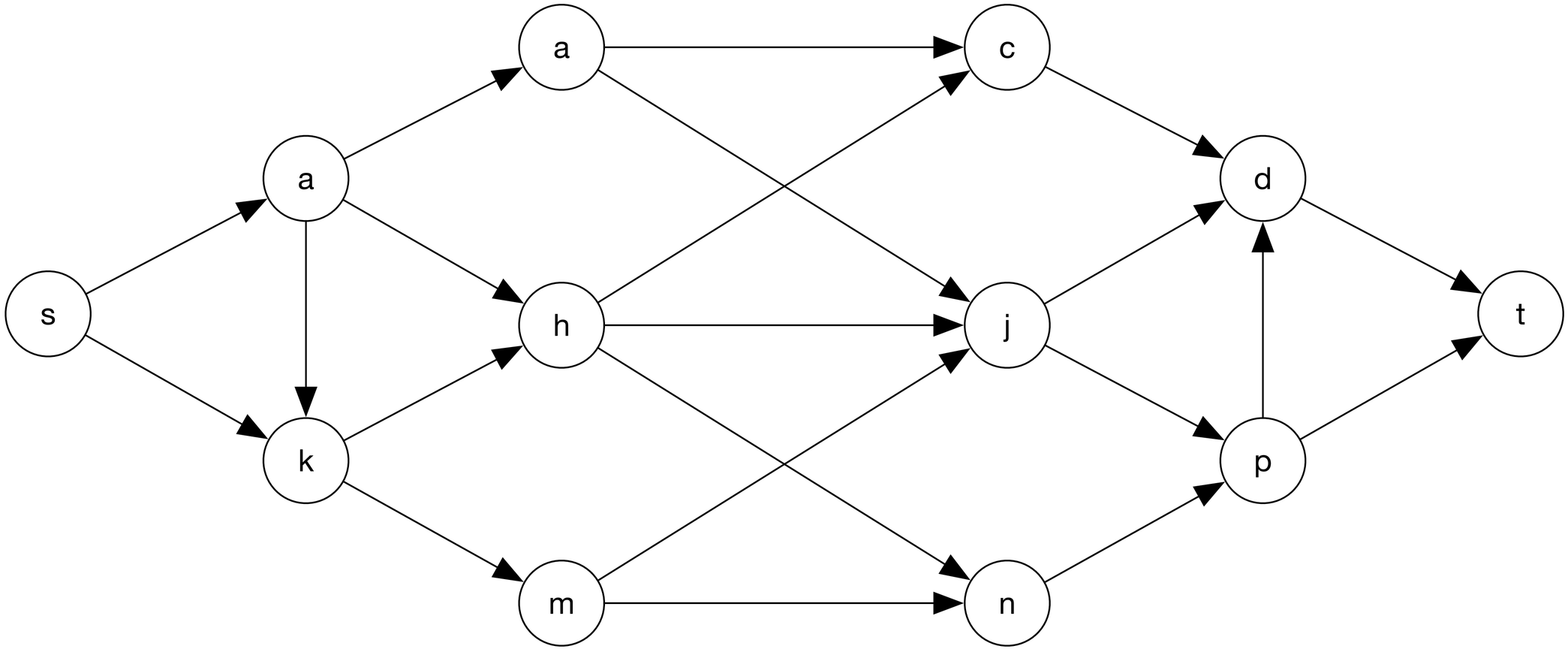

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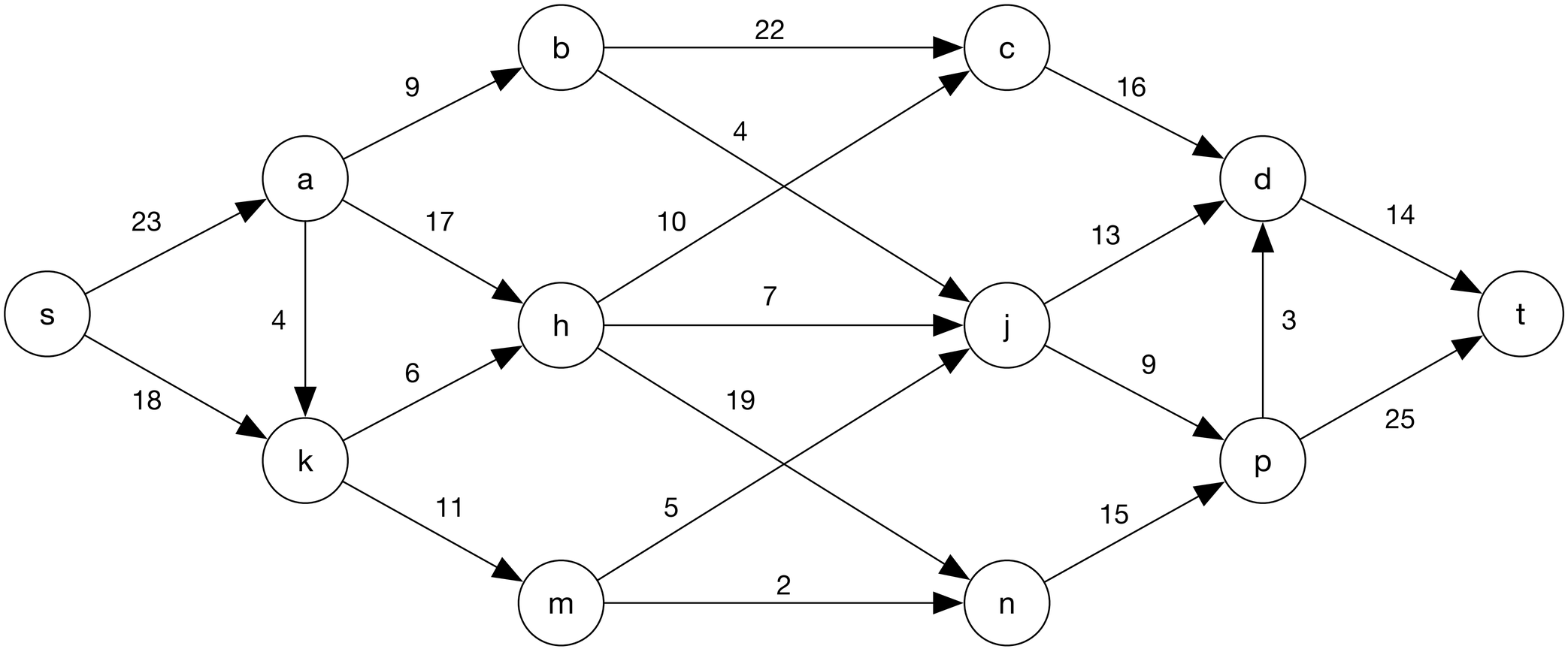

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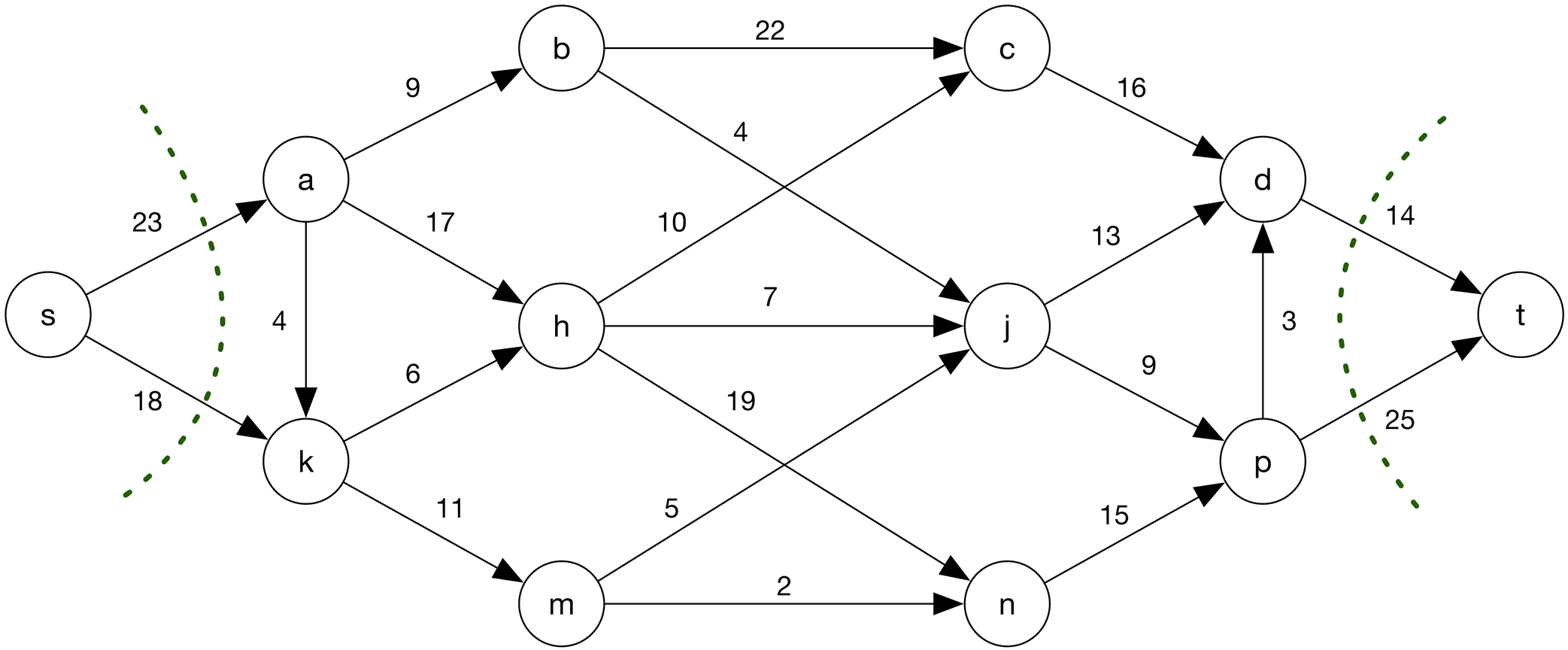

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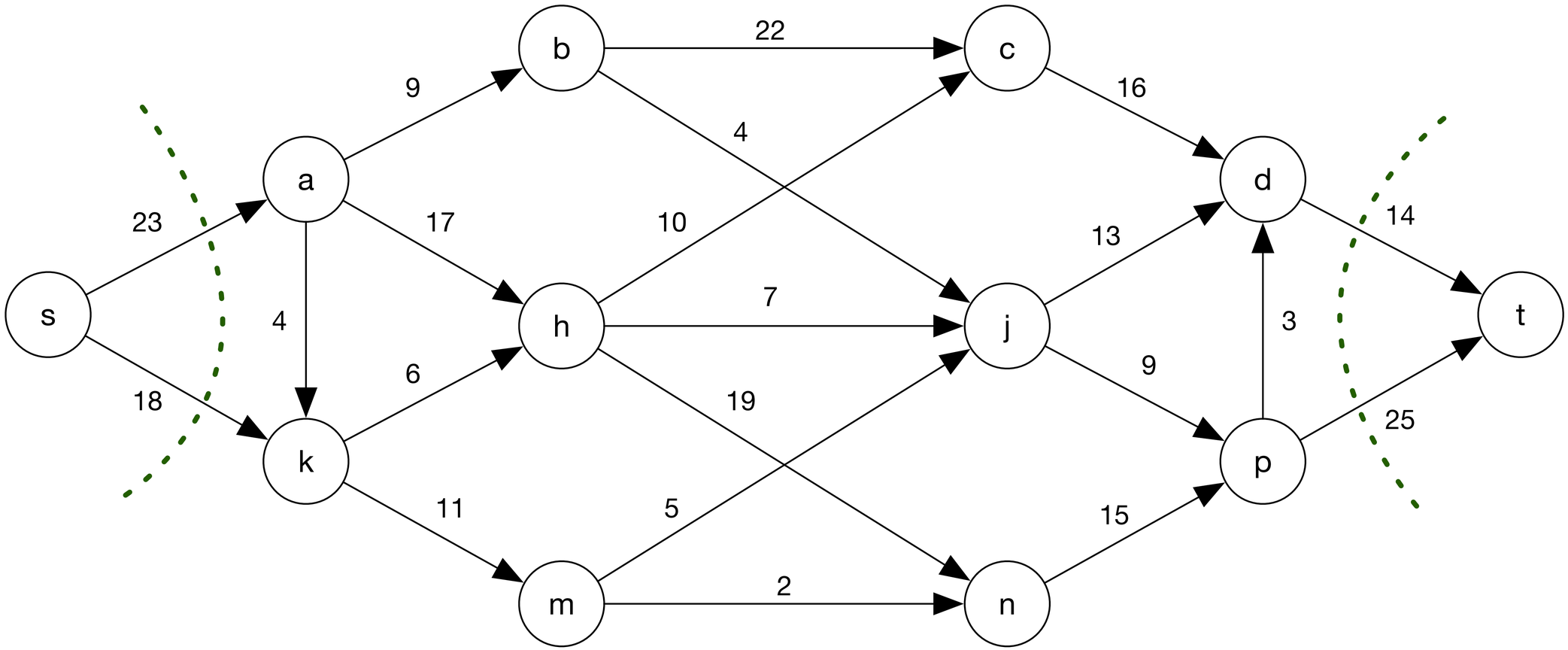

