THE UNIVERSITY OF VERMONT
COLLEGE OF ENGINEERING &
MATHEMATICAL SCIENCES

Introduction to Graphs

CS 124 / Department of Computer Science

Introduction to graphs

TBH, we've already had an introduction to graphs.

Introduction to graphs

TBH, we've already had an introduction to graphs.

Trees are graphs.

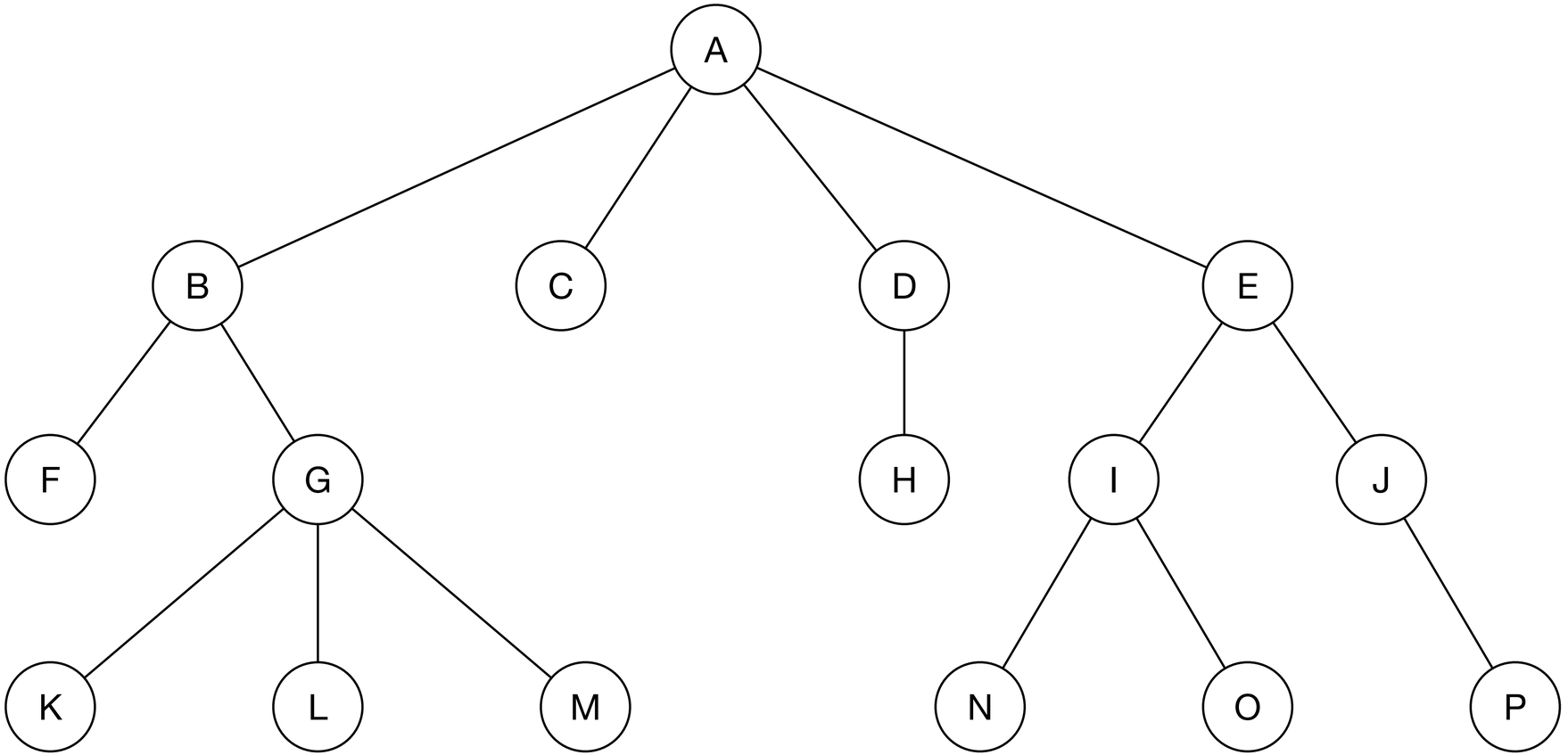
Introduction to graphs

Terms we've introduced before

- Node (or vertex)
- Edge
- Degree (# of edges incident to a node)
- Path
- Cycle

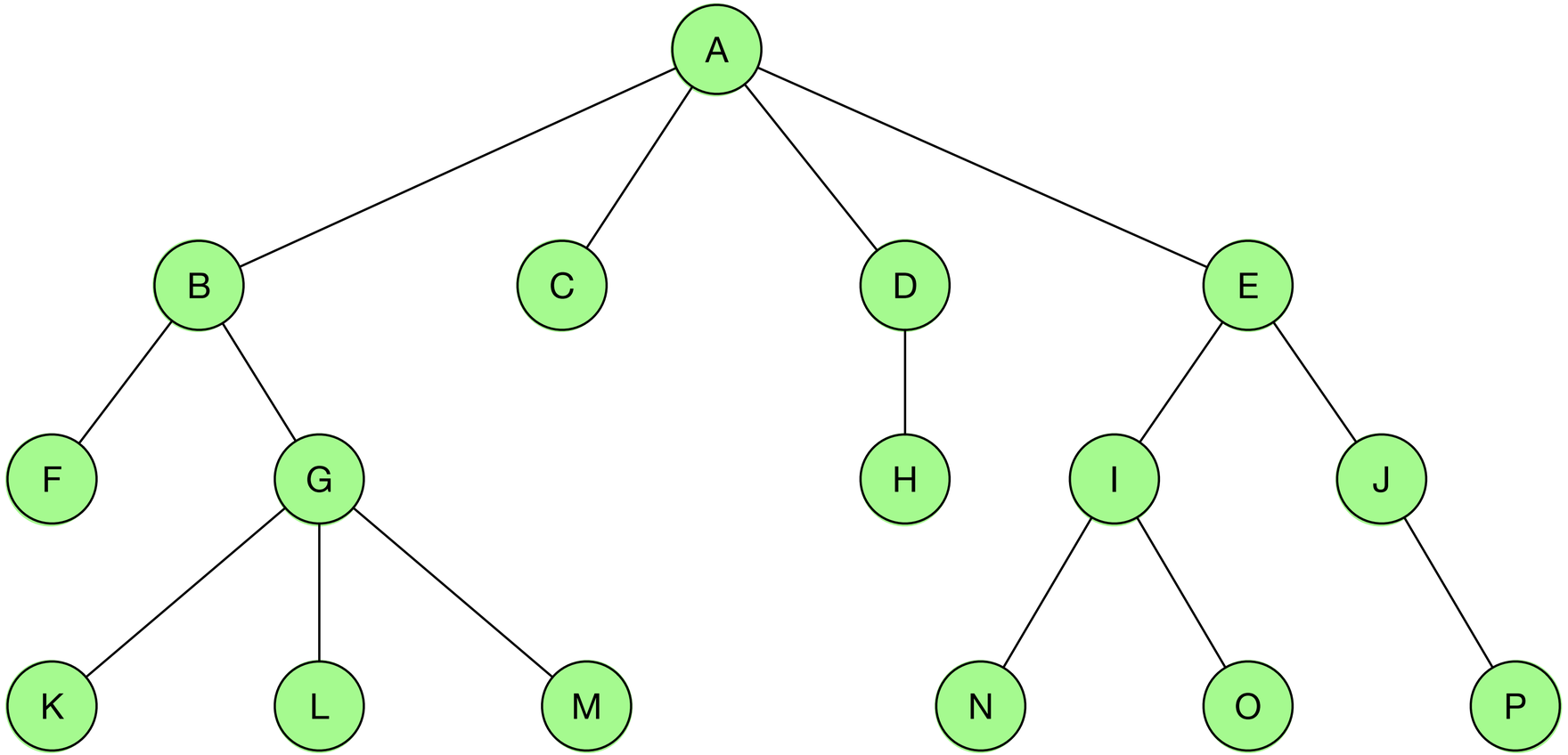
What is a tree?

A tree is a structure consisting of *nodes* (a.k.a. vertices) and *edges*.



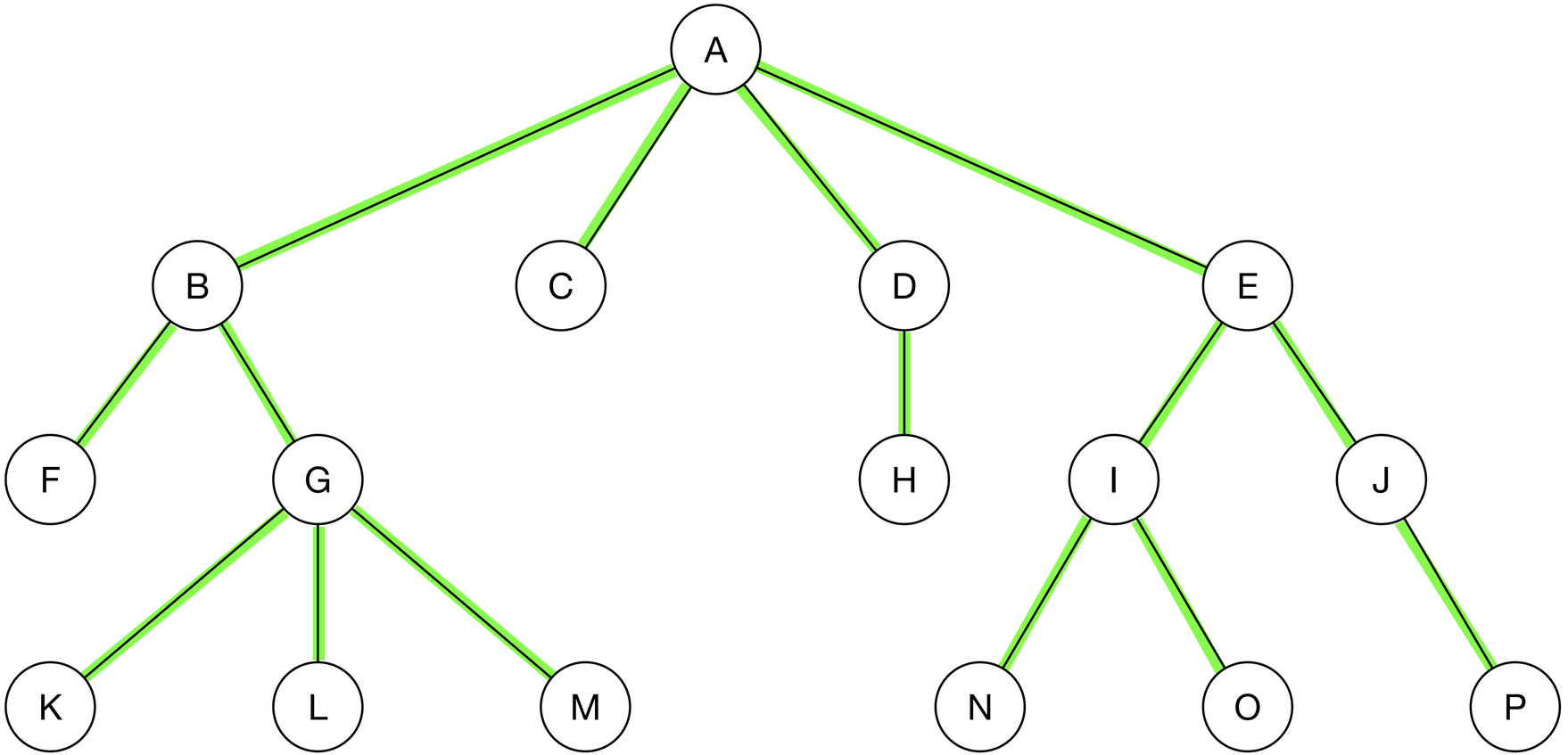
What is a tree?

These are the nodes...



What is a tree?

...and these are the edges.



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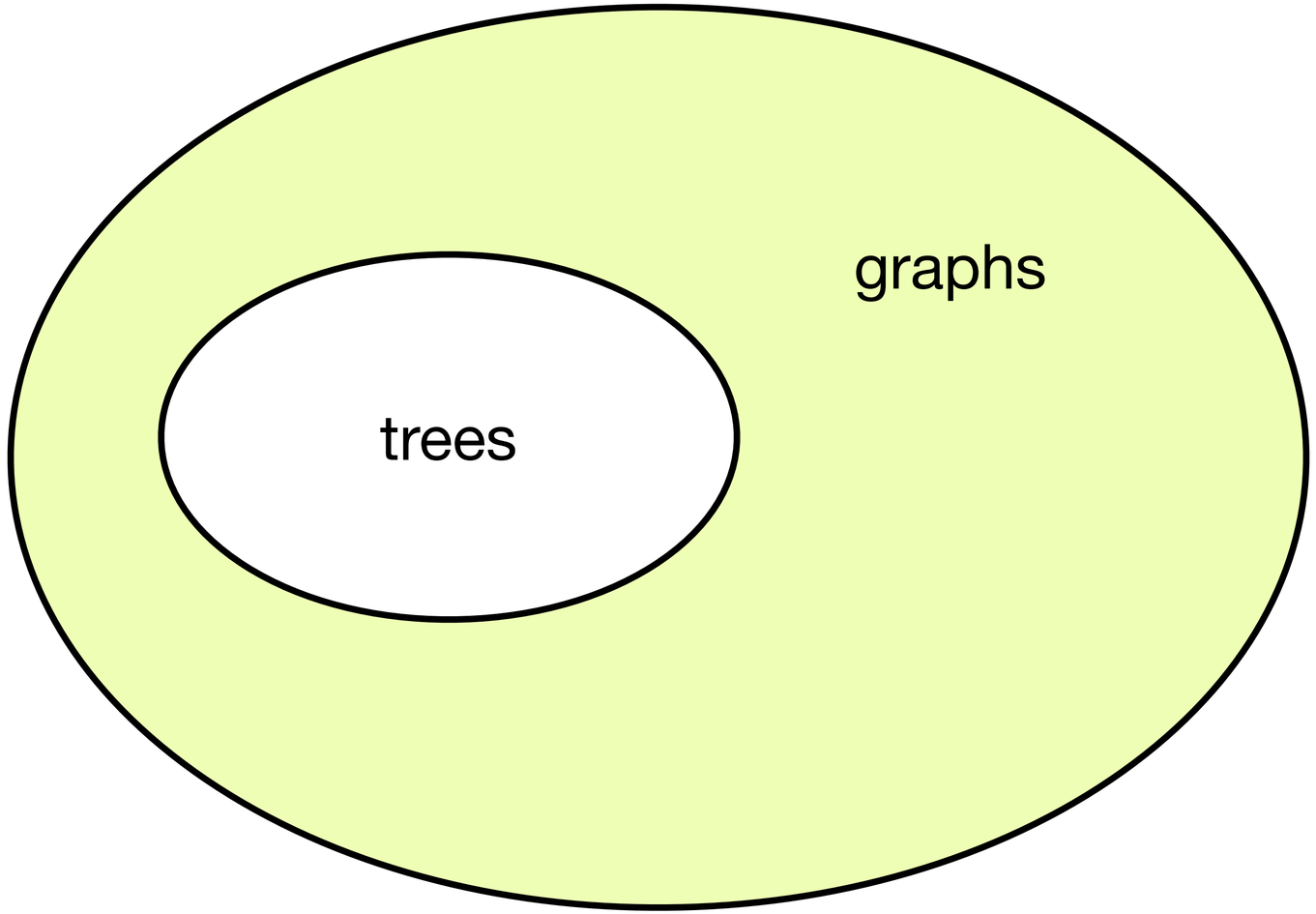
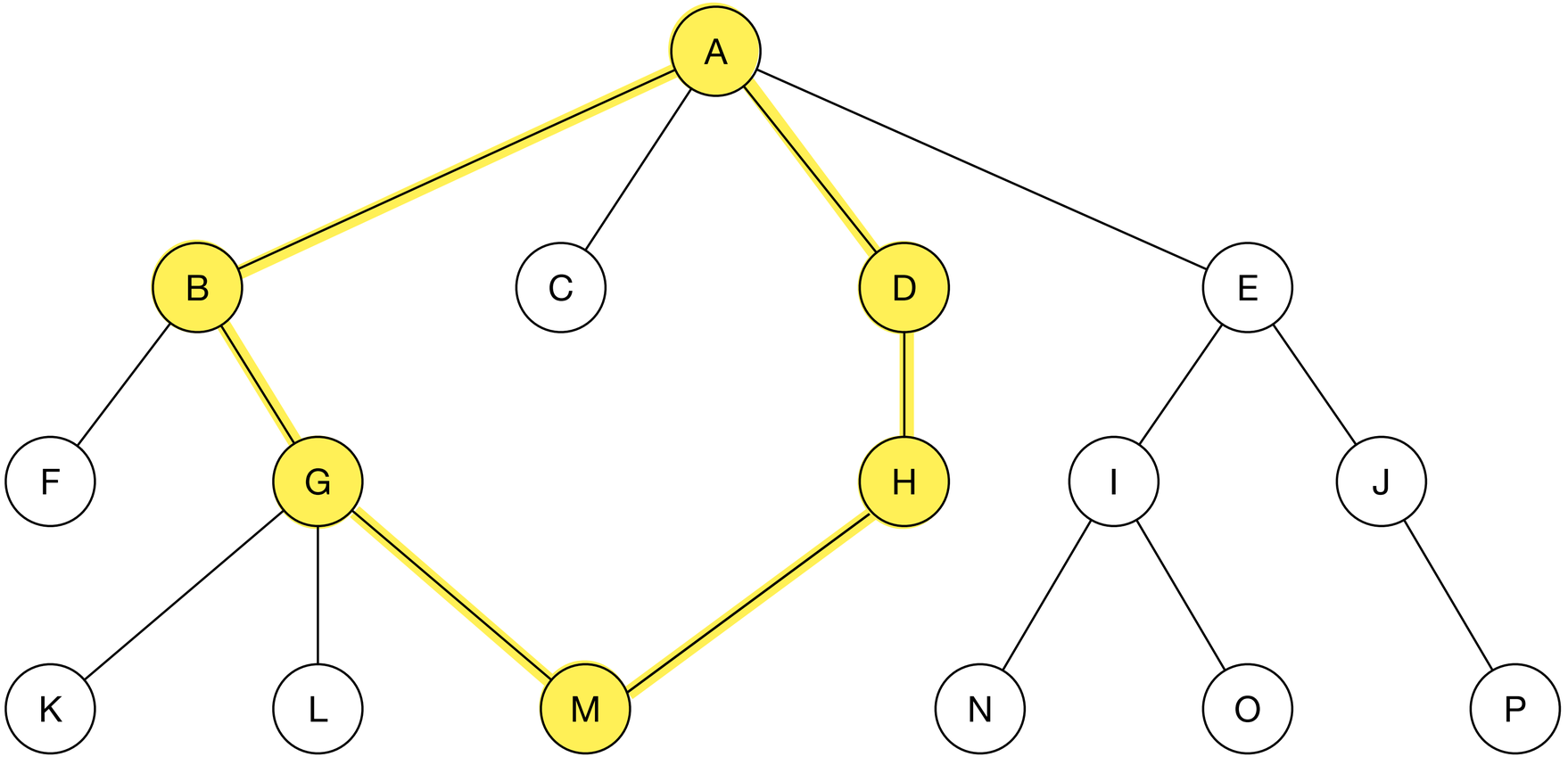


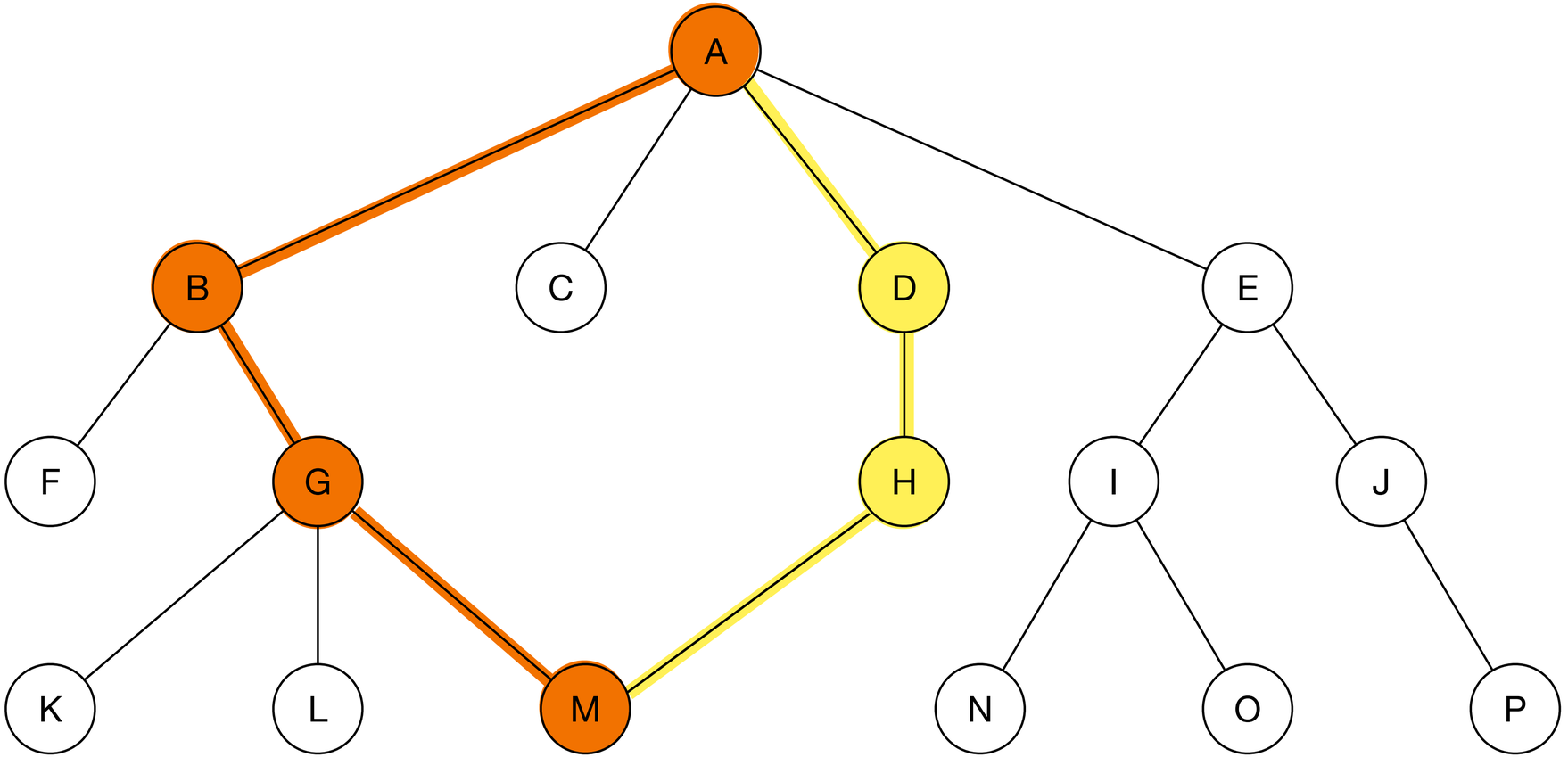
What is a tree?

There must be exactly path between any pair of nodes.



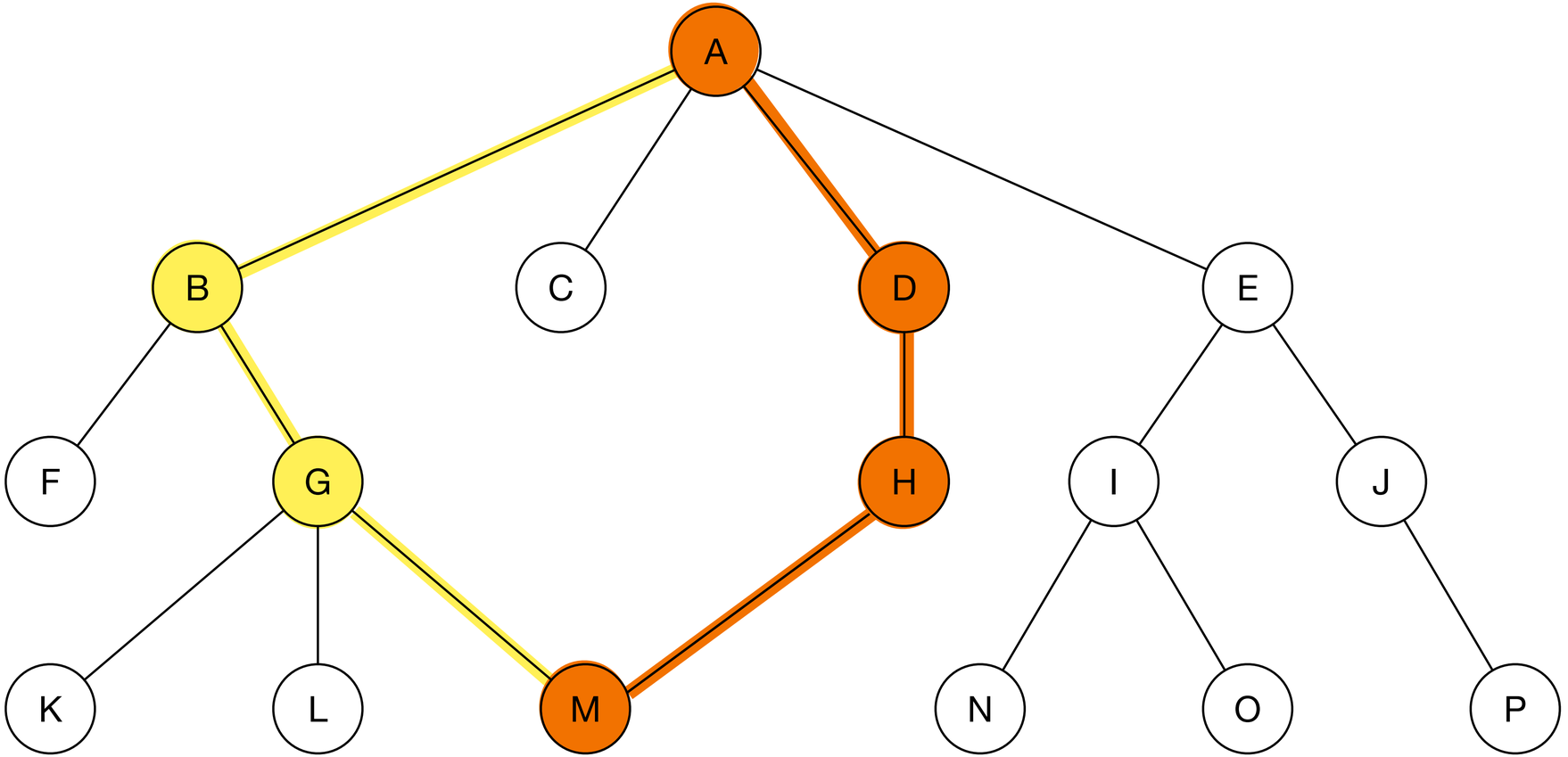
What is a tree?

Here's one path from A to M passing through B and G...



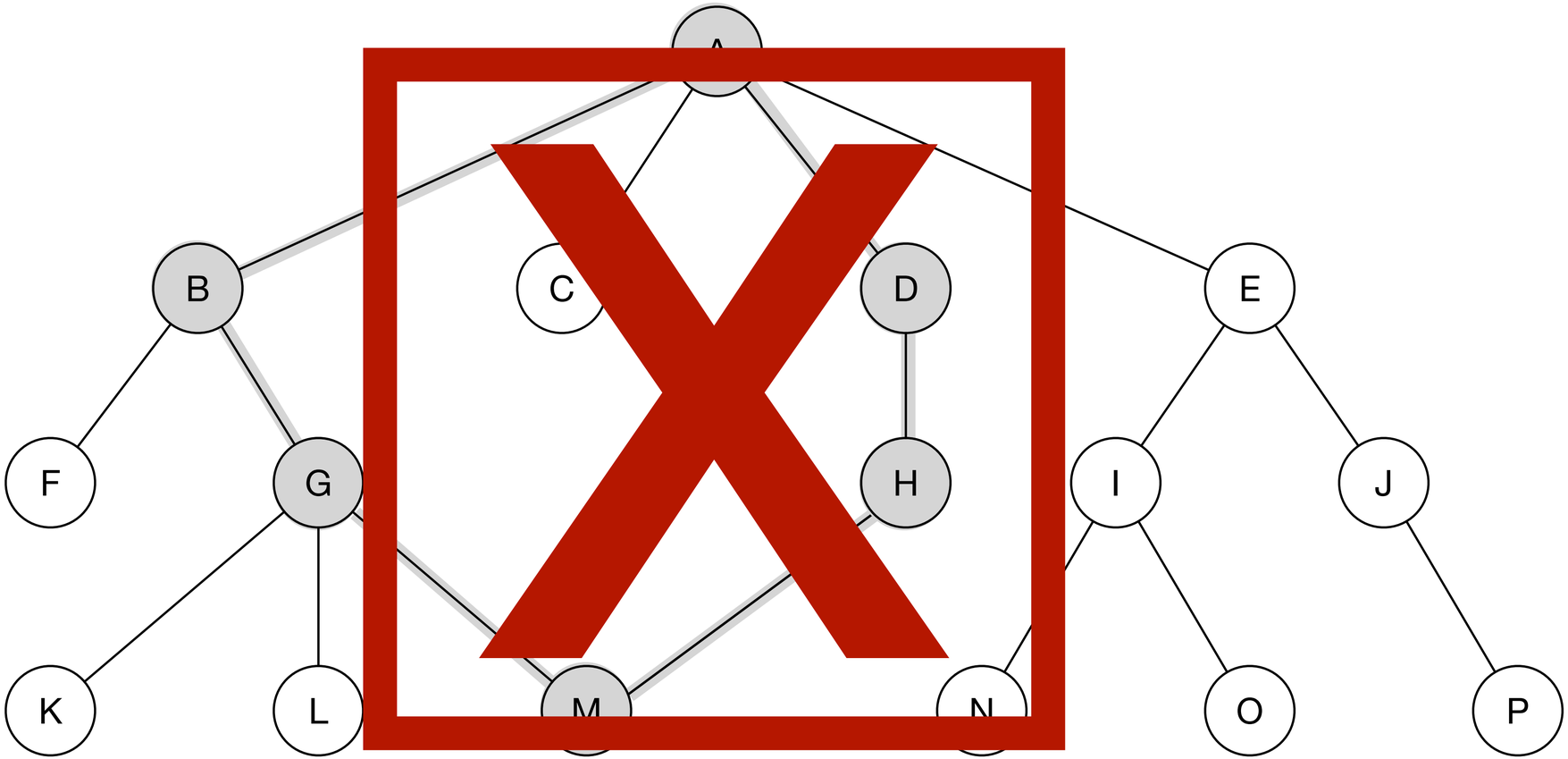
What is a tree?