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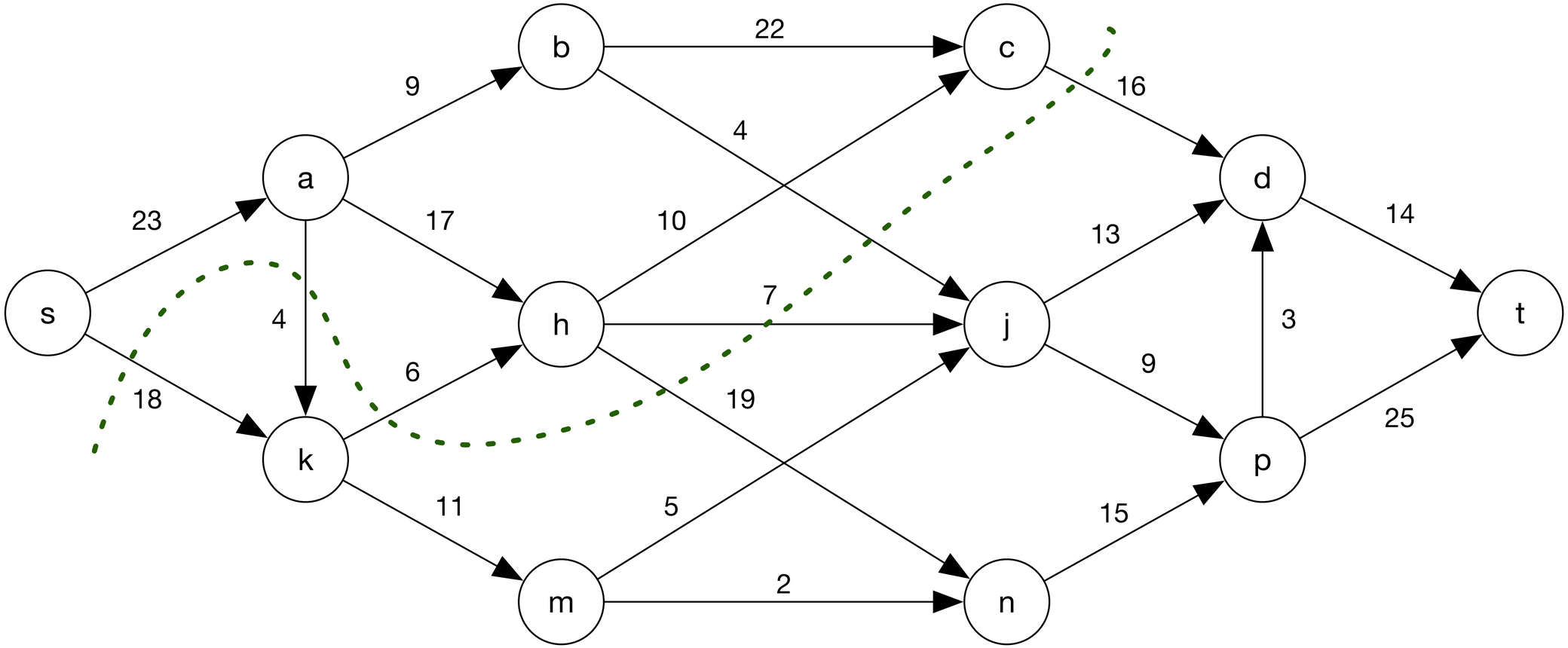

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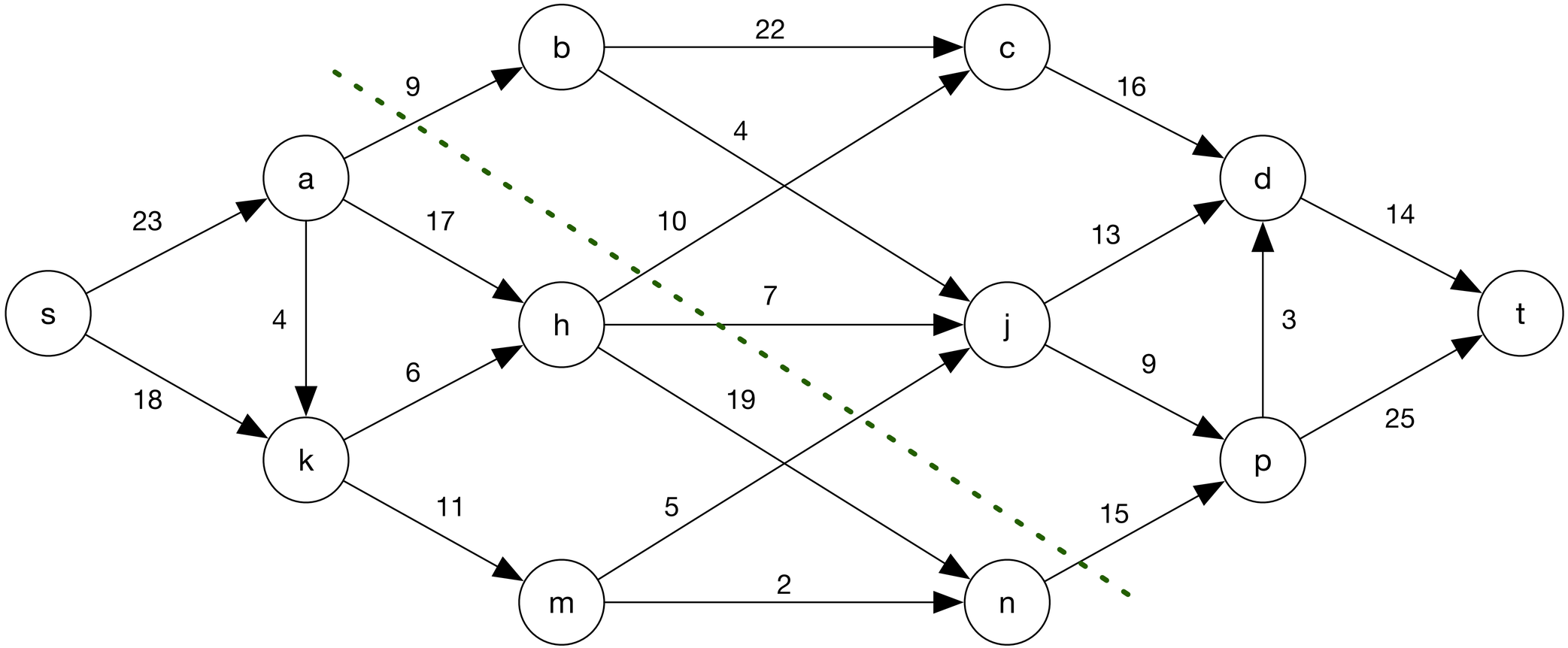

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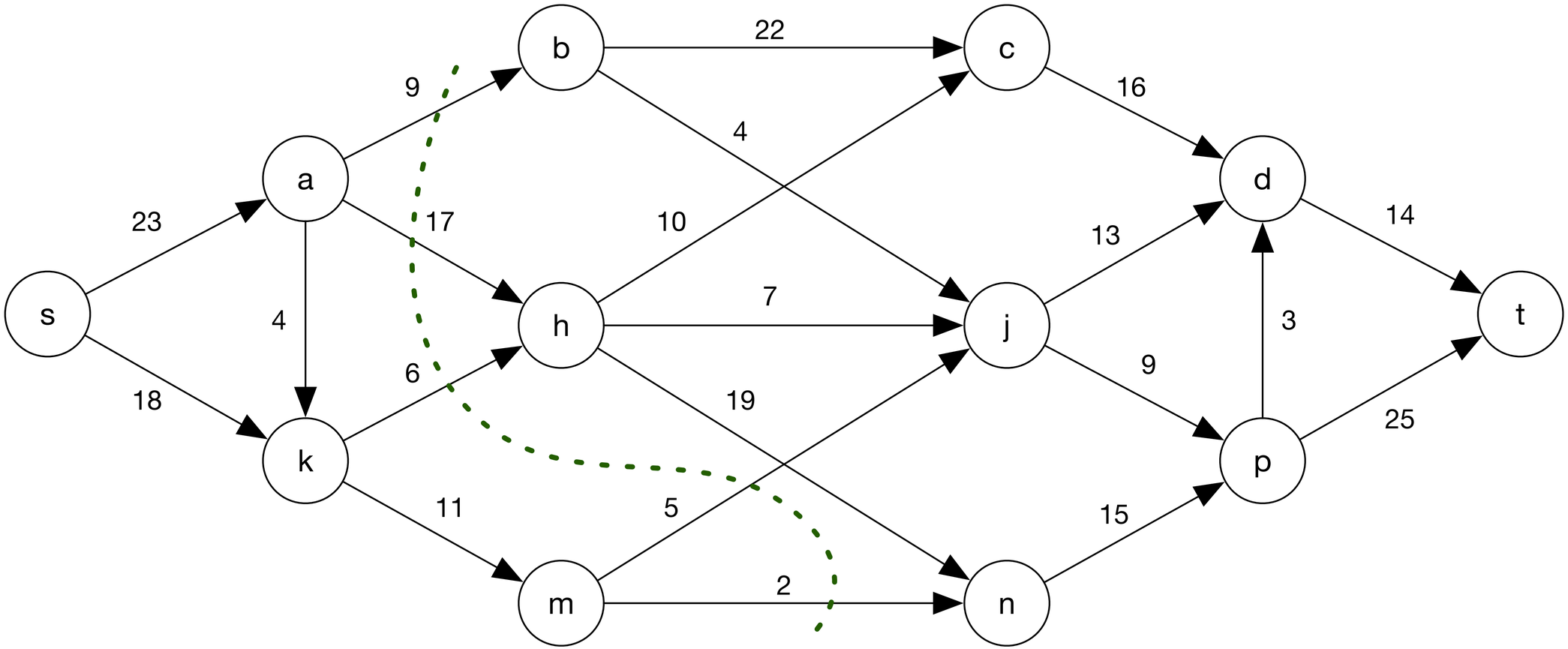