...and here's another passing through D and H.



What is a tree?

However, there must be only one path between any pair of nodes.

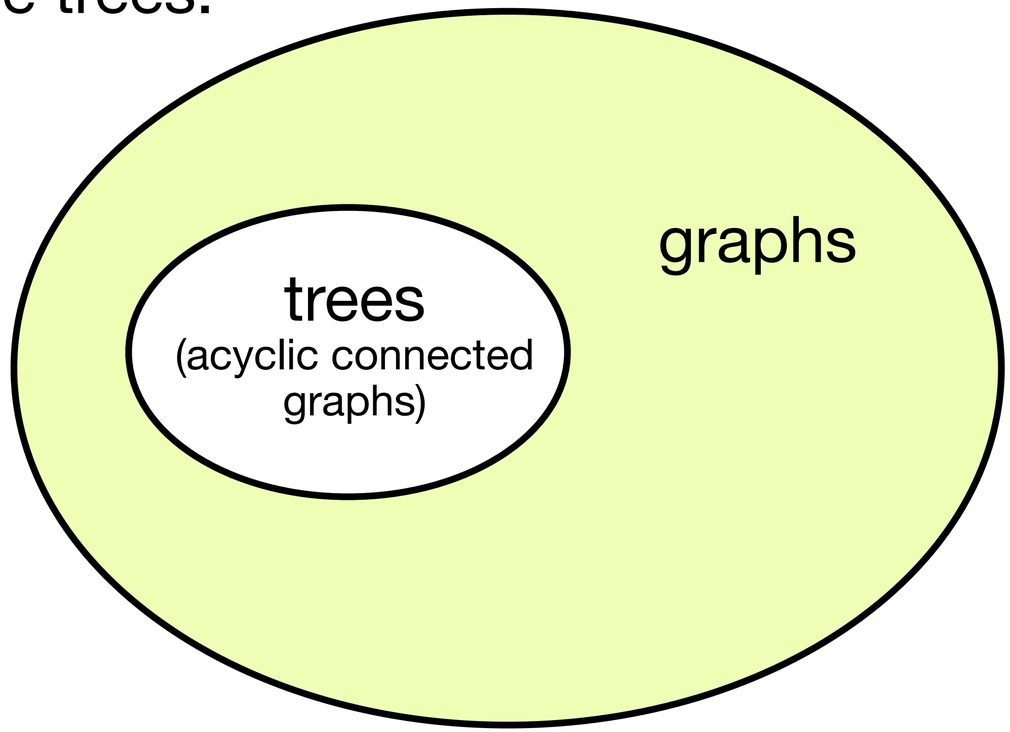


Introduction to graphs

If we relax the condition that there must be exactly one path between any pair of nodes, then we have a graph.

All trees are graphs but not all graphs are trees.

Trees are acyclic, connected graphs.



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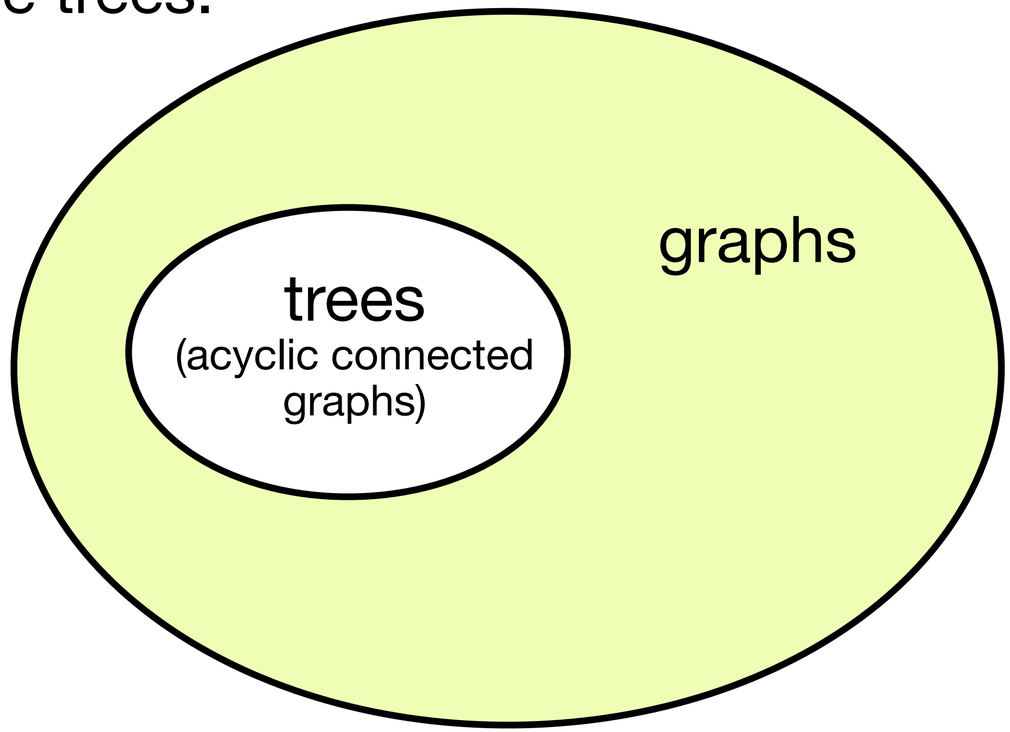


Graphs may or may not contain cycles.

Graphs may or may not be connected.

Introduction to graphs

V = {set of all nodes (or vertices)}

E = {set of all edges}

|V| = number of nodes in a graph

|E| = number of edges in a graph

Introduction to graphs

V = {set of all nodes (or vertices)}

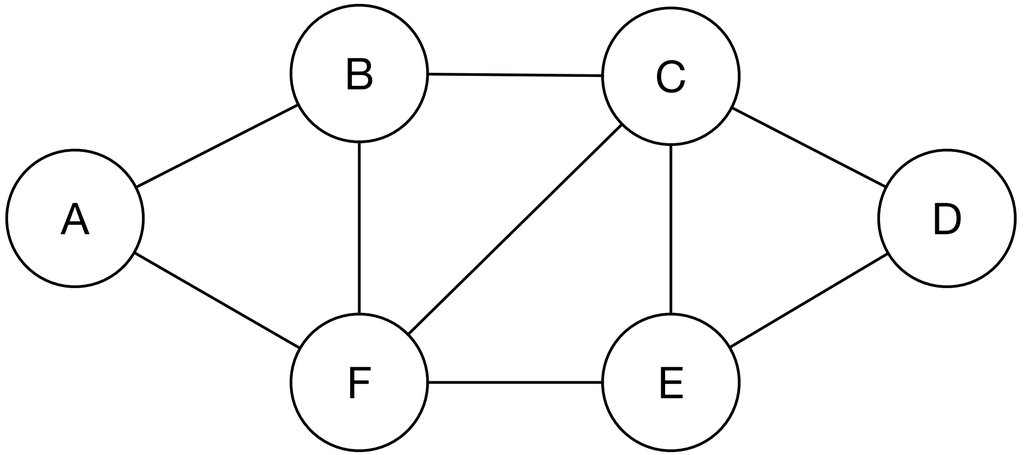
E = {set of all edges}

Should have used script $\mathcal{V}$ and $\mathcal{E}$ to distinguish from labels of vertices in graph, e.g., $\mathcal{E}$ is set of all edges, E is the label of one of the vertices in the graph. My bad.

|V| = number of nodes in a graph

|E| = number of edges in a graph

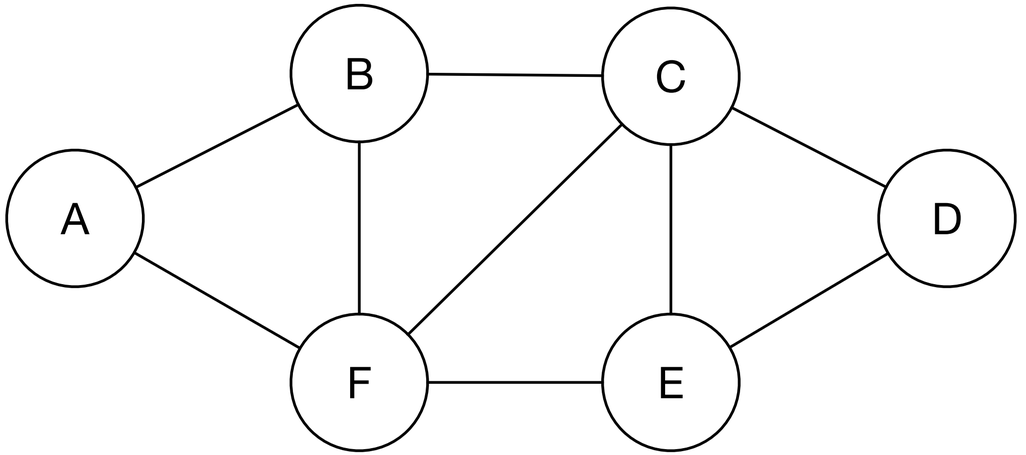
$|\mathcal{E}|$



Introduction to graphs

V = {A, B, C, D, E, F}

E = {set of all edges}



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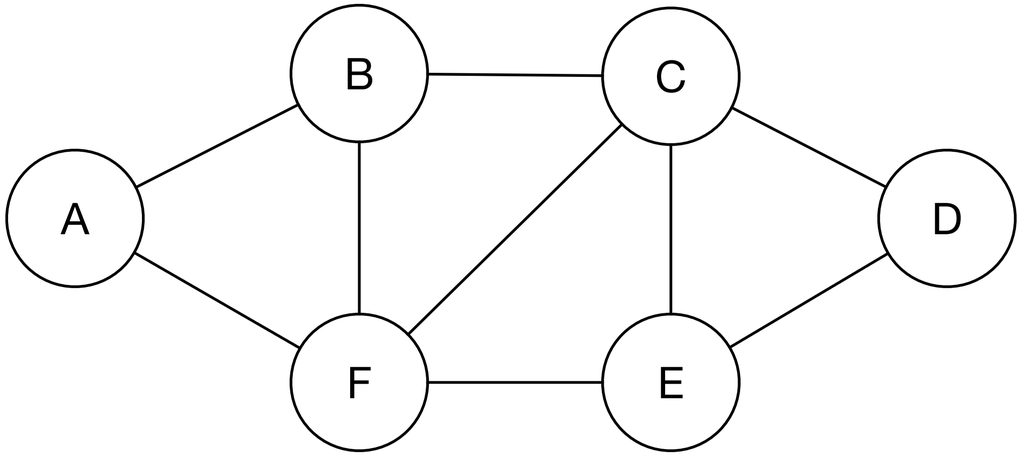

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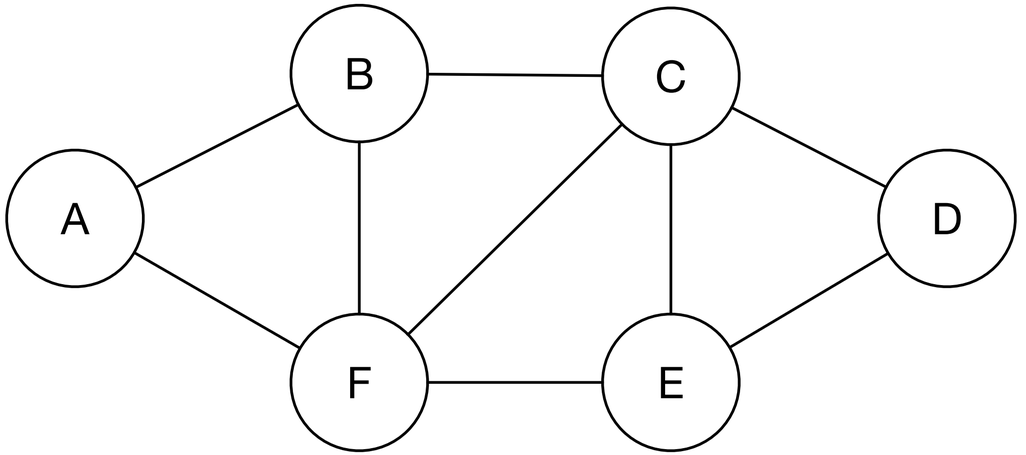