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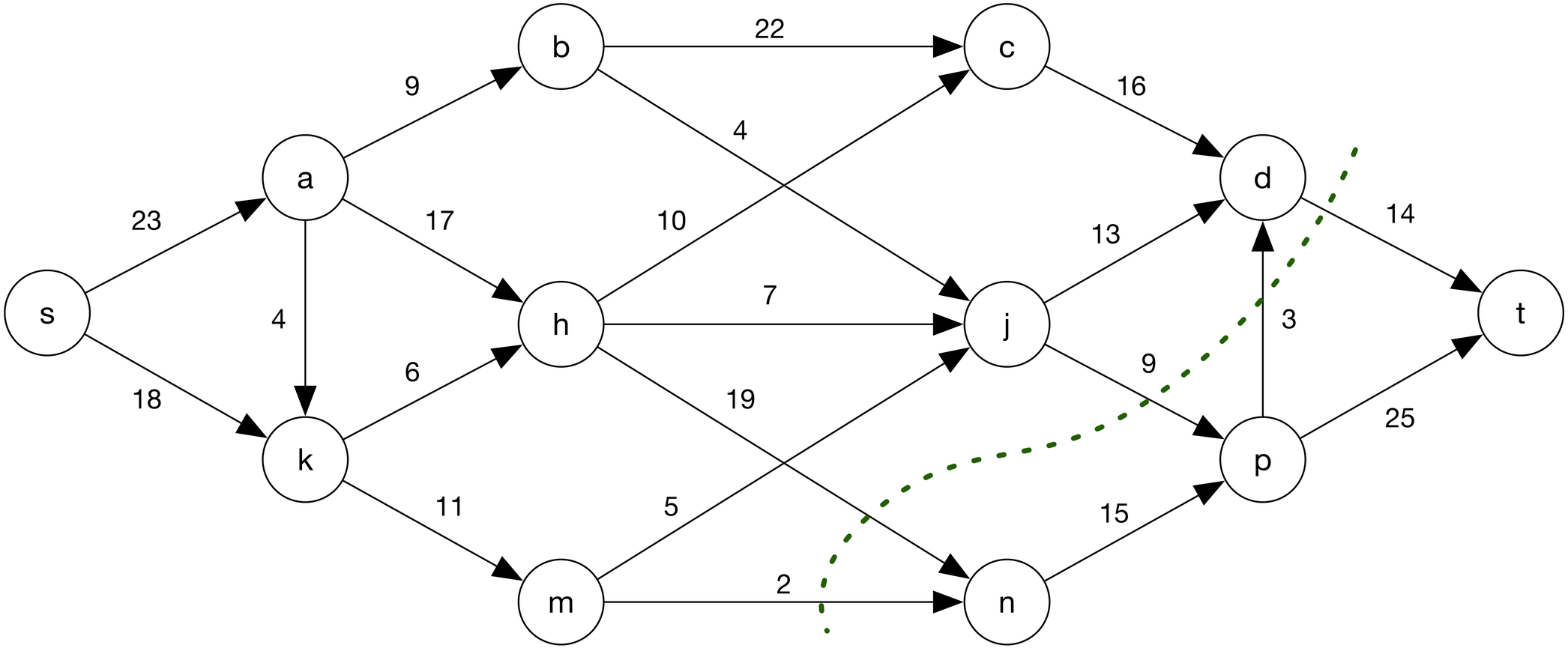

Network flow

Ford-Fulkerson algorithm

Edmonds-Karp algorithm

Karger-Stein algorithm

Network flow

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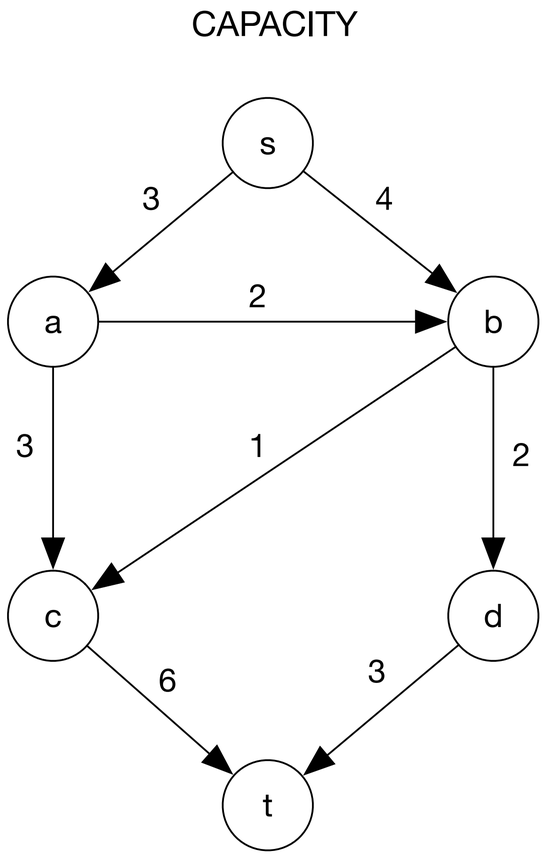
Network flow



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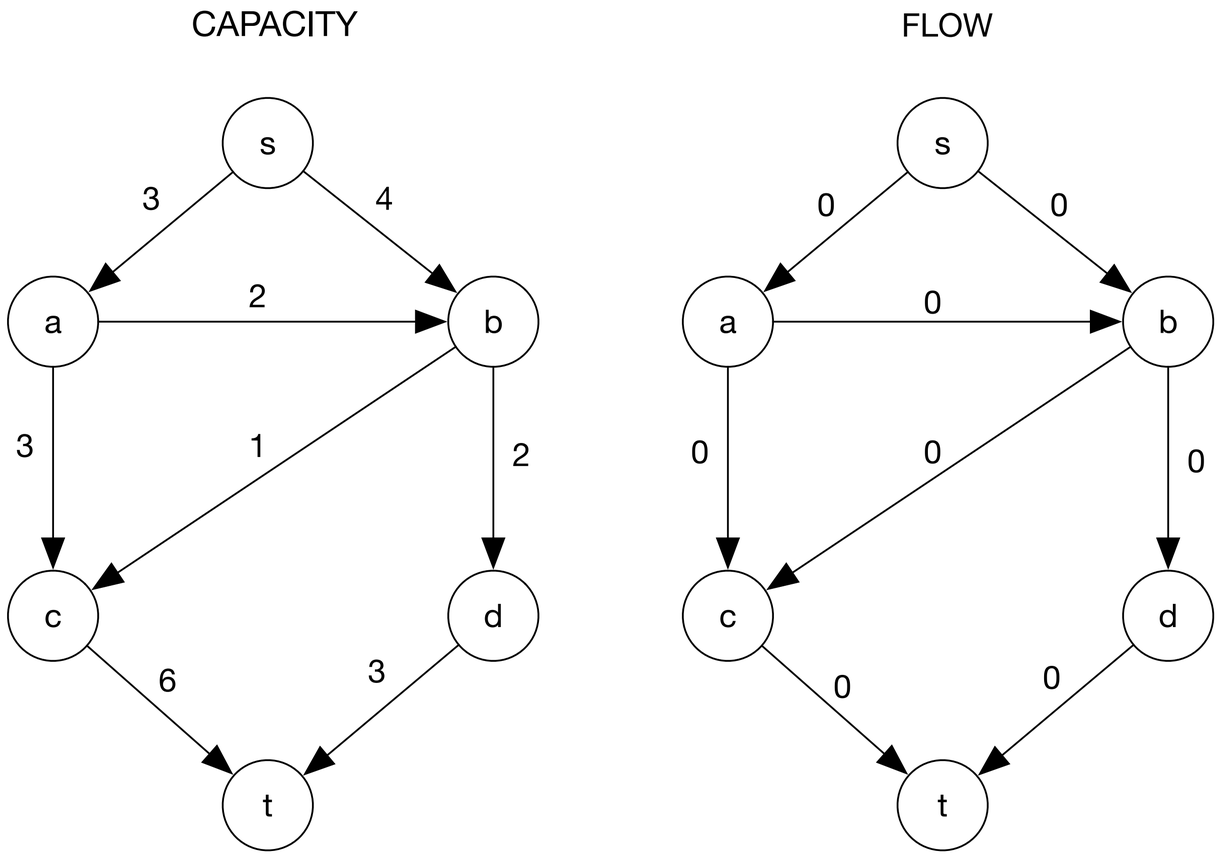

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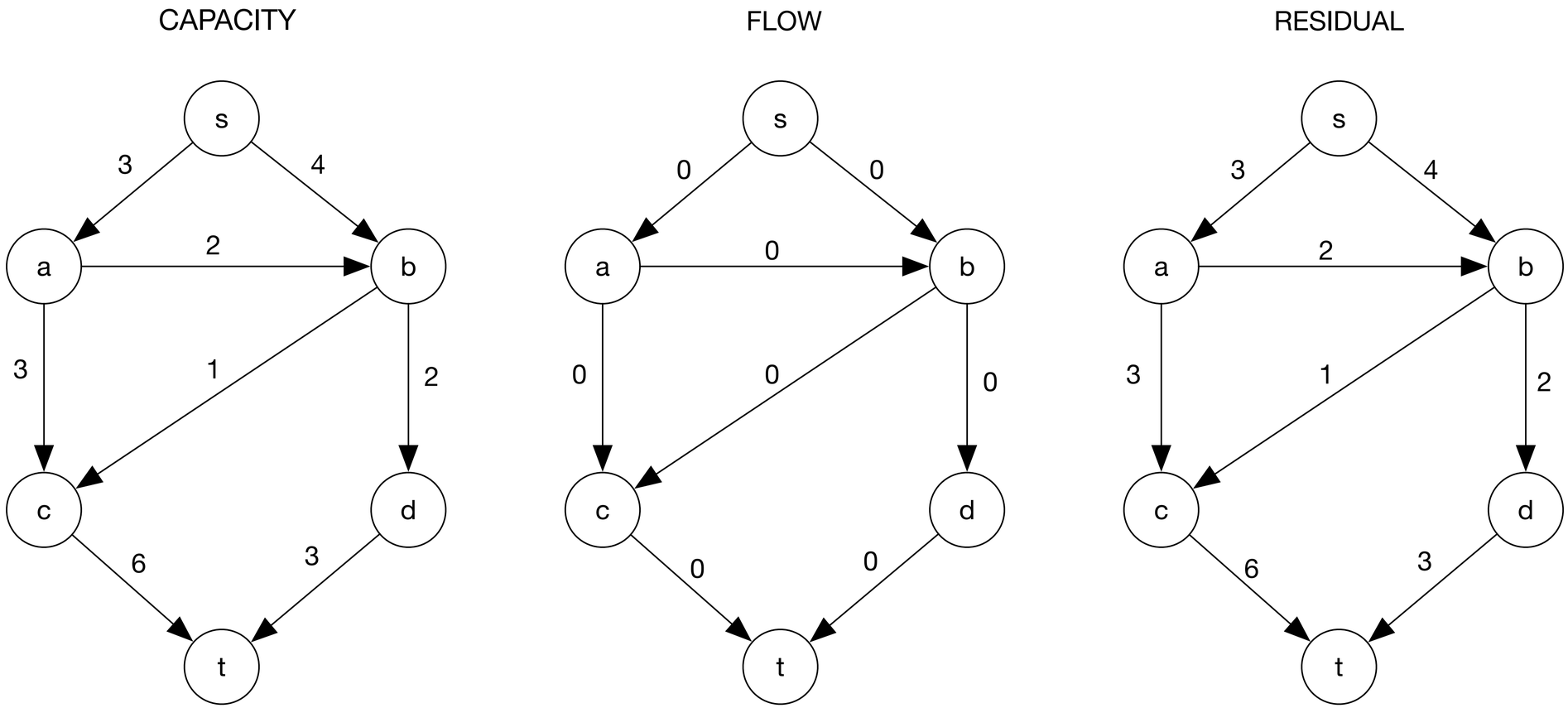

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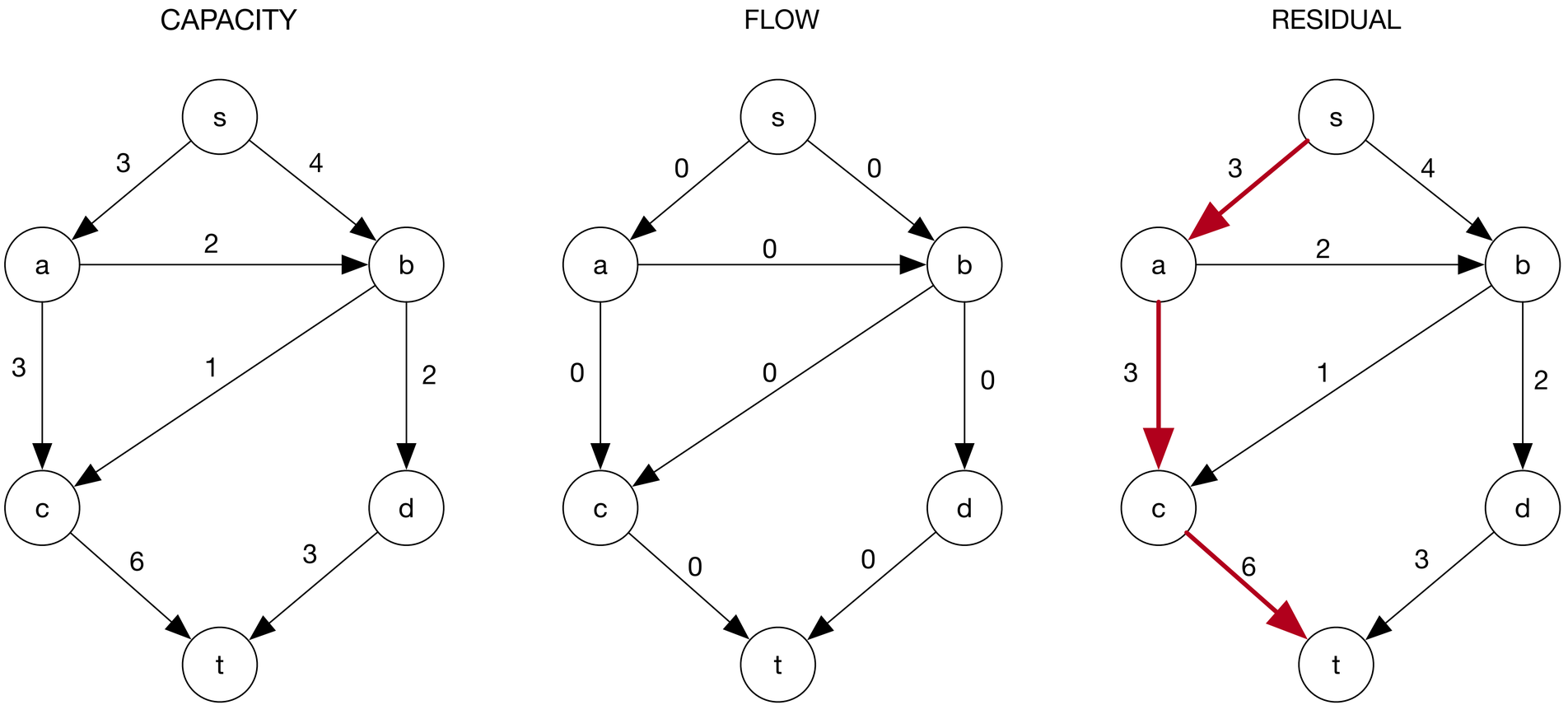

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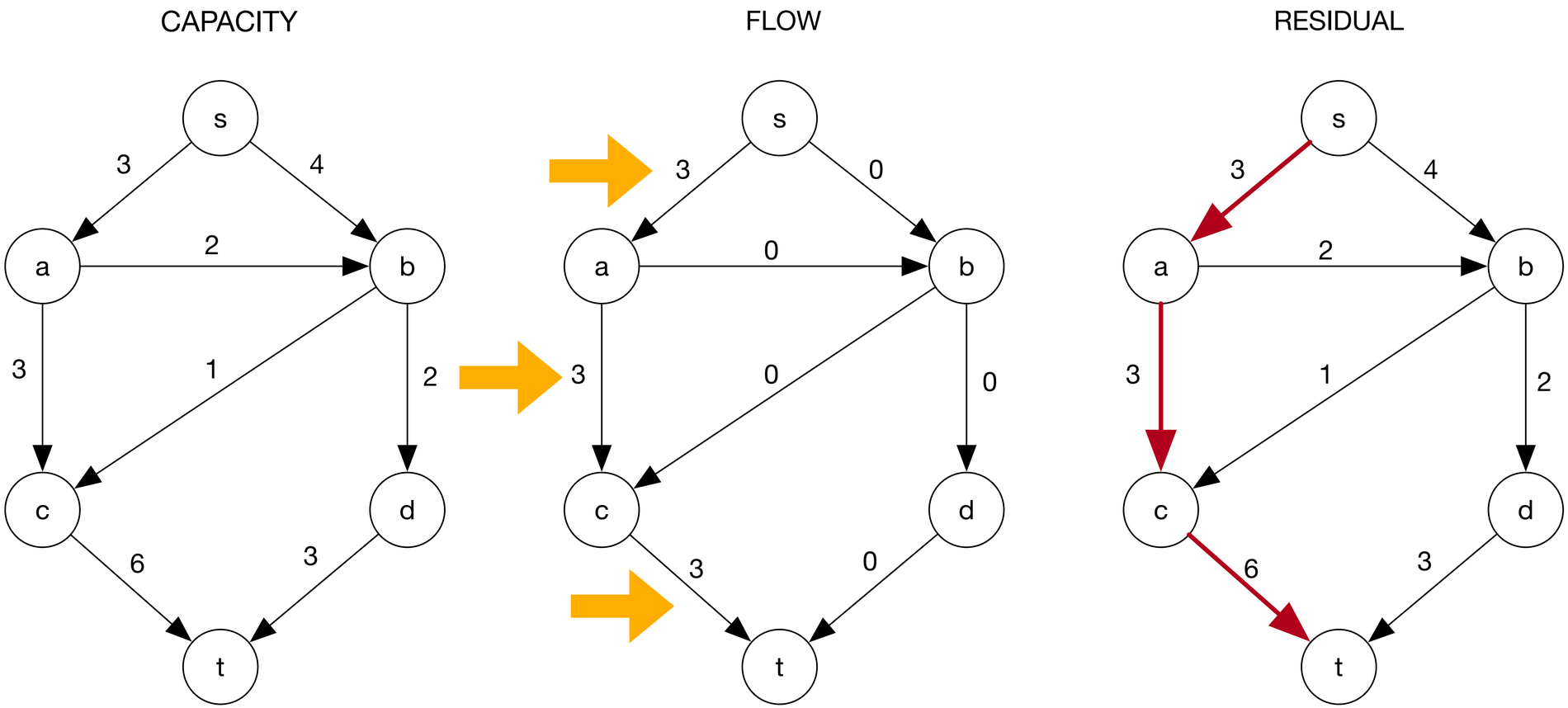

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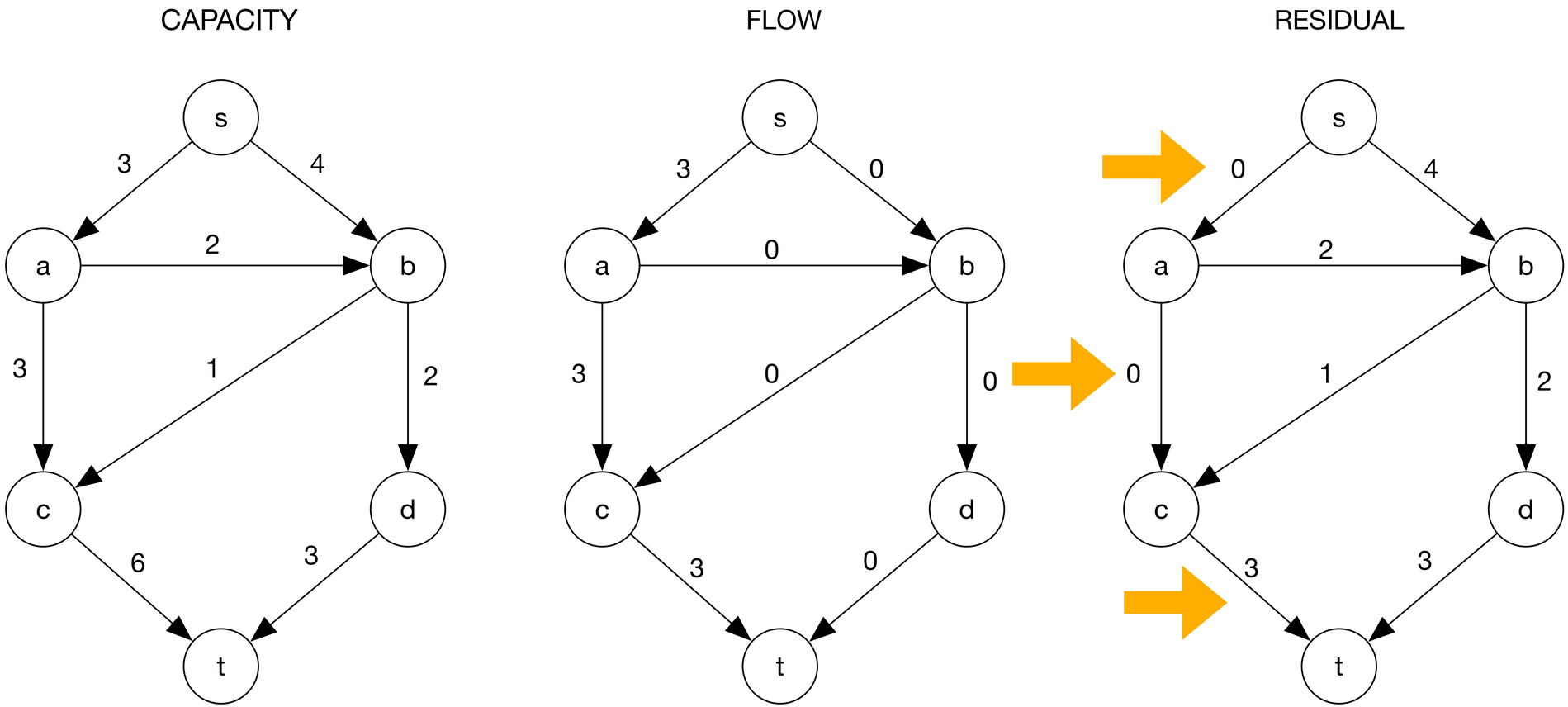

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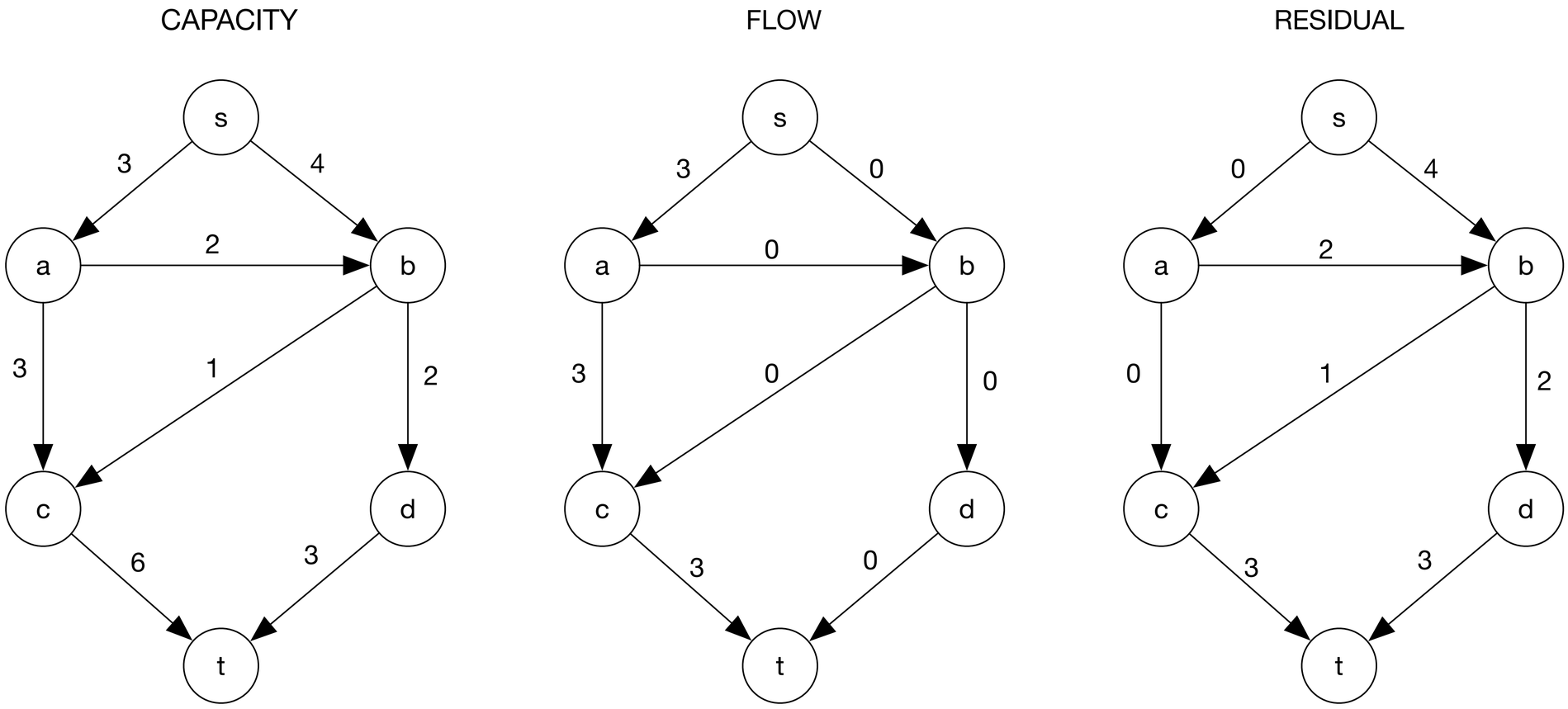