Introduction to graphs

V = {A, B, C, D, E, F}

E = {AB, AF, BC, BF, CF,

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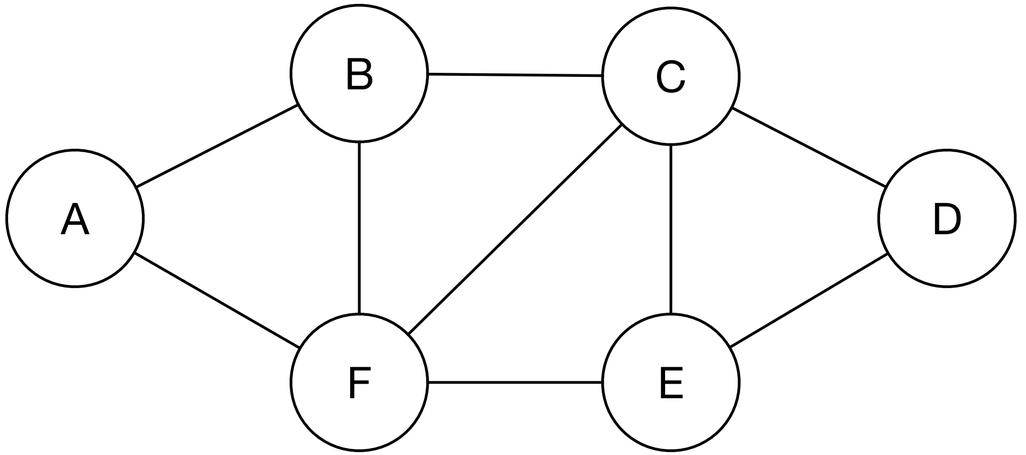
|V| = 6

|E| = number of edges in a graph



Introduction to graphs

V = {A, B, C, D, E, F}

E = {AB, AF, BC, BF, CF,

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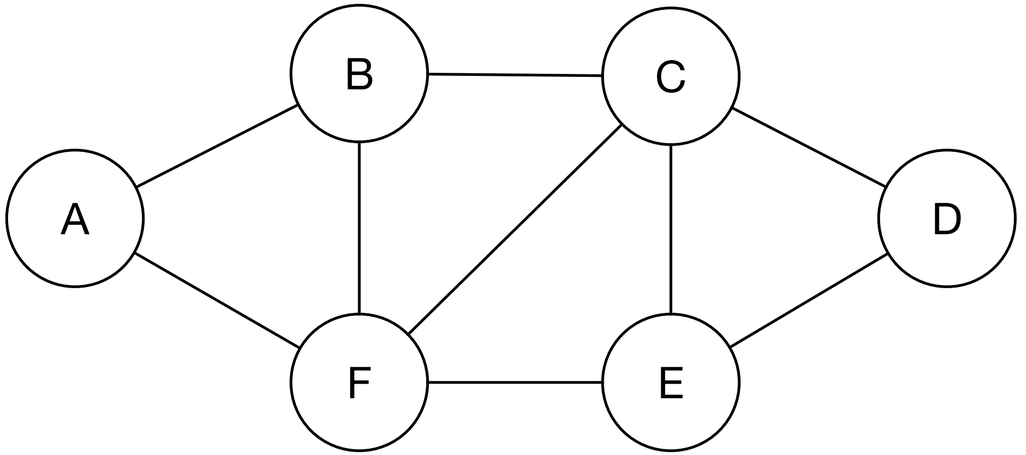
|V| = 6

|E| = 9



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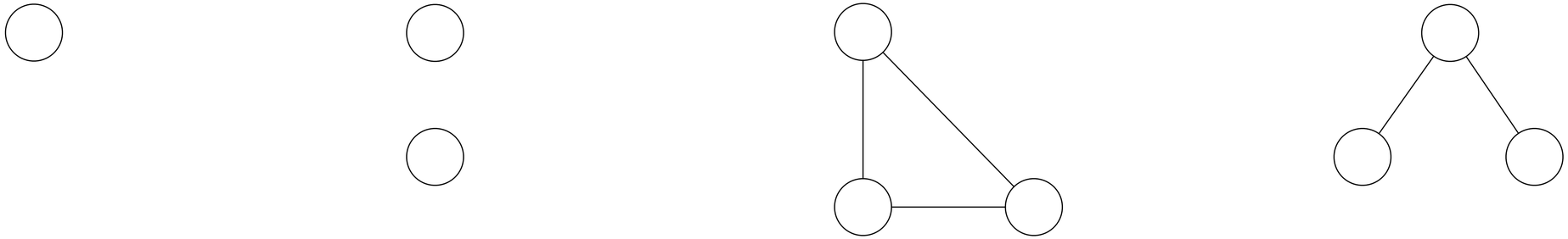
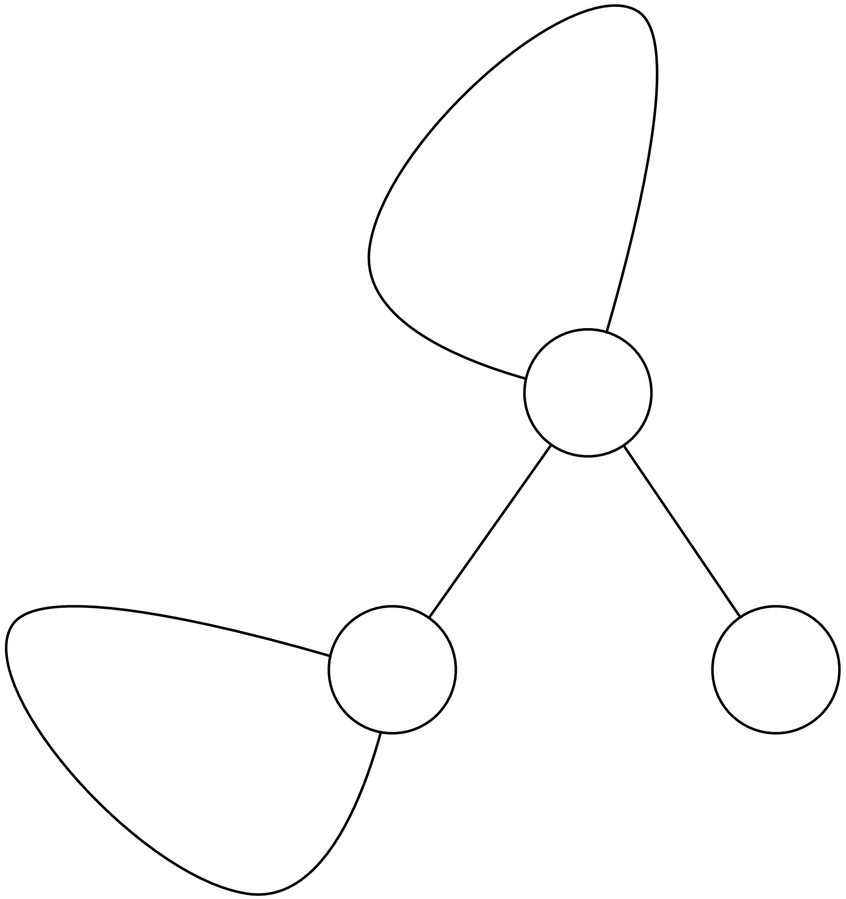


Introduction to graphs

Graphs may include
self-loops



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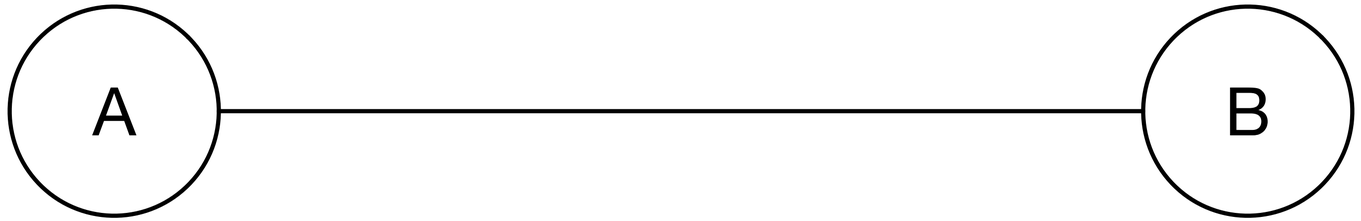

What does it mean to be adjacent?
A direct path exists from one node to another
(not passing through any other node)

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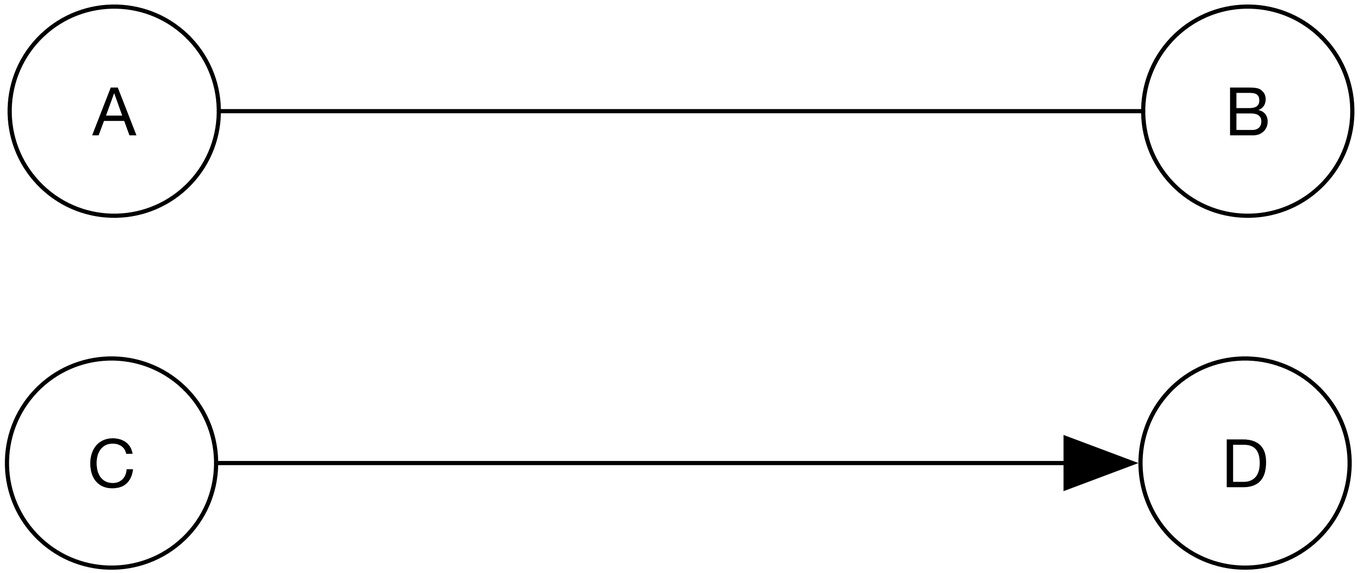

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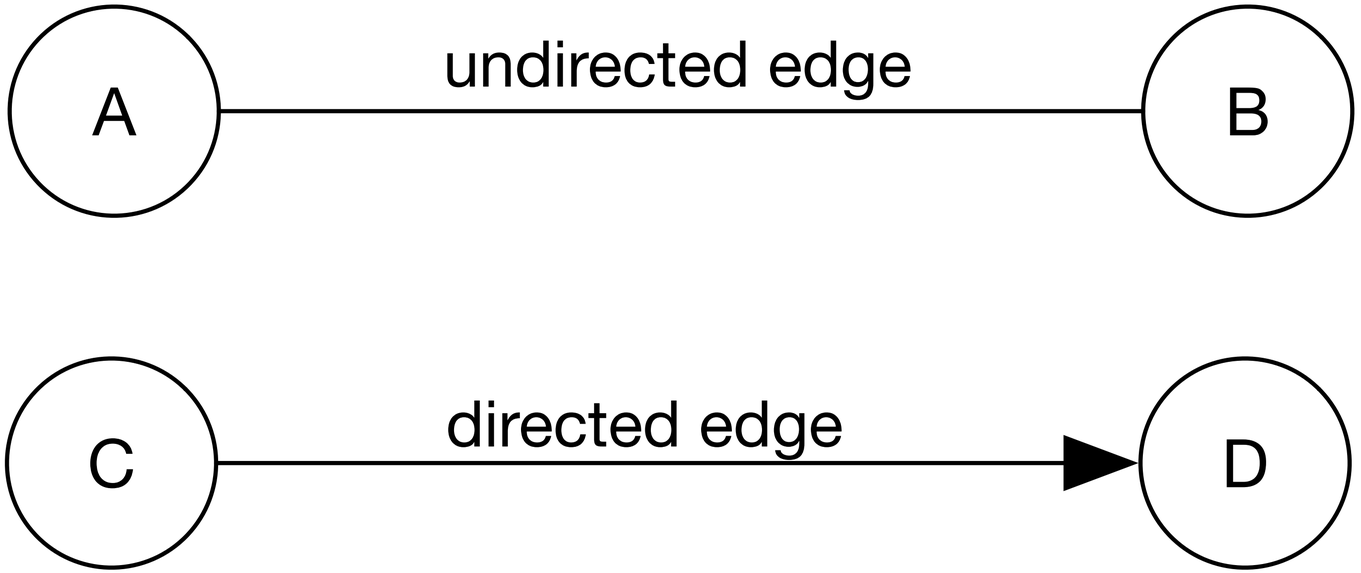

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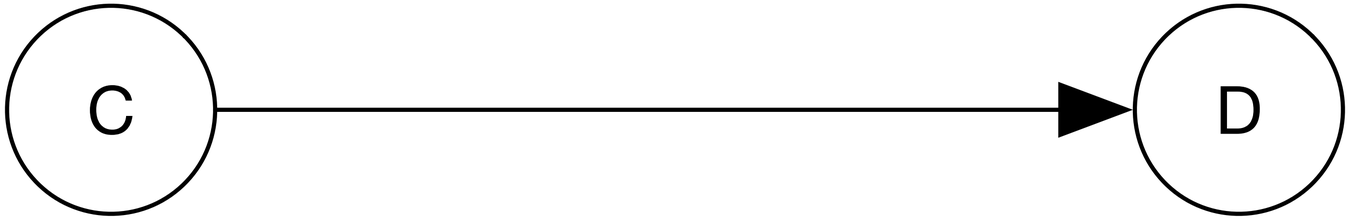

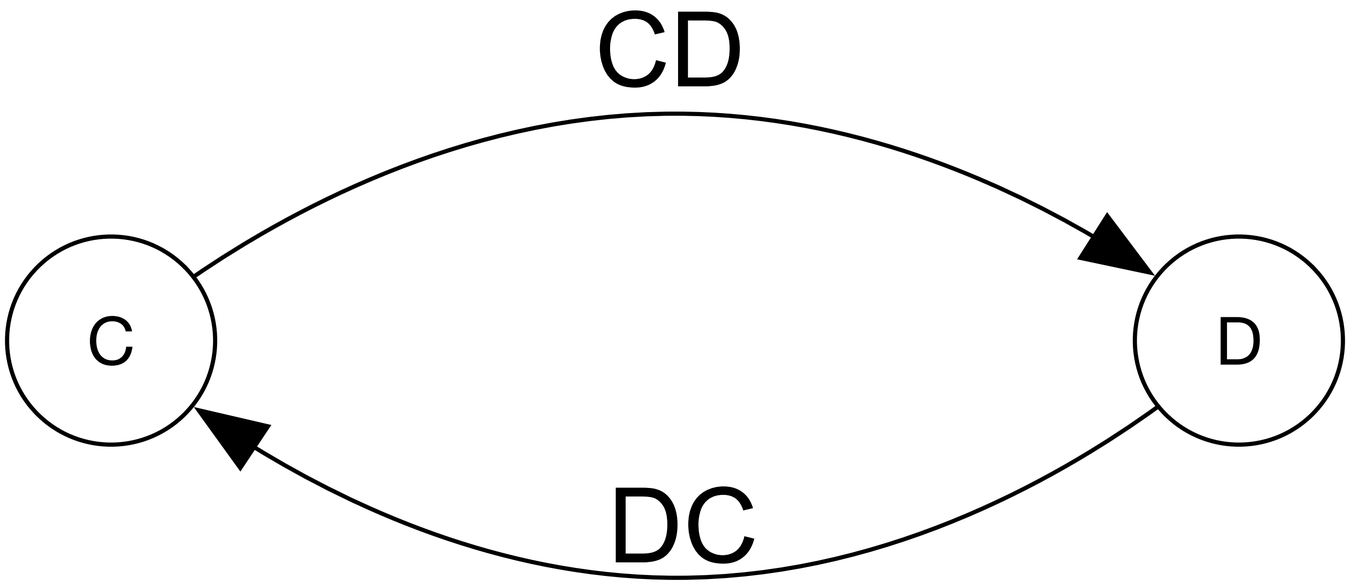

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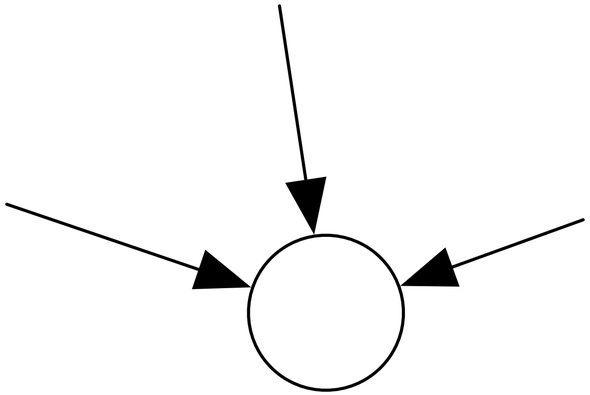
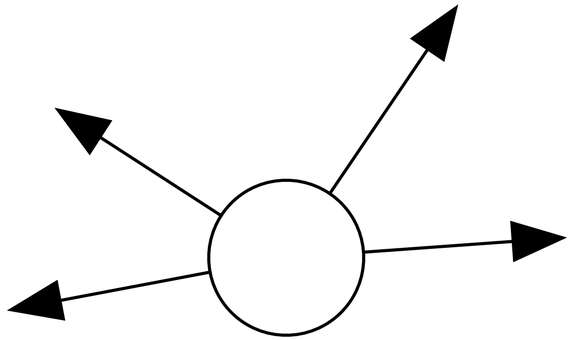


in degree = 3



out degree = 4

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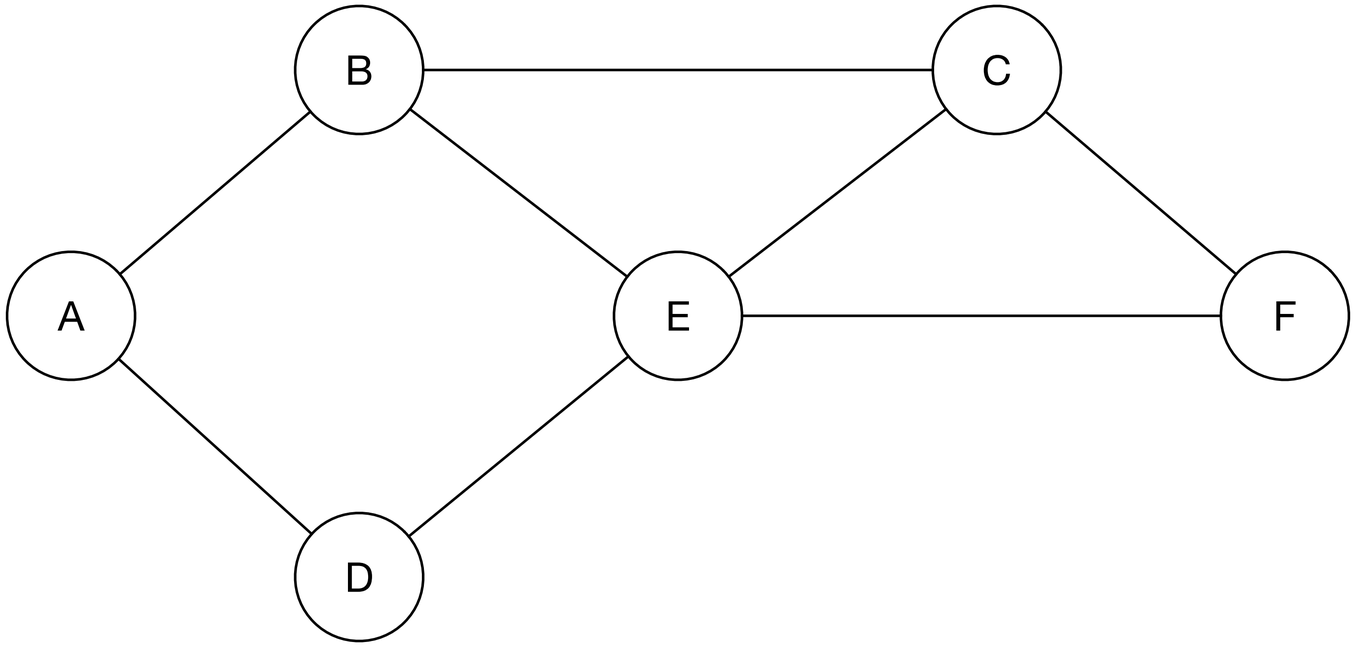

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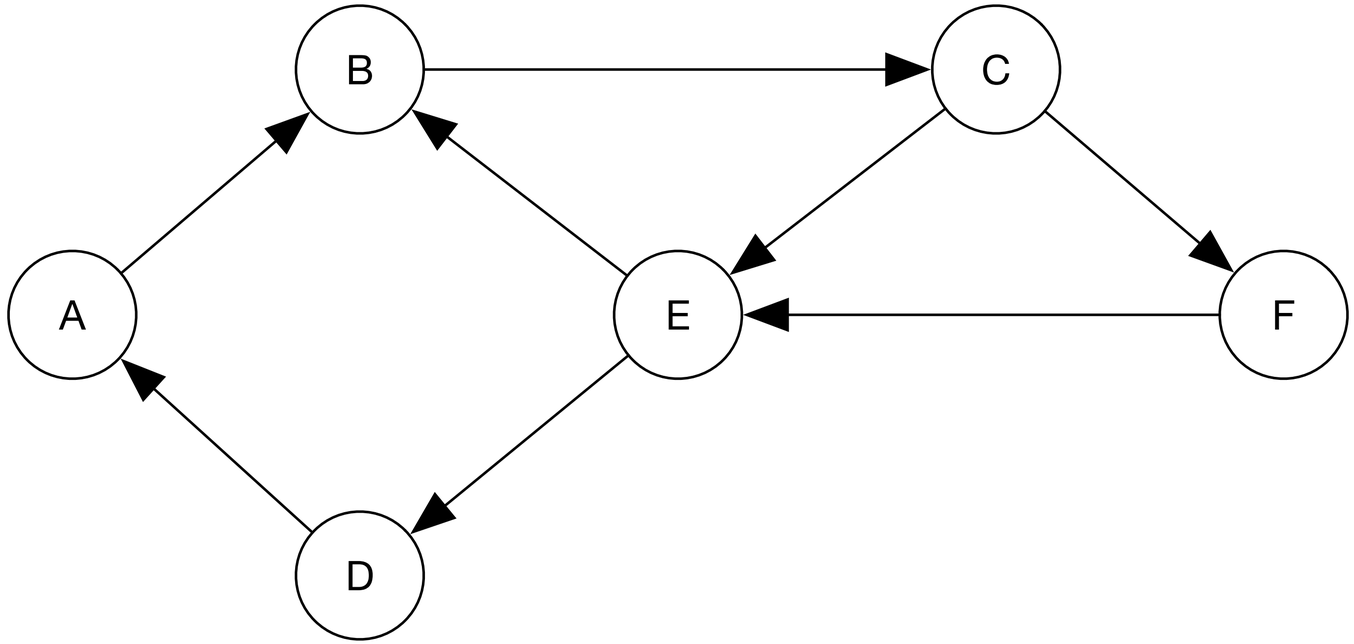

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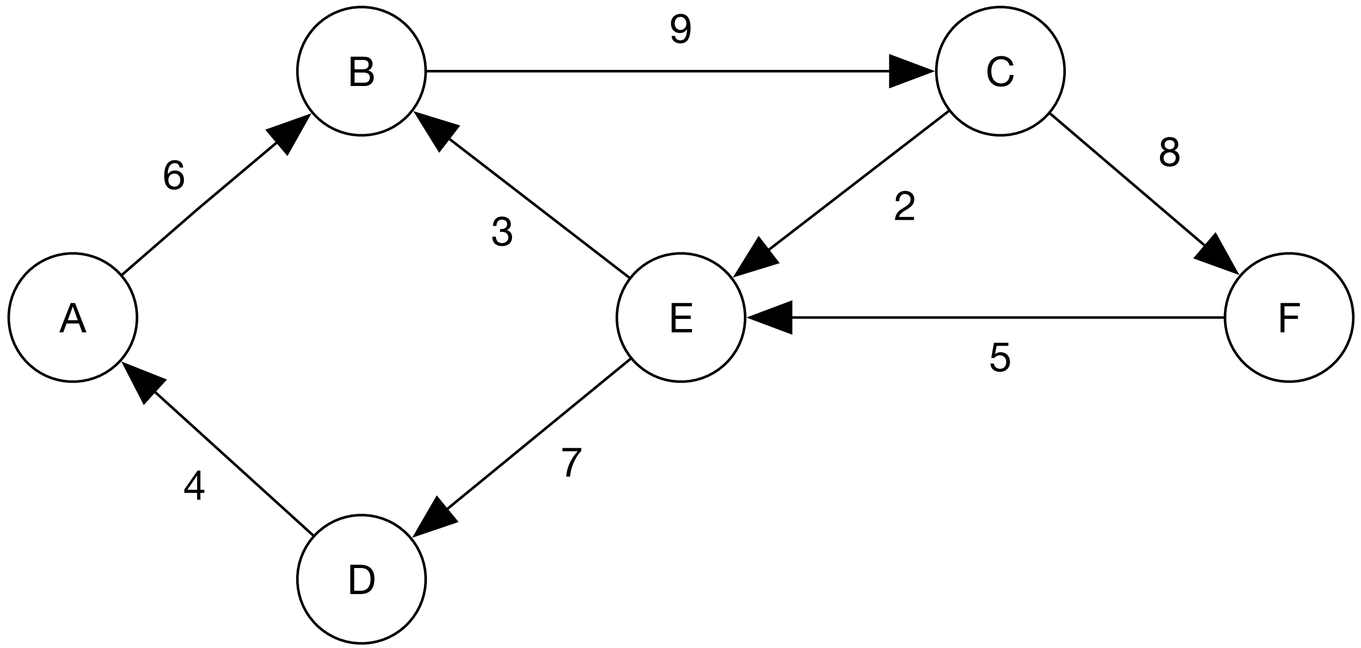

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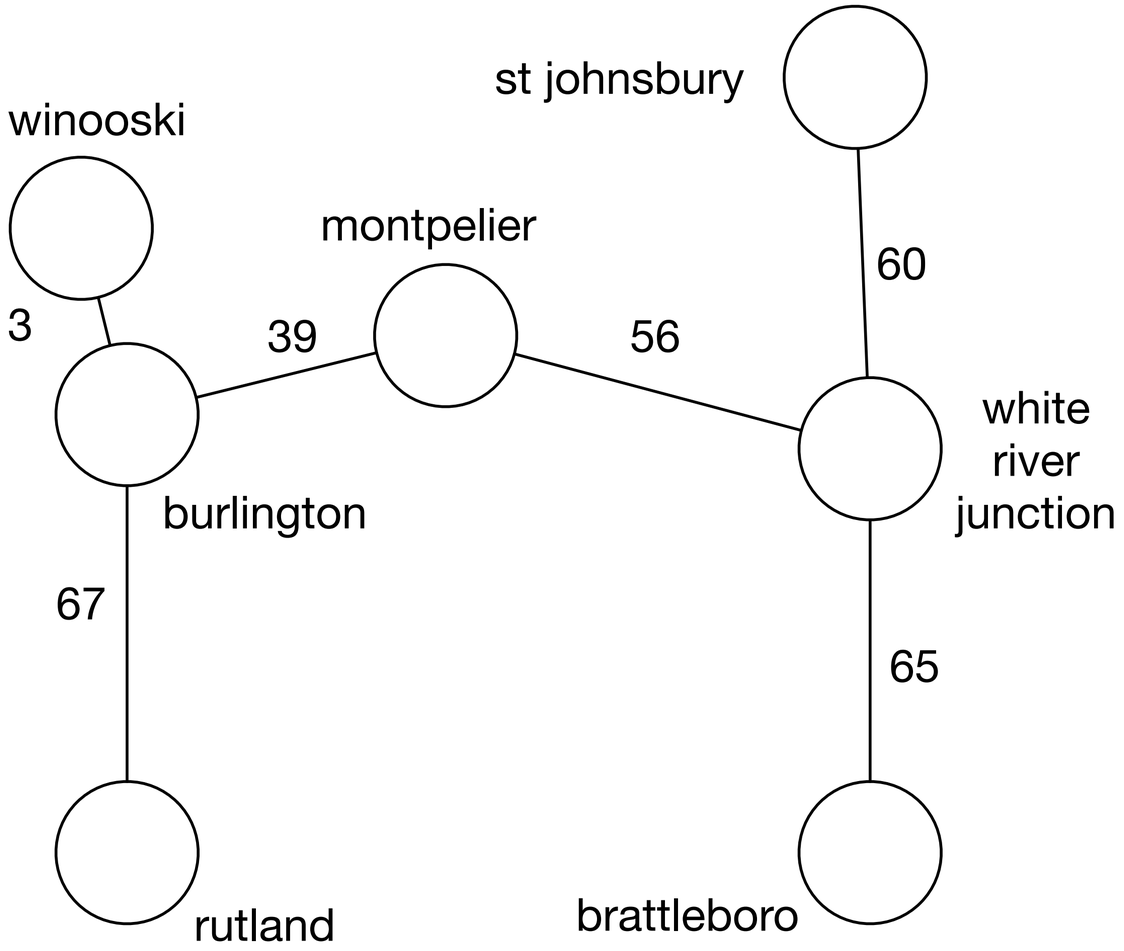

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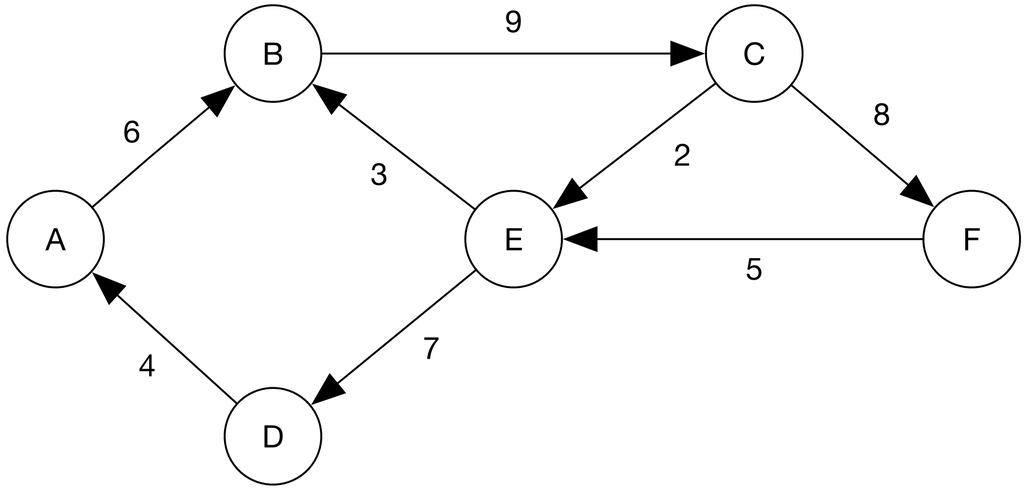