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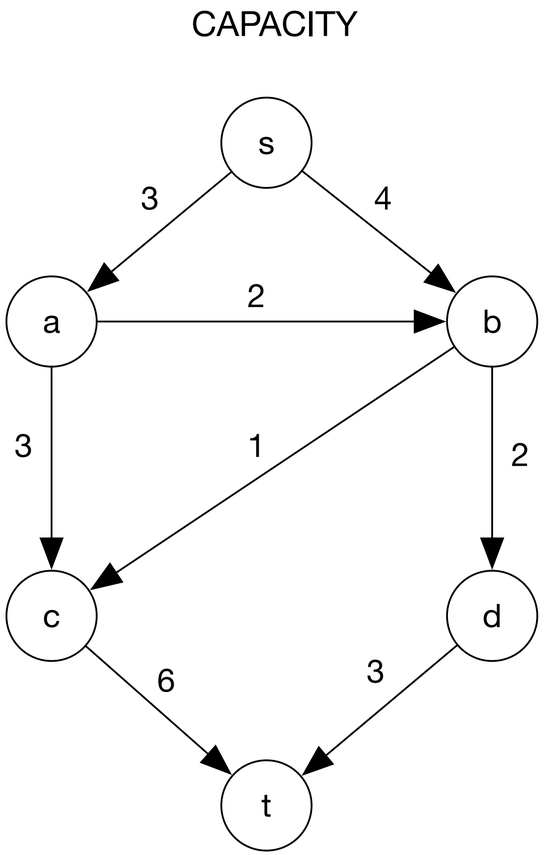

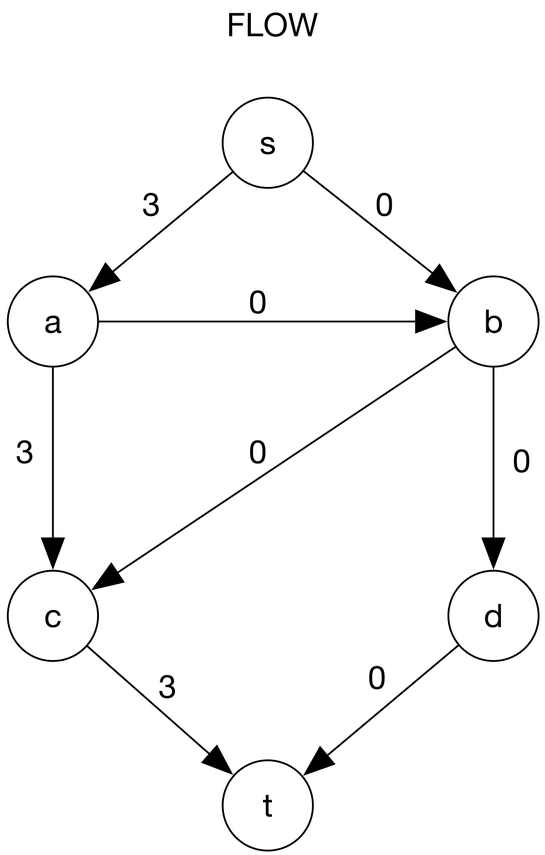

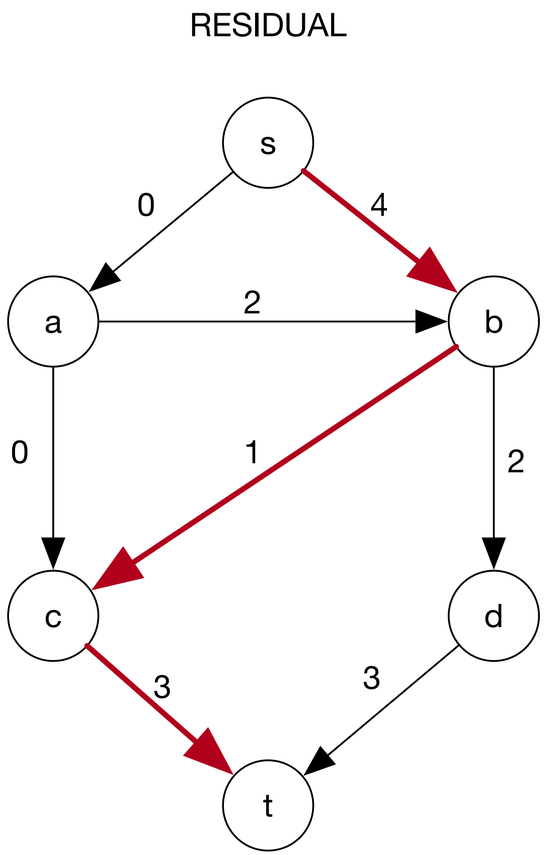

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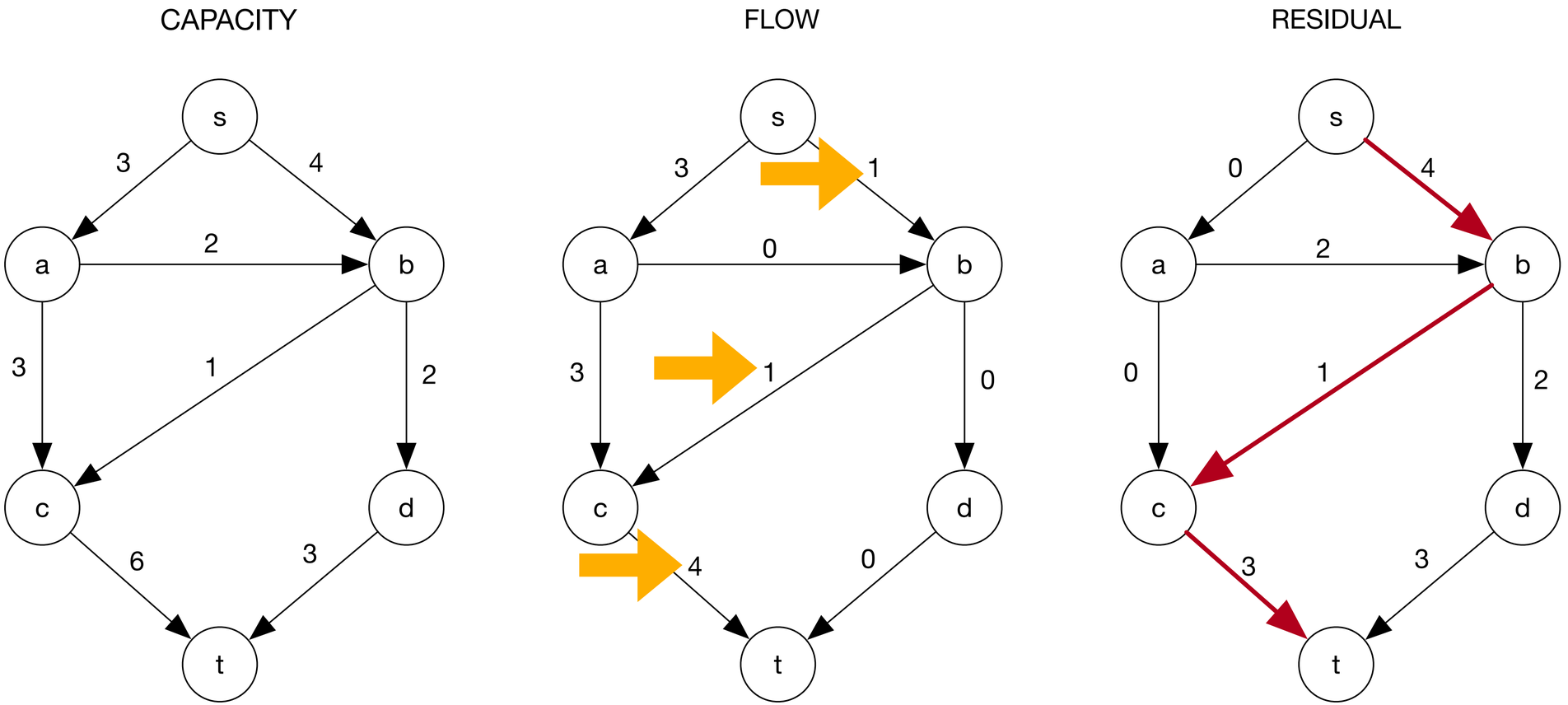

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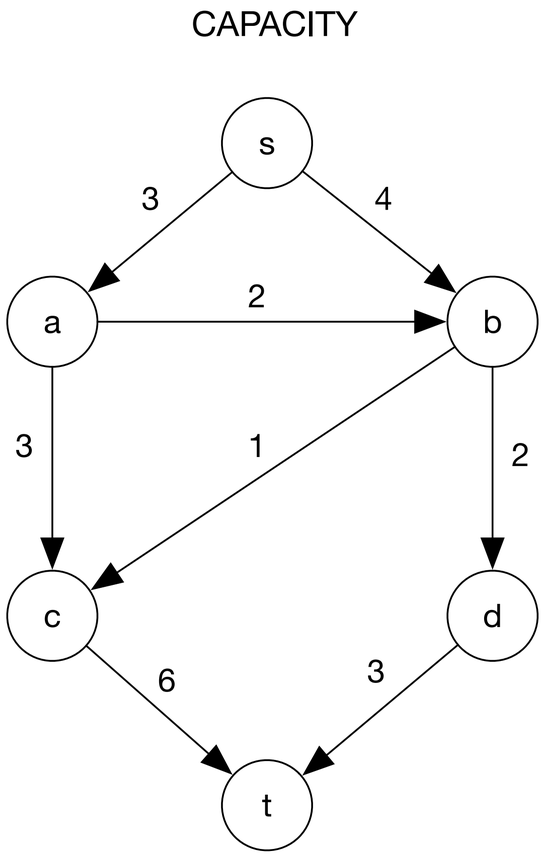
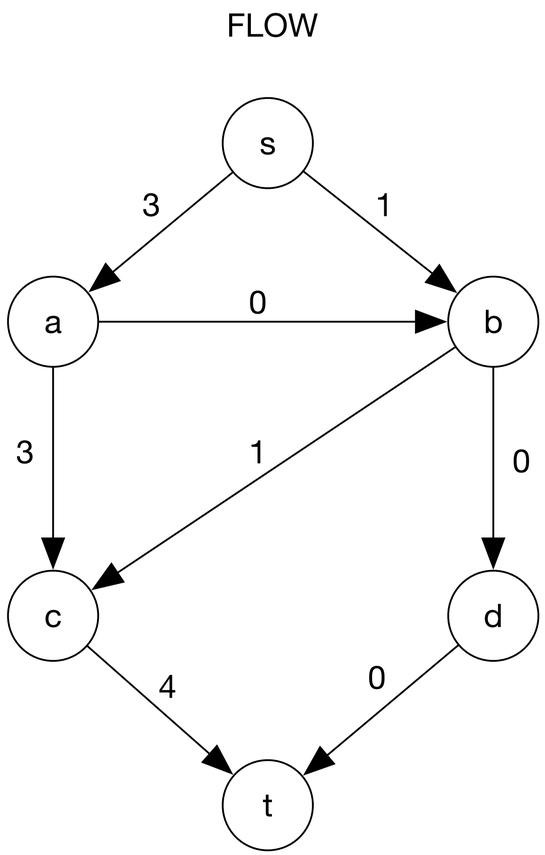
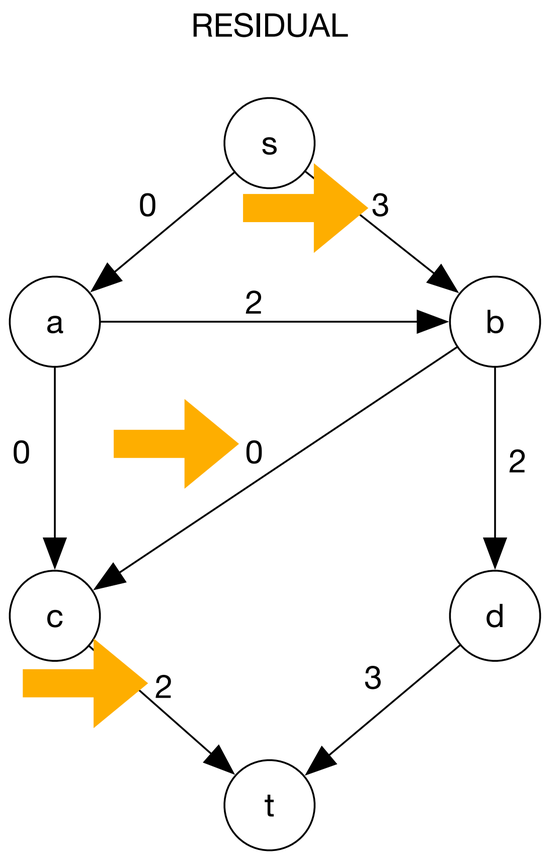
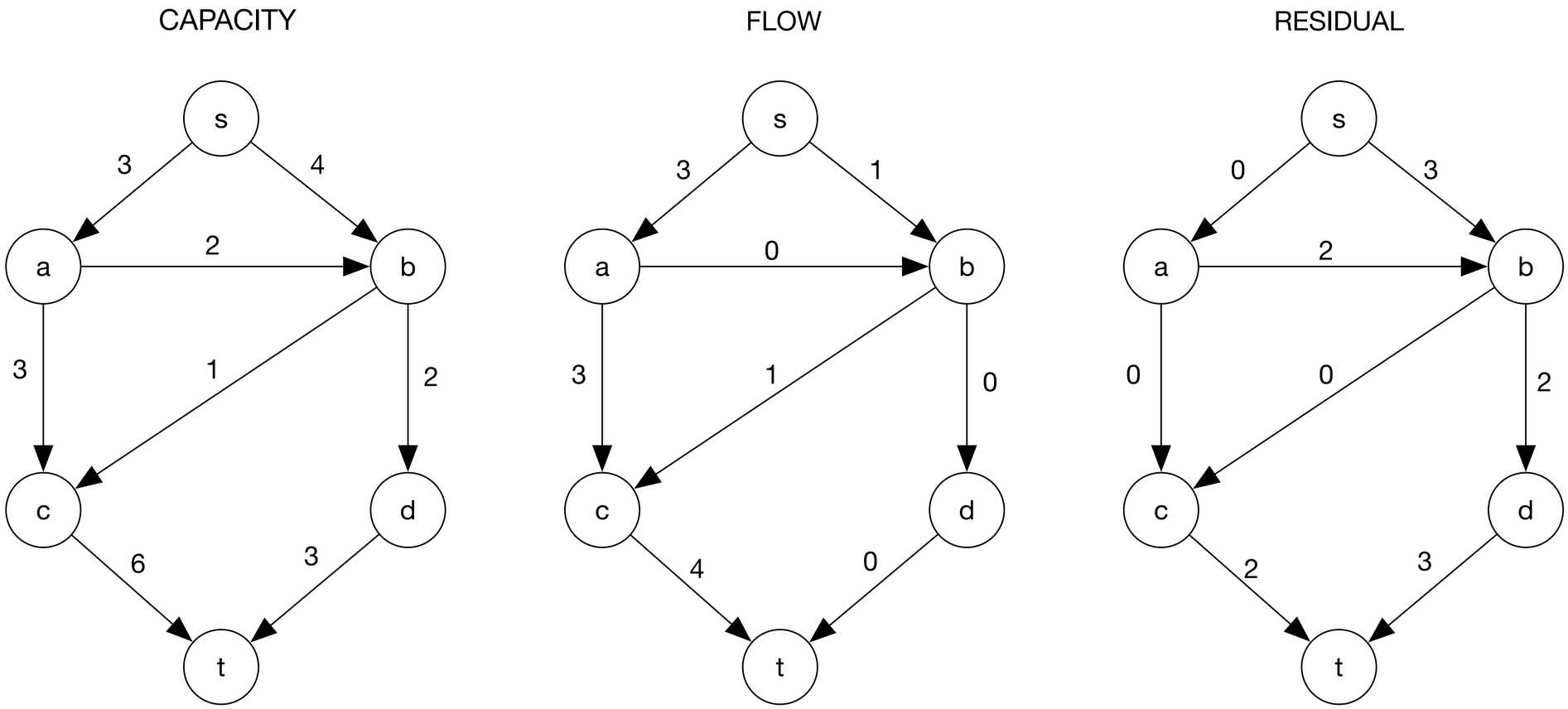