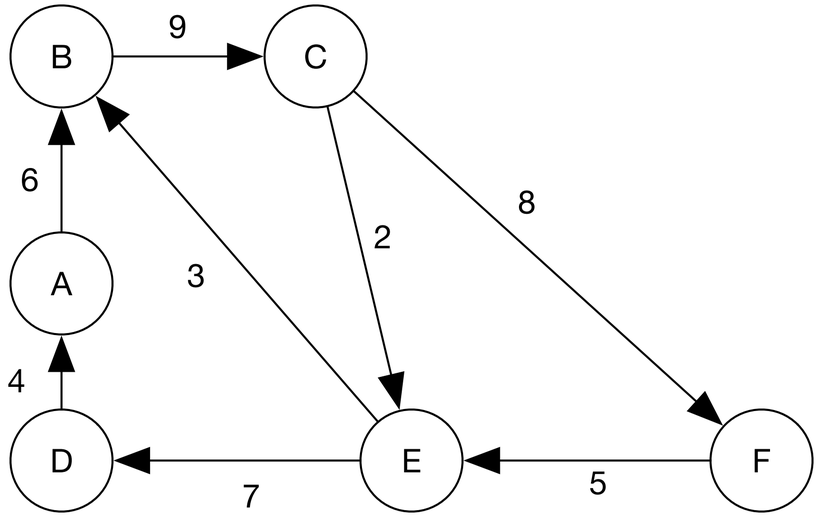

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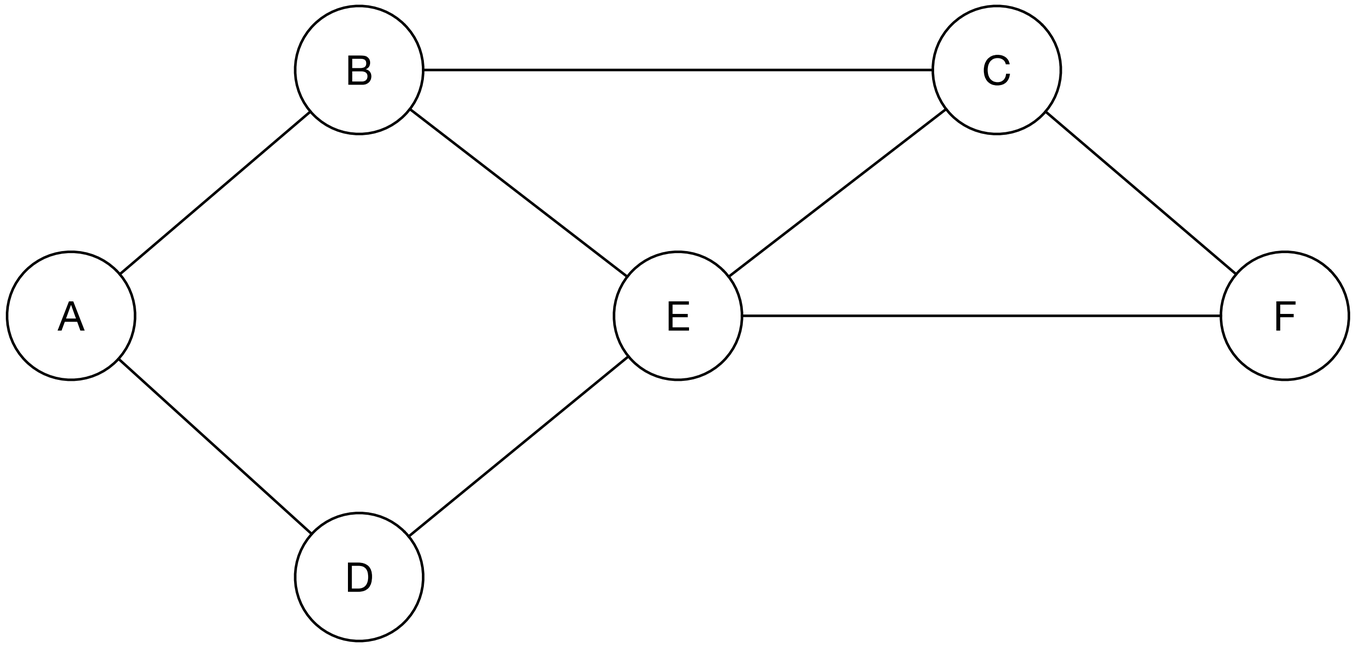

What kind of data structure should we use to represent a graph?

Introduction to graphs

Adjacency list

Adjacency matrix

Incidence matrix

Introduction to graphs

Adjacency list

Adjacency matrix

Incidence matrix

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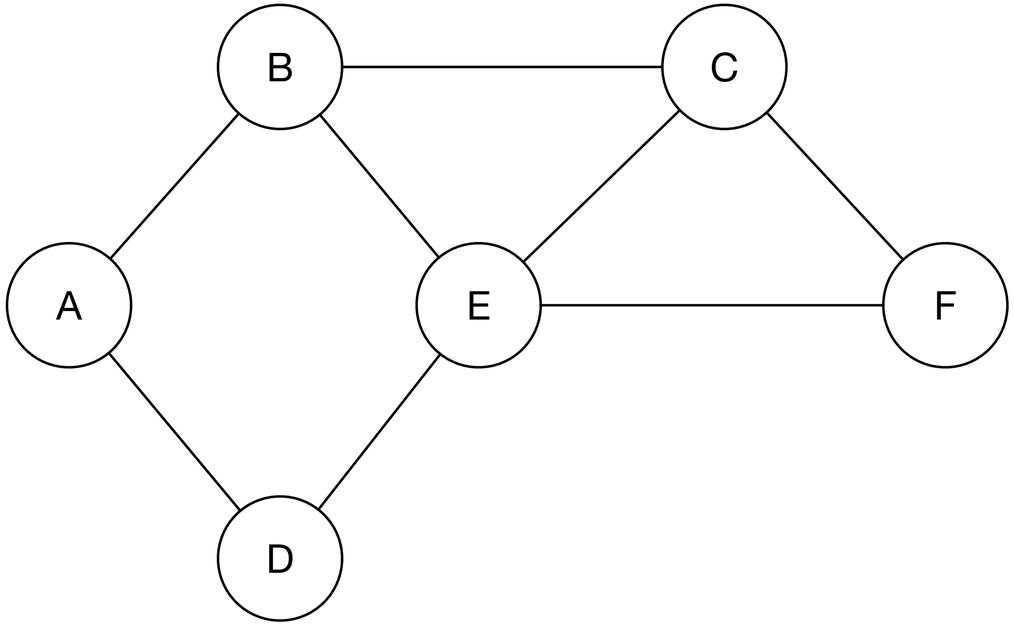

Adjacency list

A	
B	
C	
D	
E	
F	

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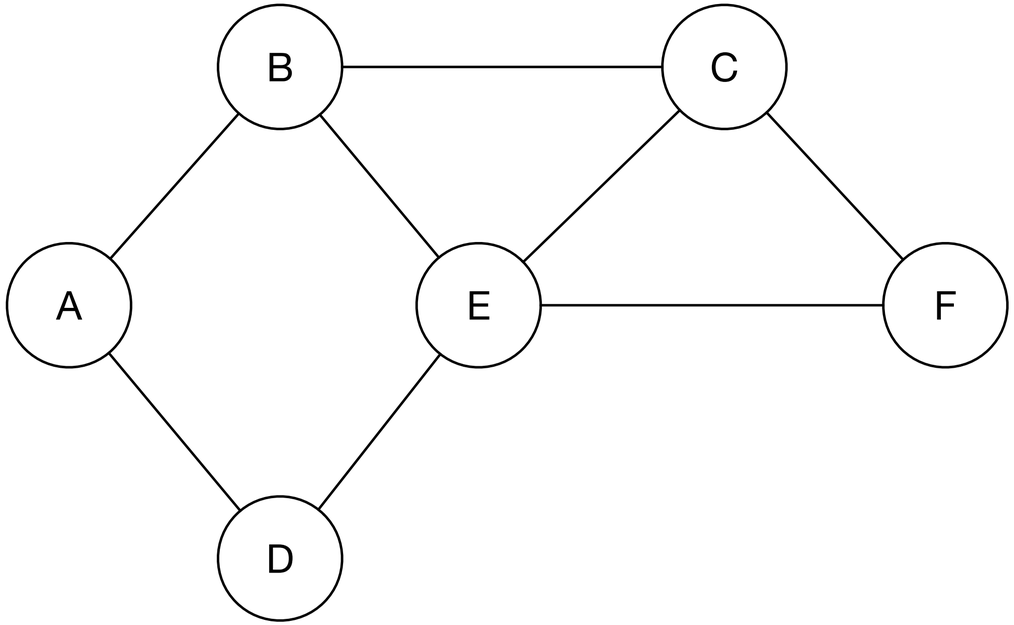

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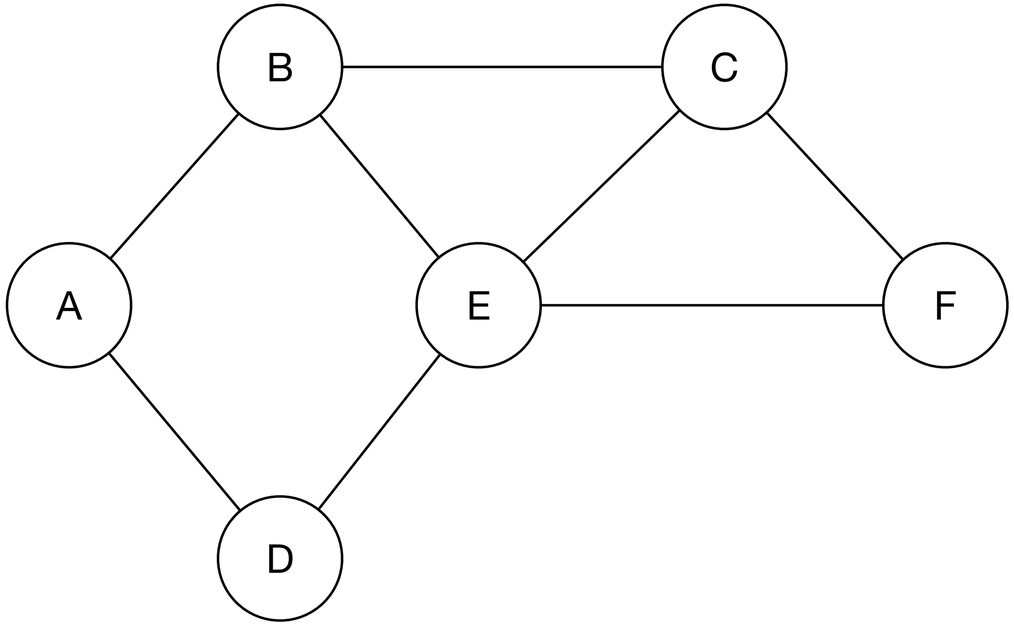

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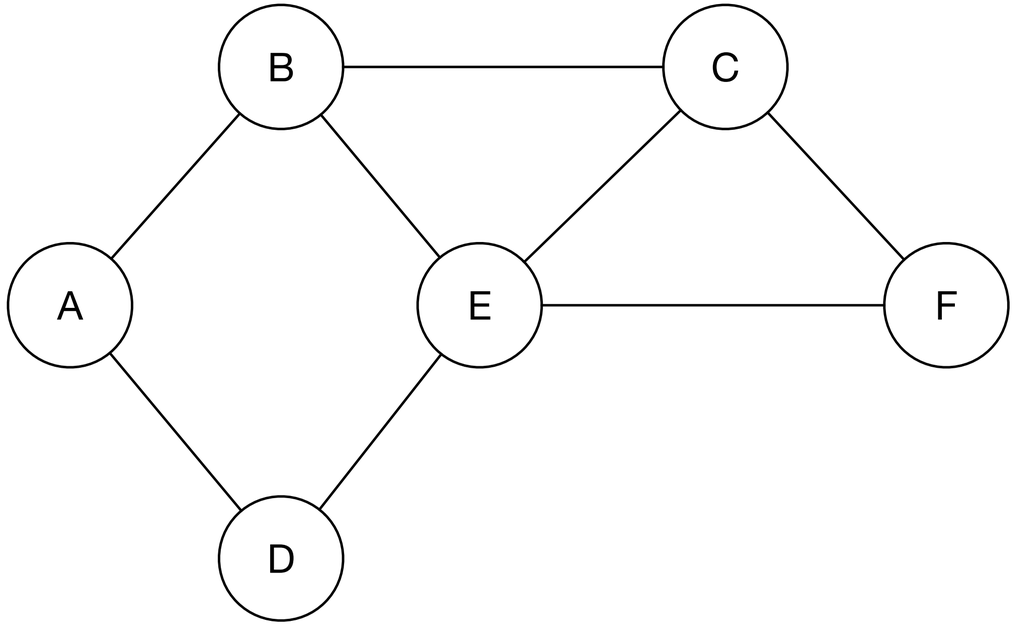

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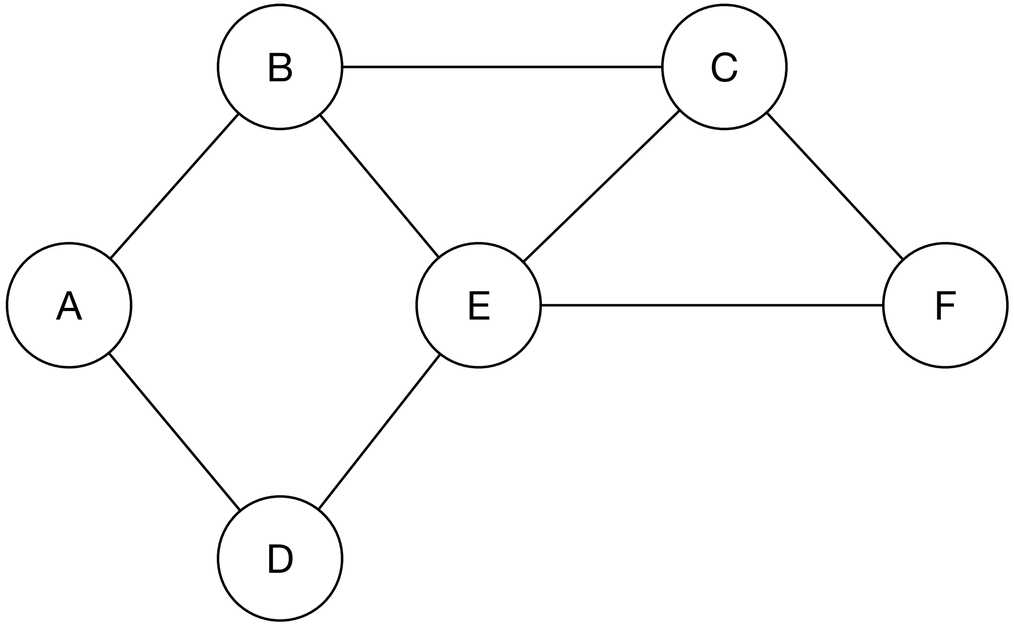

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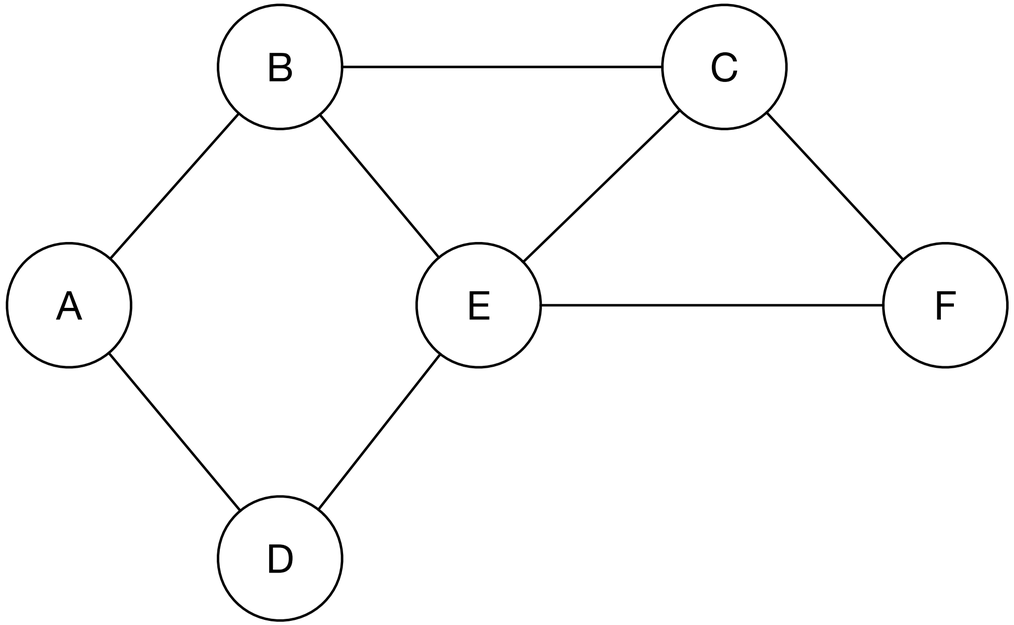

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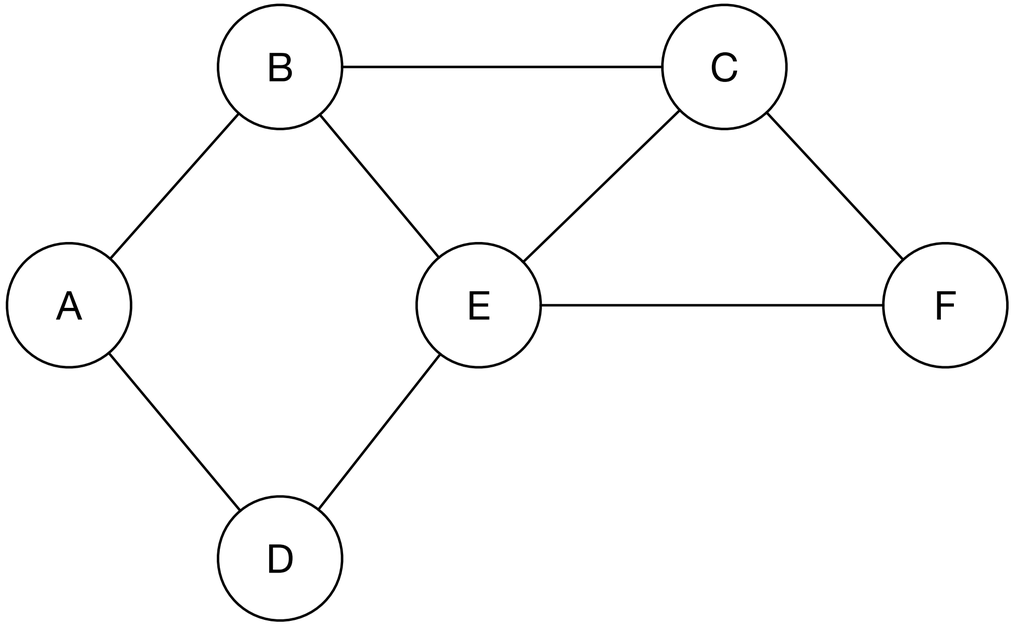

Adjacency list

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Introduction to graphs

✓ Adjacency list

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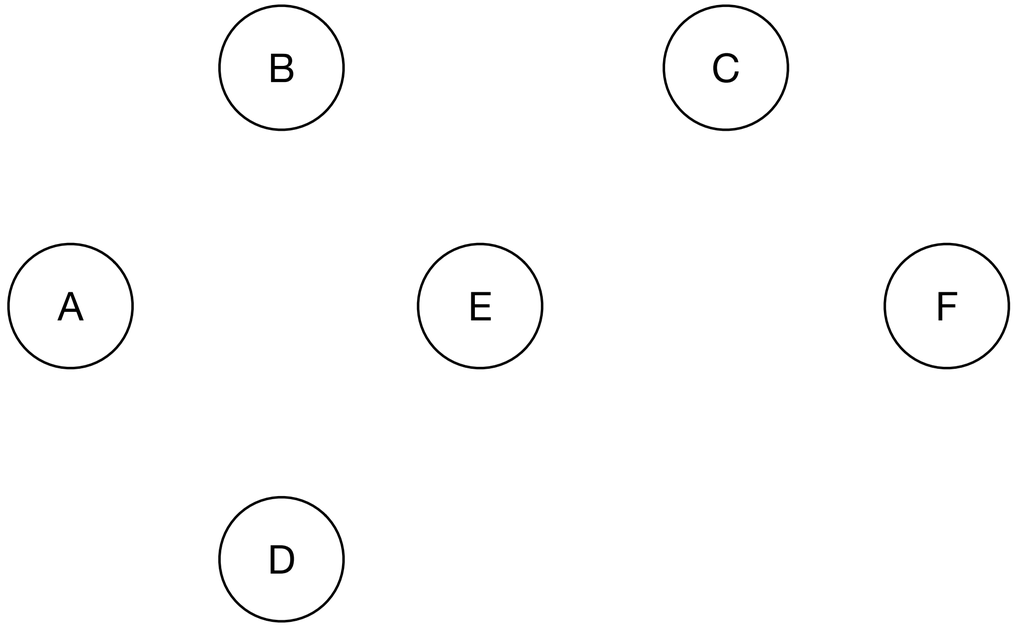
Introduction to graphs



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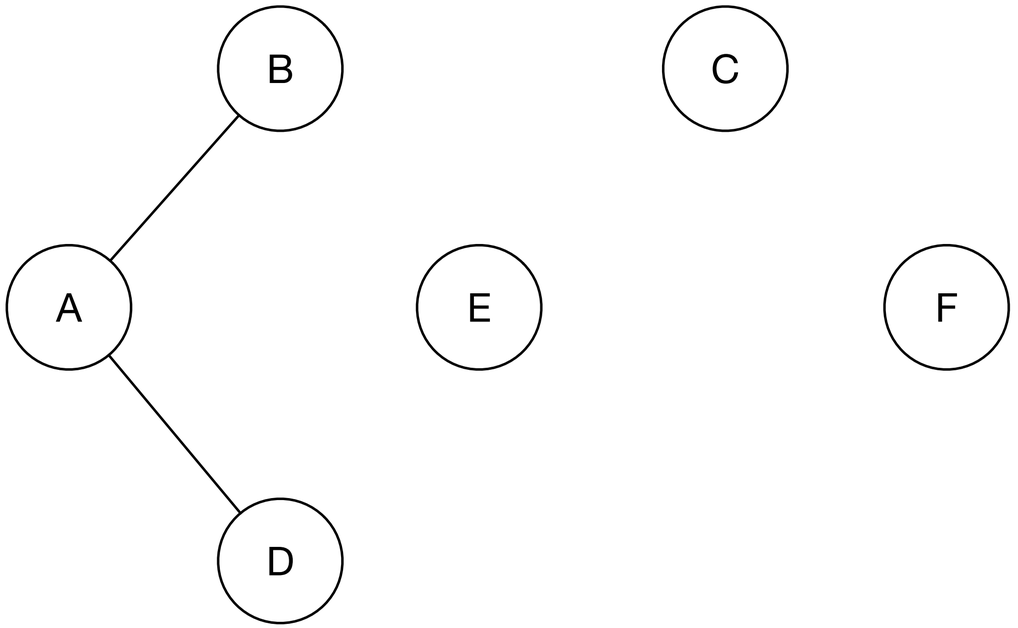

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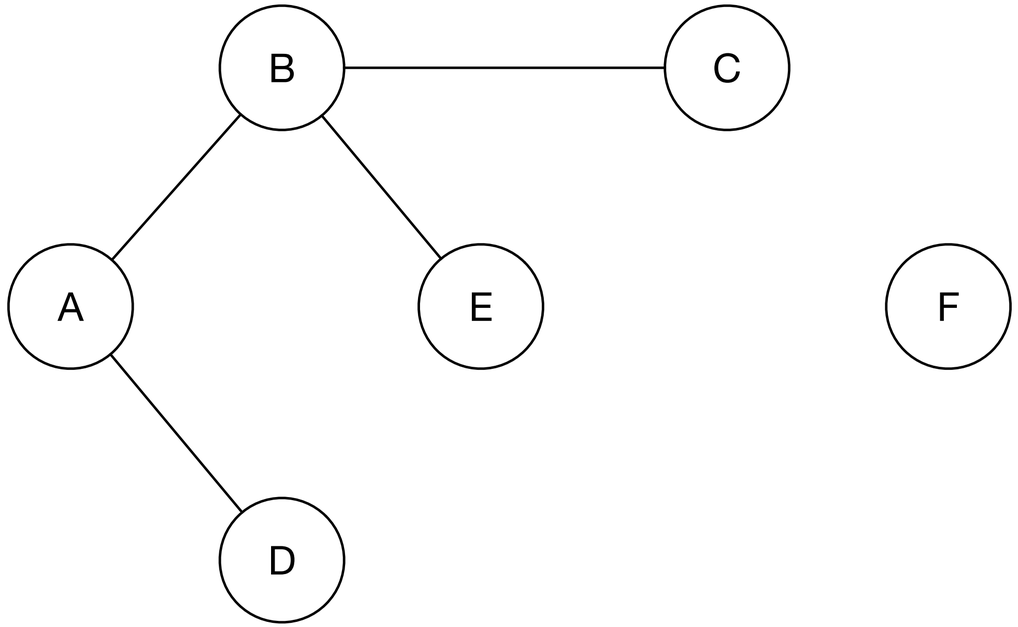

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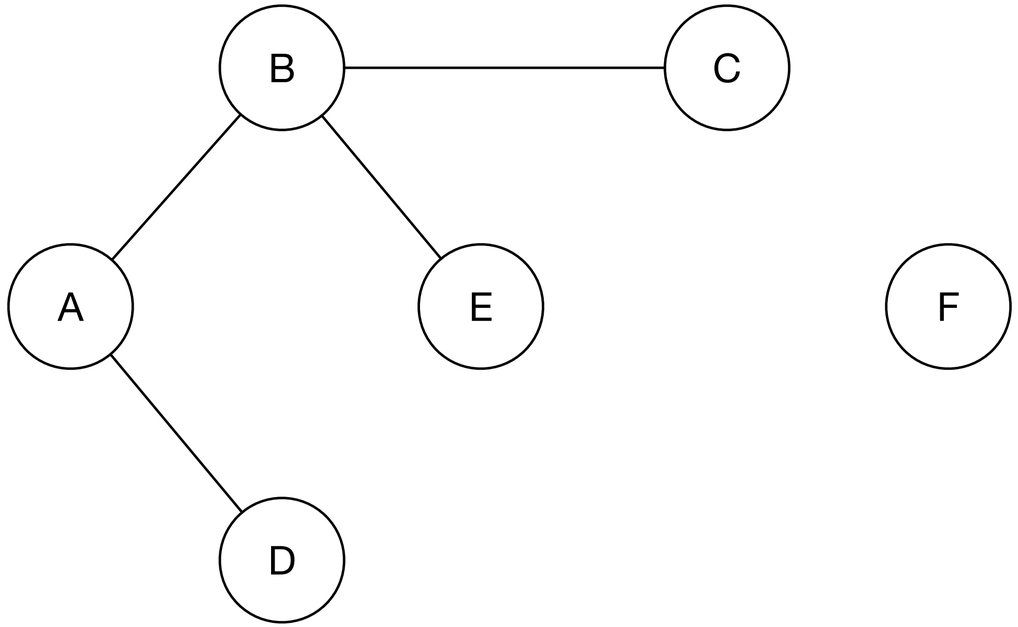

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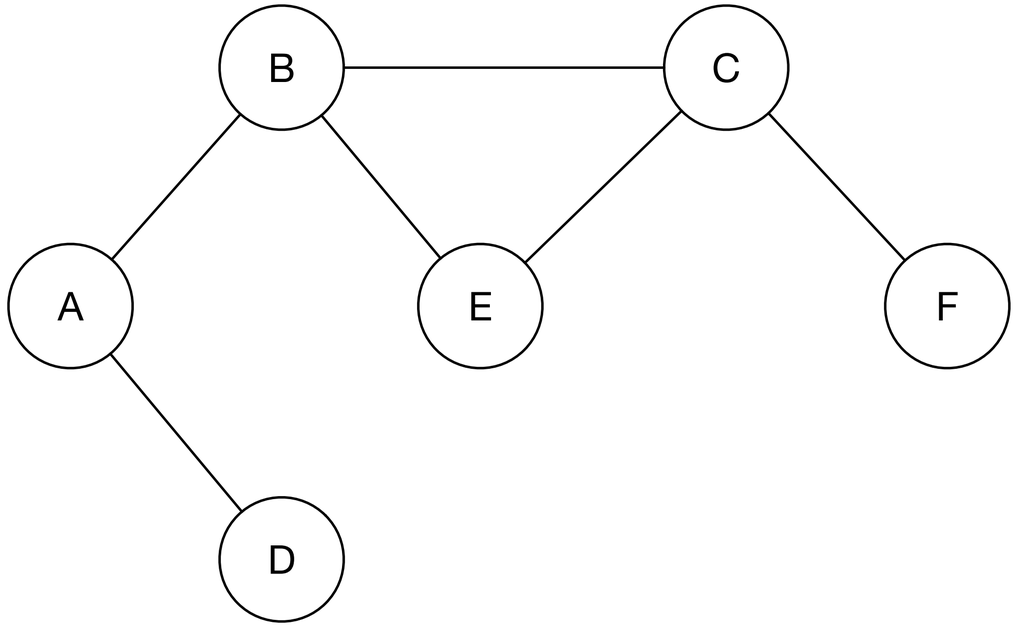