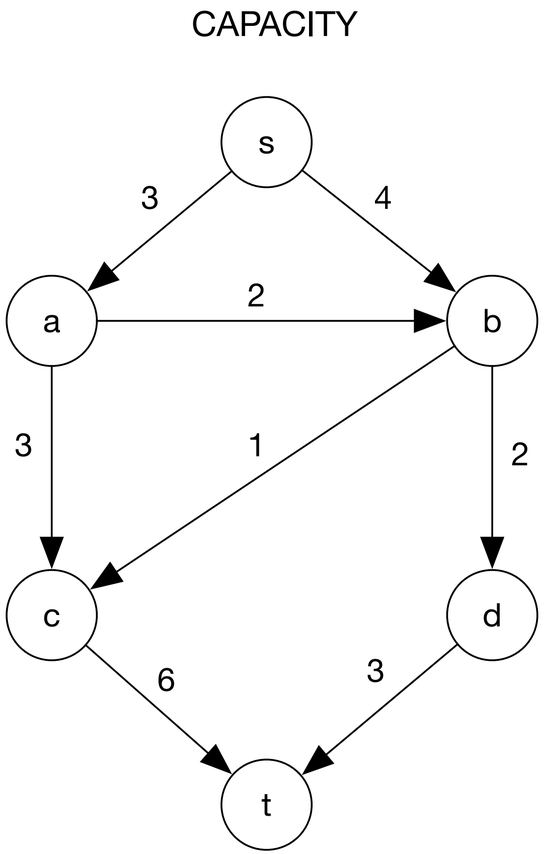


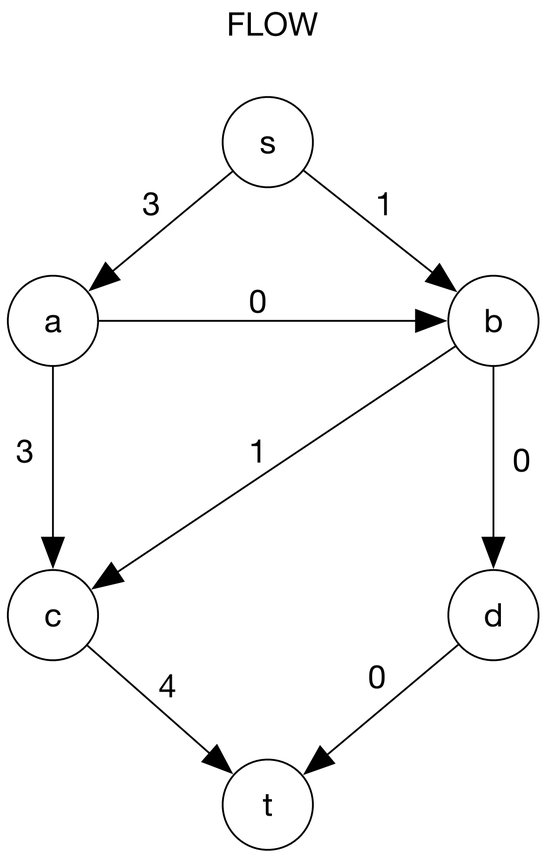


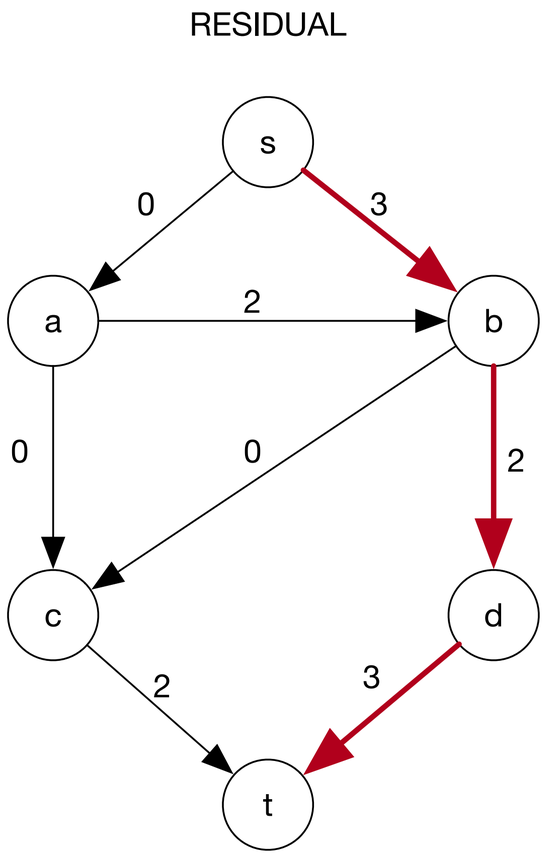
Network flow



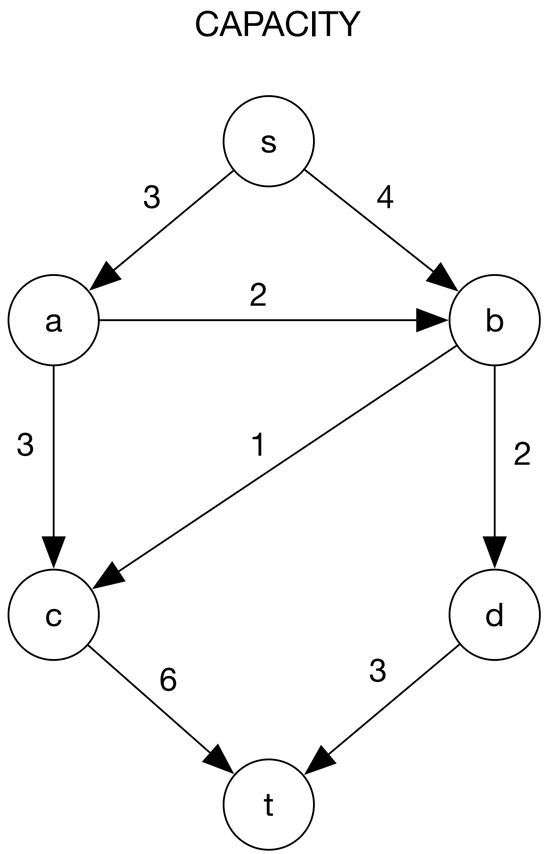
Network flow

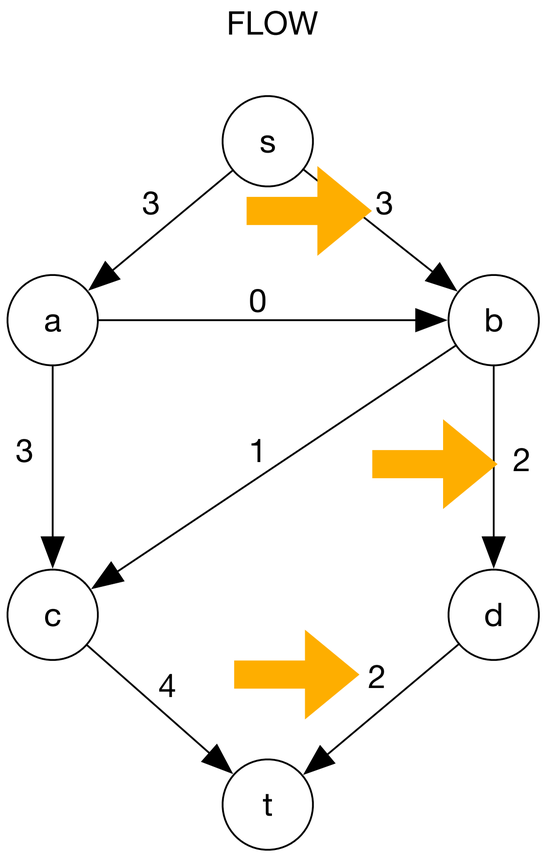


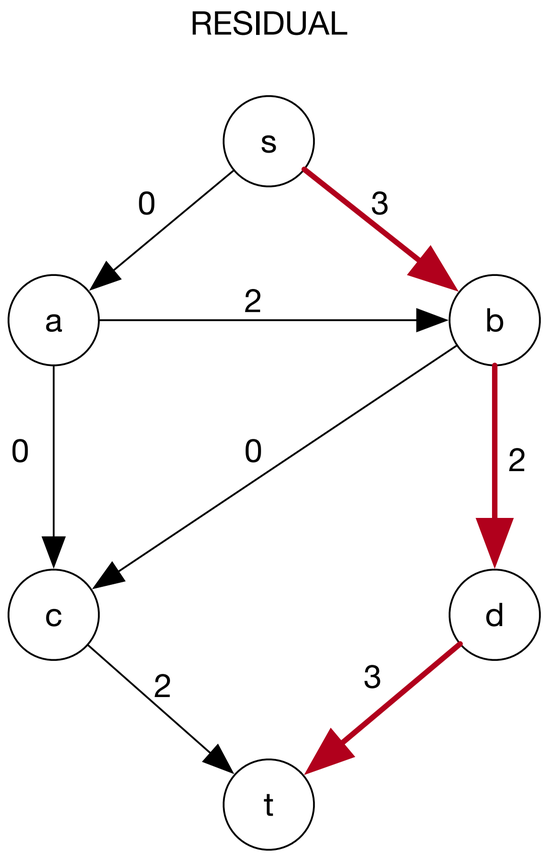




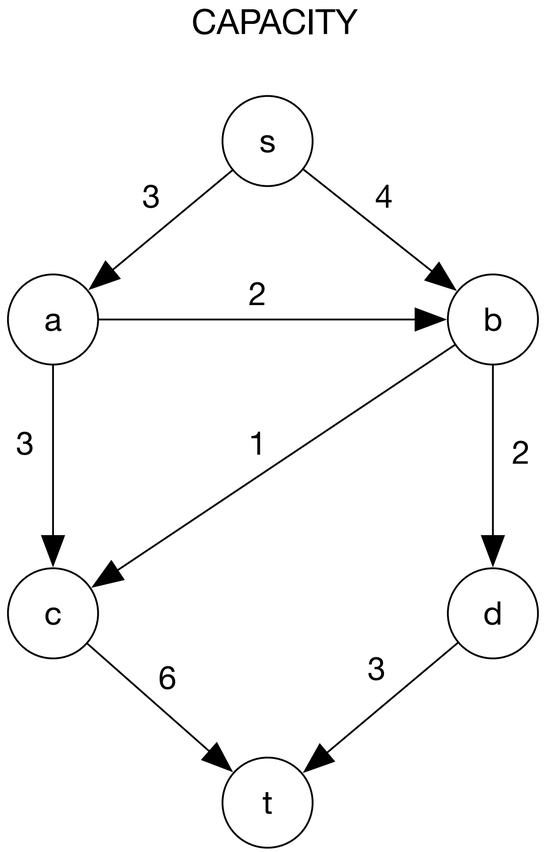
Network flow

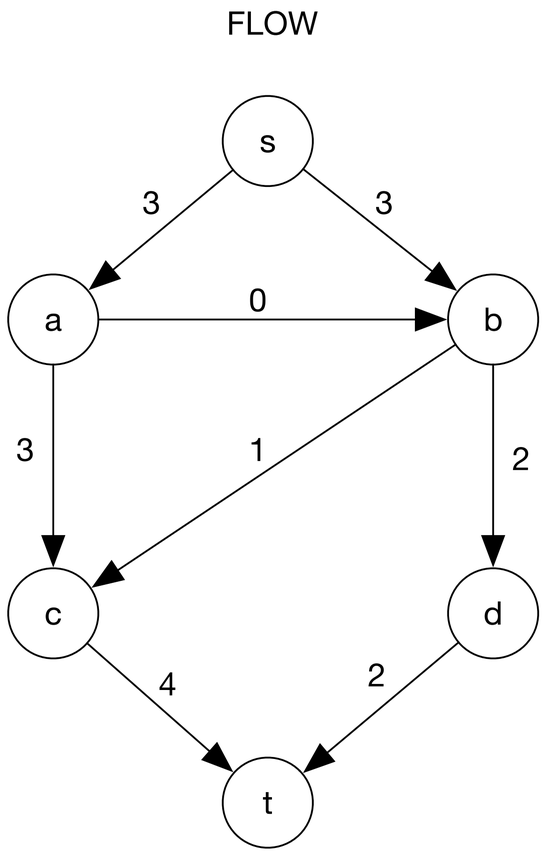


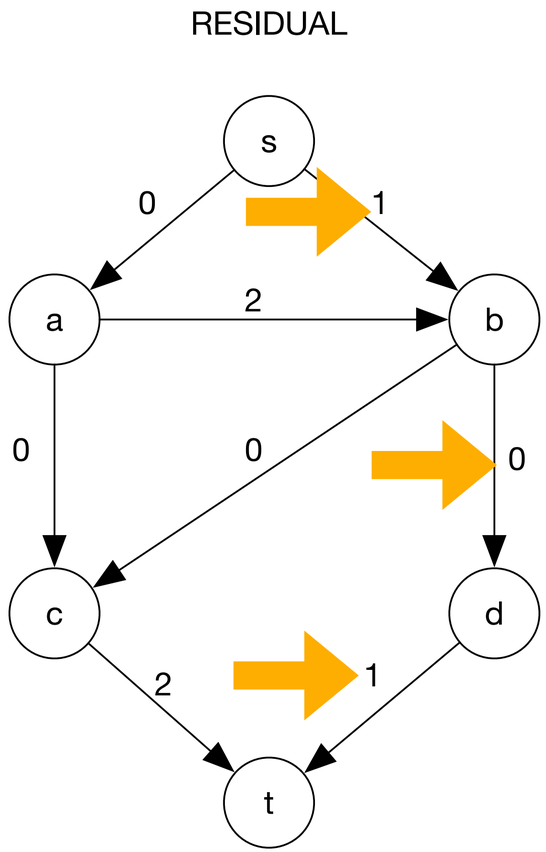




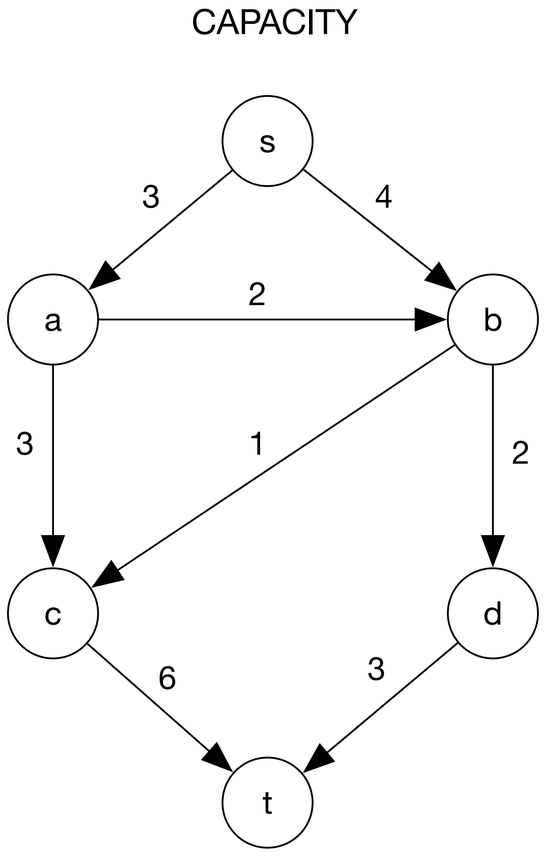
Network flow

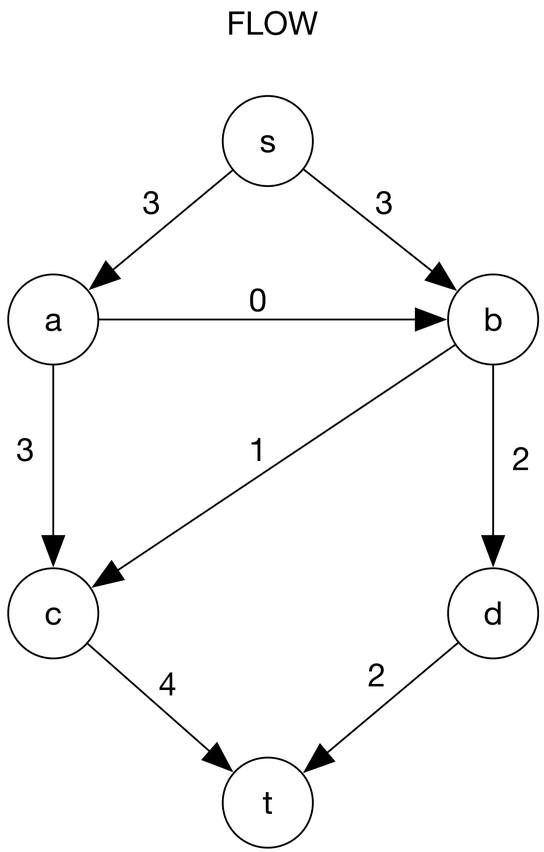


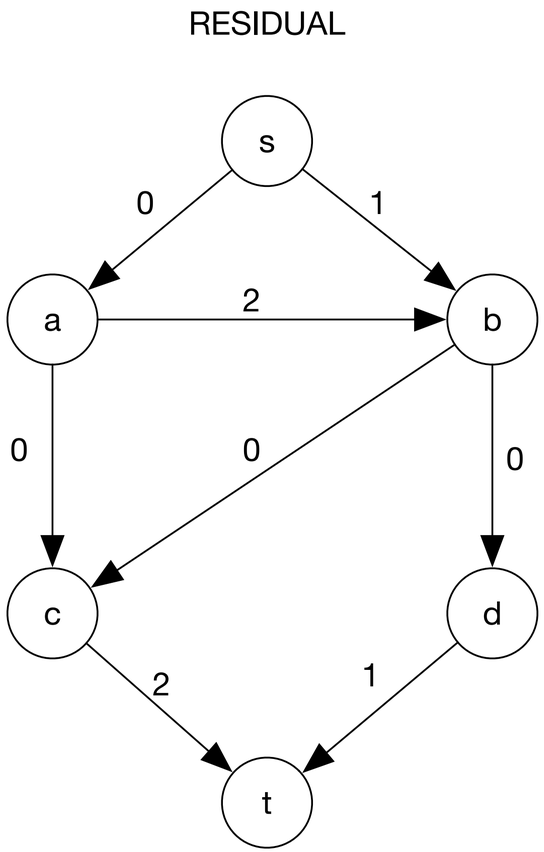




Network flow

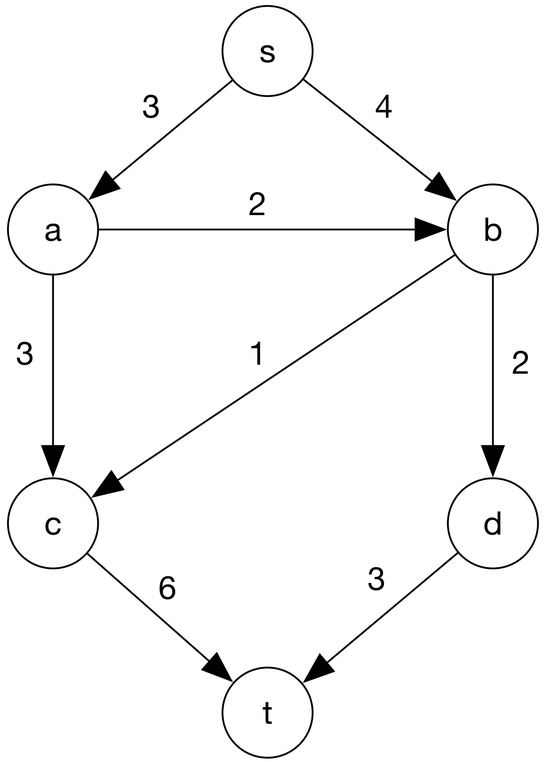




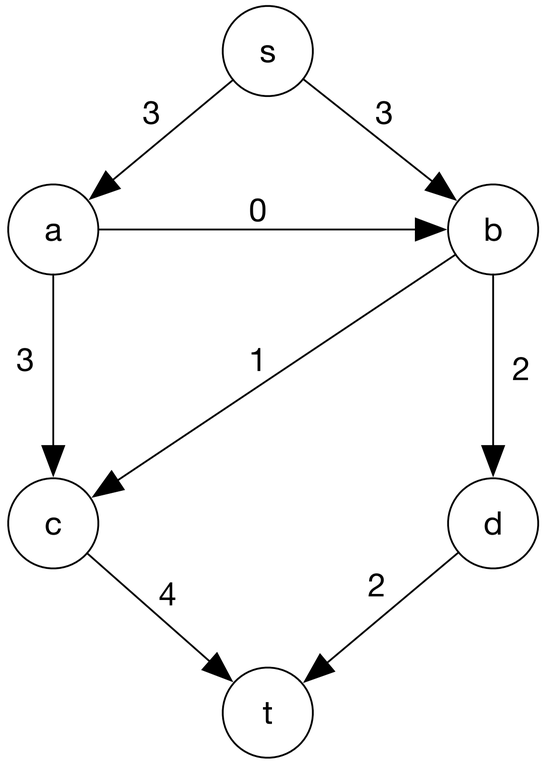


Network flow

CAPACITY



FLOW

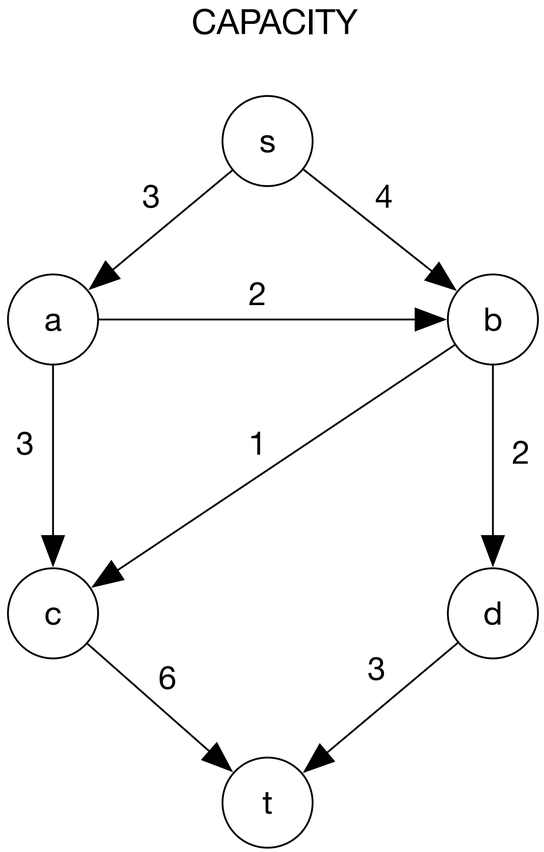


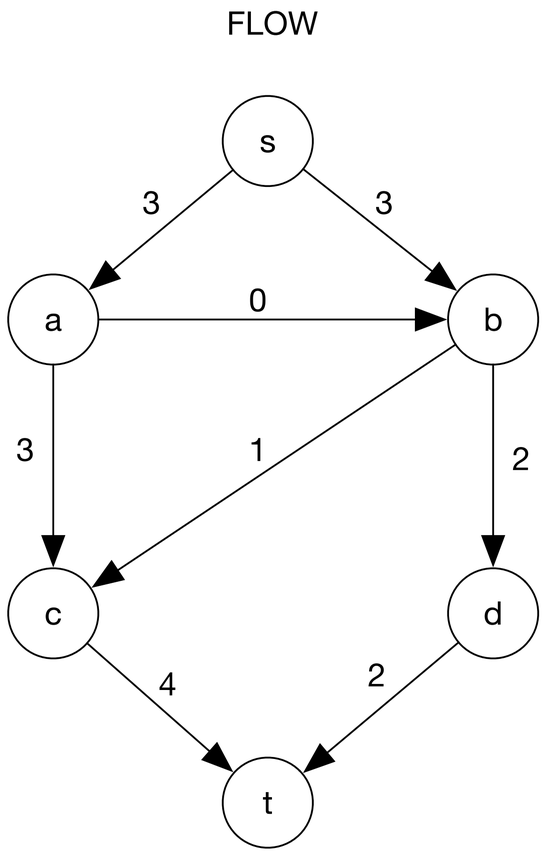
$$f_{u,v} \leq c_{u,v}$$

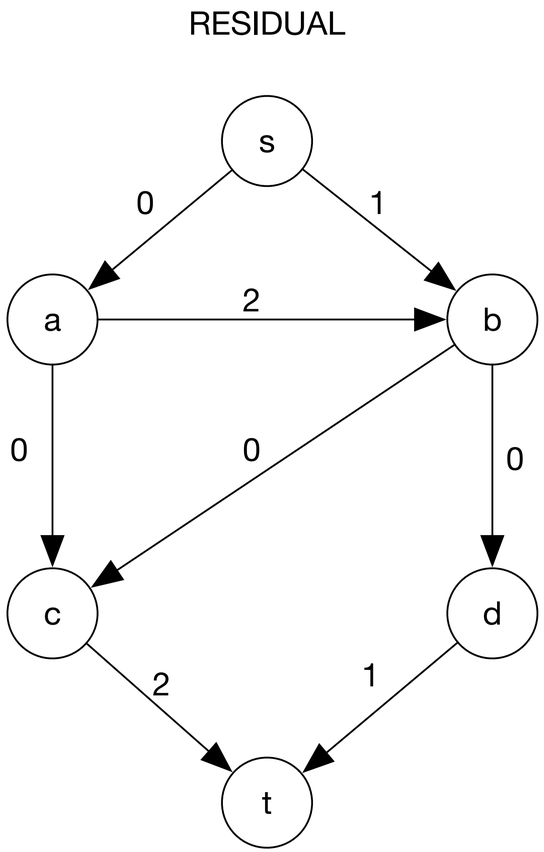
$$\sum_{u:(u,v)\in E} f_{u,v} = \sum_{w:(v,w)\in E} f_{v,w}$$

$$\sum_{u:(s,u)\in E} = \sum_{w:(w,t)\in E}$$

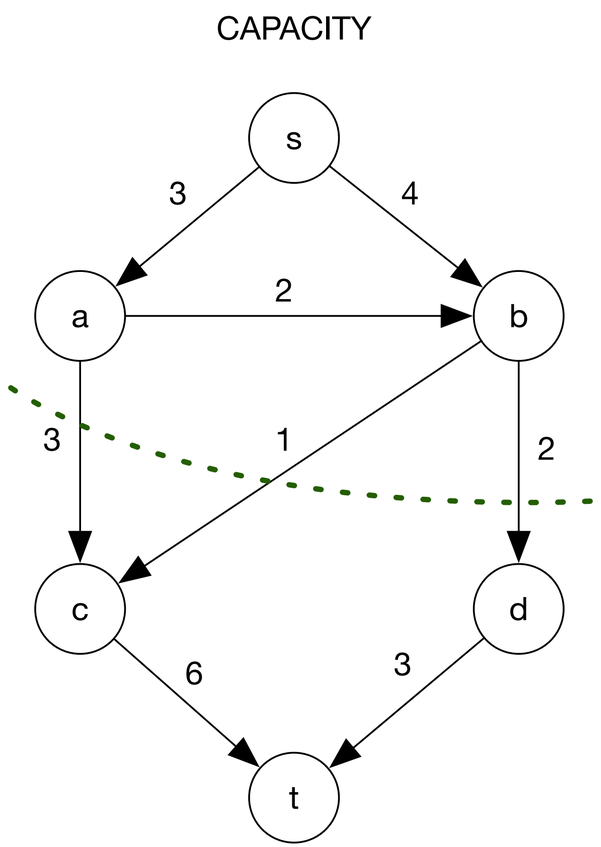
Network flow

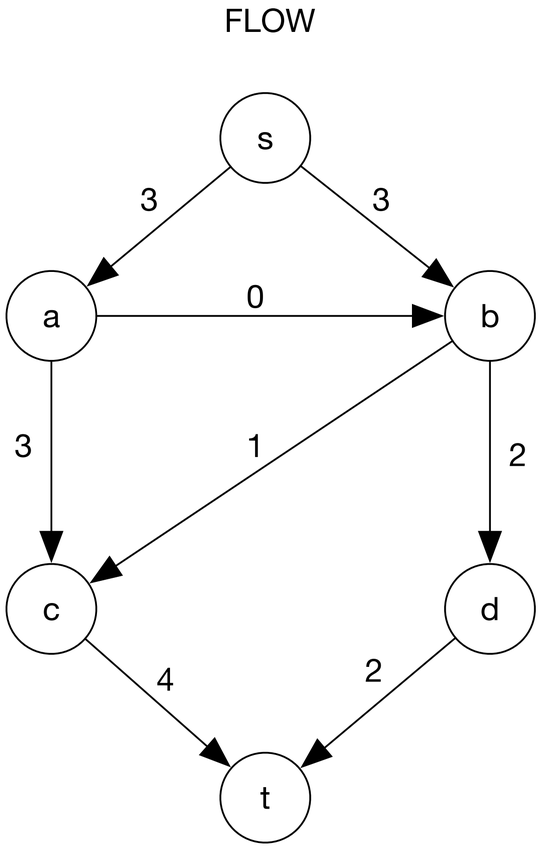


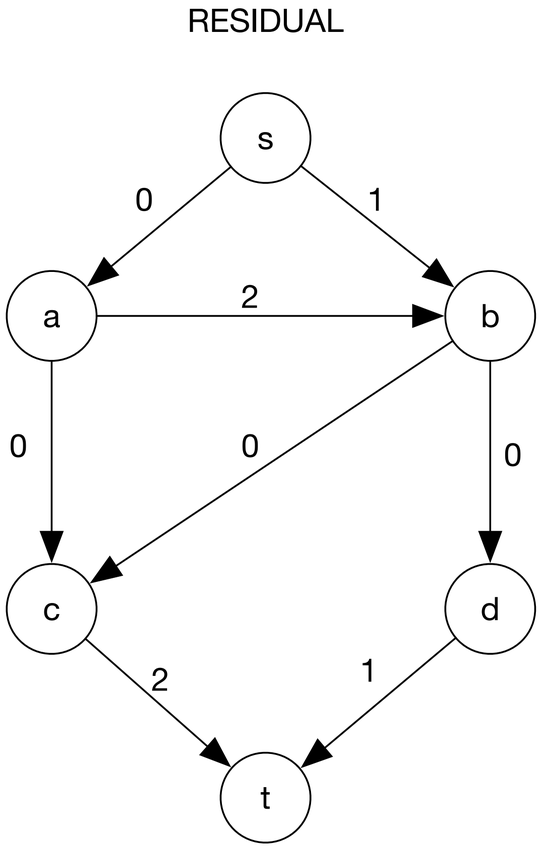




Network flow







Network flow

Is this a greedy algorithm?

Network flow

Is this a greedy algorithm? Yes!

Network flow

Is this a greedy algorithm? Yes!

What's the complexity?

Network flow

Is this a greedy algorithm? Yes!

What's the complexity? $\mathcal{O}(|E|f)$

Network flow

Is this a greedy algorithm? Yes!

What's the complexity? $\mathcal{O}(|E|f)$

How do we find augmenting paths?

Network flow

Is this a greedy algorithm? Yes!

What's the complexity? $\mathcal{O}(|E|f)$

How do we find augmenting paths? It depends.