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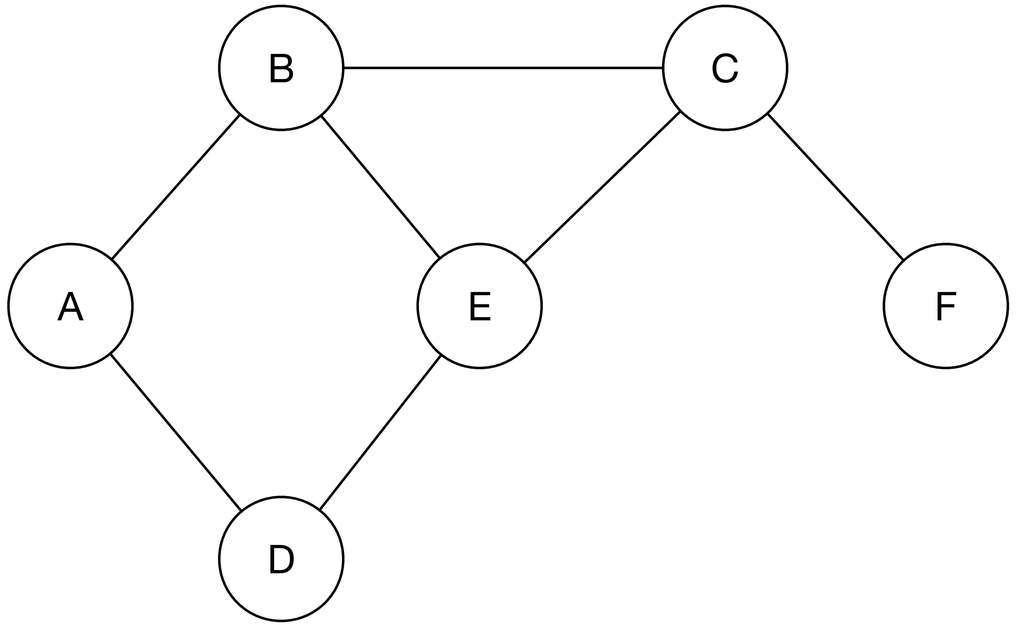

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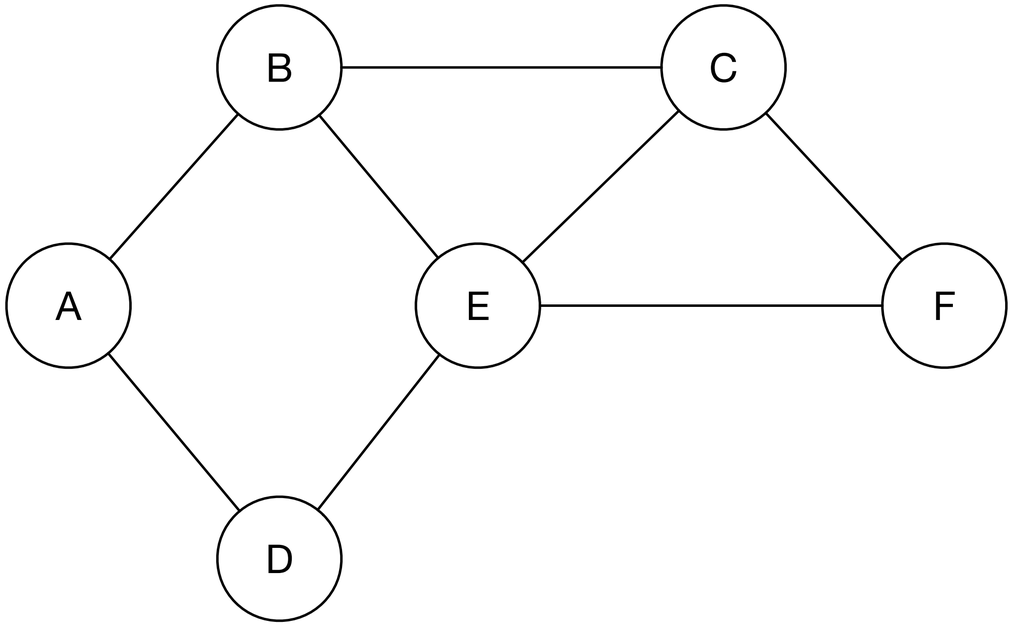

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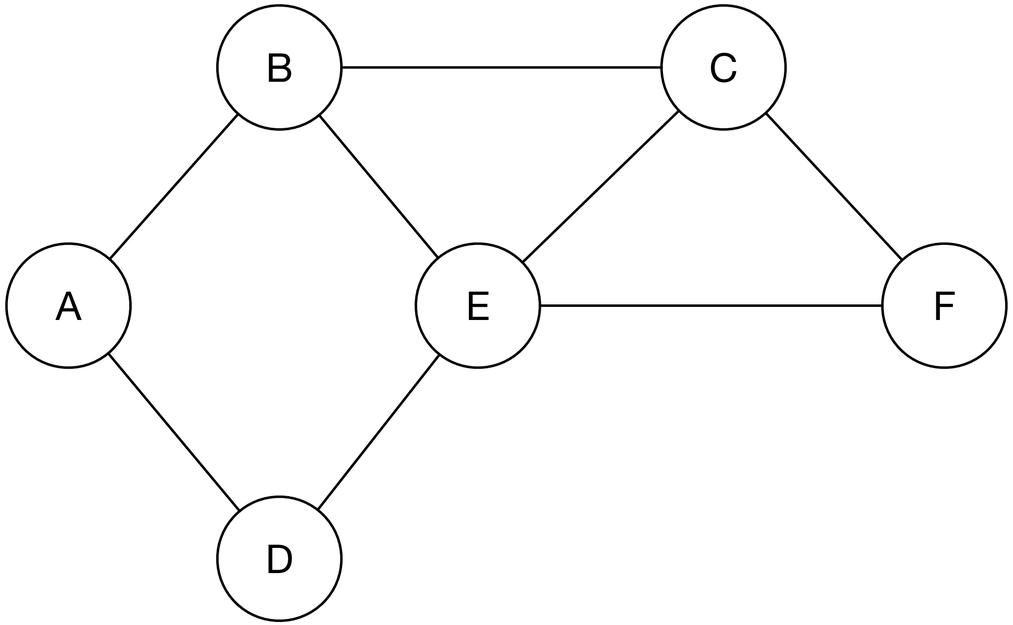

Adjacency list

✓		
✓	A	B, D
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✓	D	A, E
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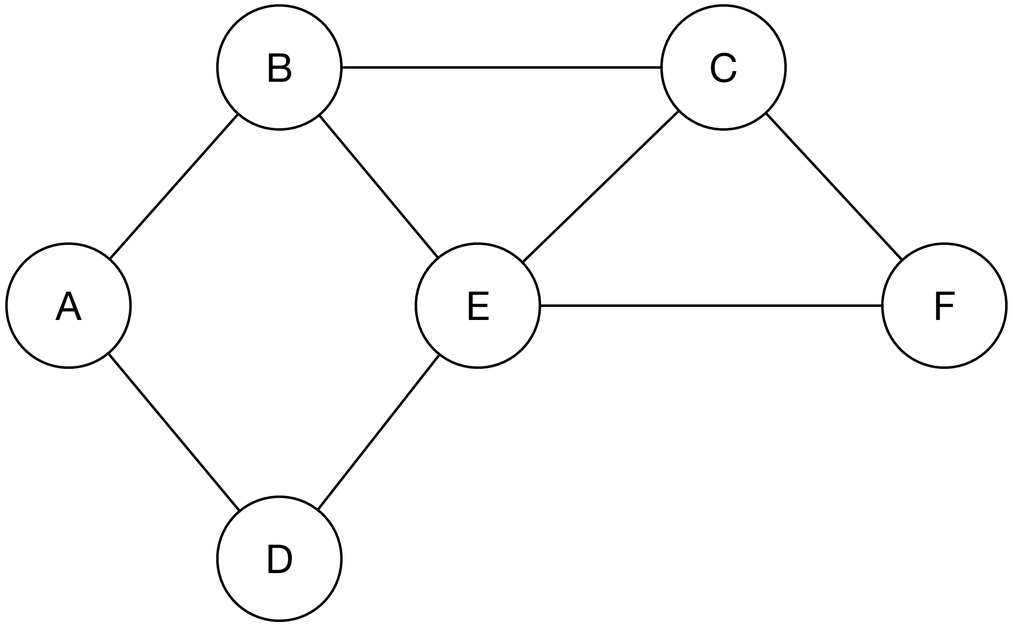
✓	F	C, E

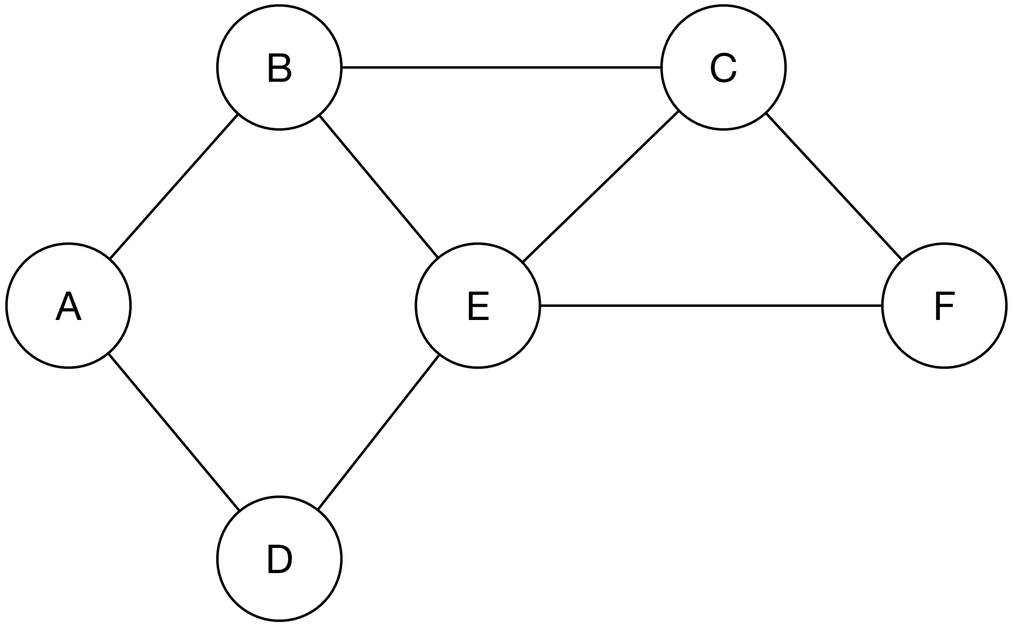
Introduction to graphs



Adjacency list

A	B, D
B	A, C, E
C	B, E, F
D	A, E
E	B, D, C, F
F	C, E

Introduction to graphs

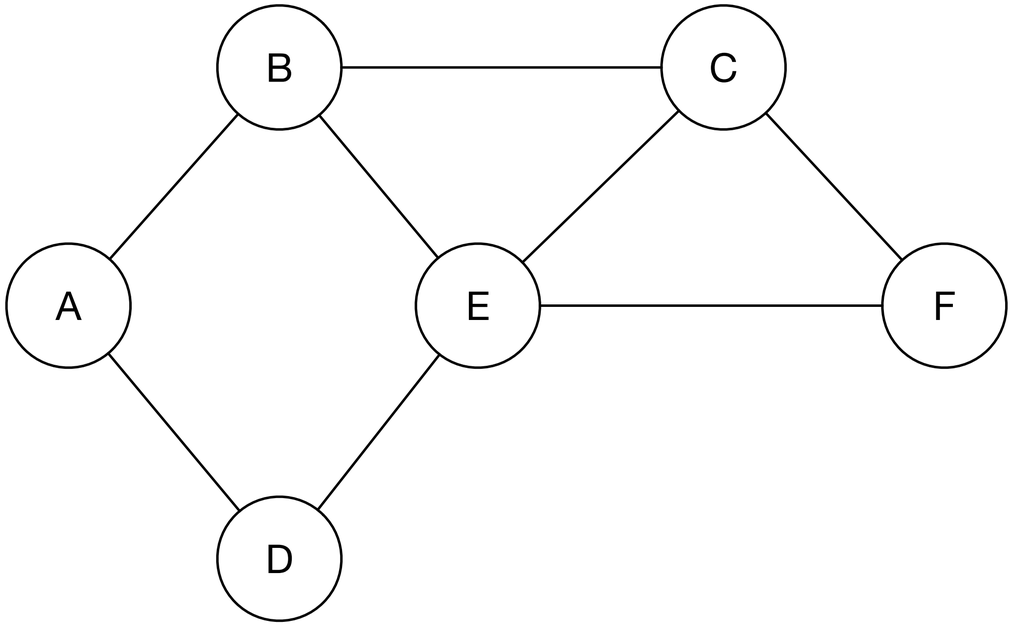


Space: $O(|V| + |E|)$

Adjacency list

A	B, D
B	A, C, E
C	B, E, F
D	A, E
E	B, D, C, F
F	C, E

Introduction to graphs

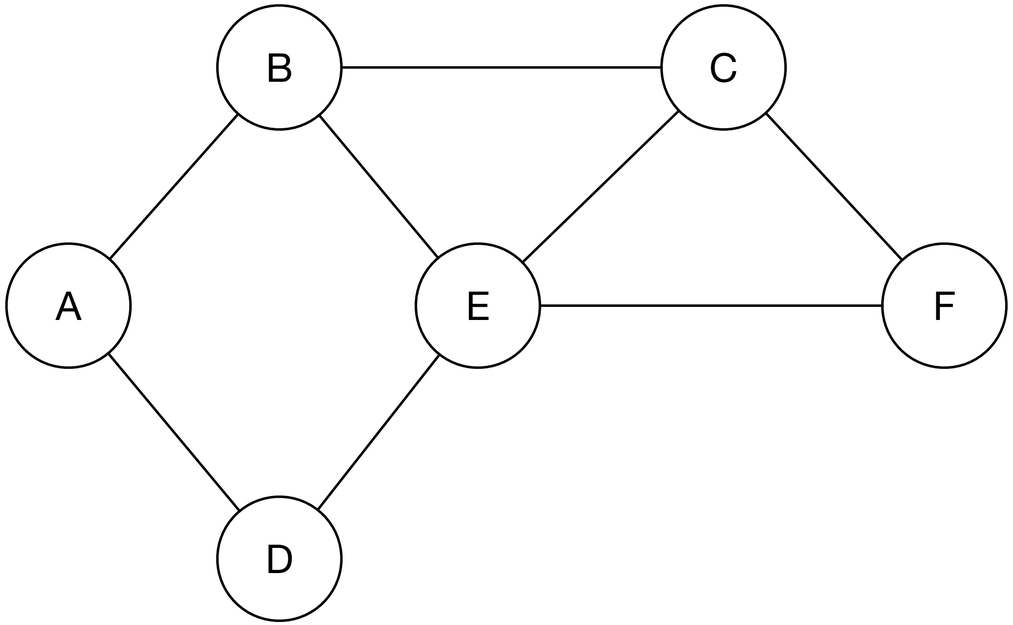


Space: $O(|V| + |E|)$

Adjacency list

A	B, D
B	A, C, E
C	B, E, F
D	A, E
E	B, D, C, F
F	C, E

Introduction to graphs

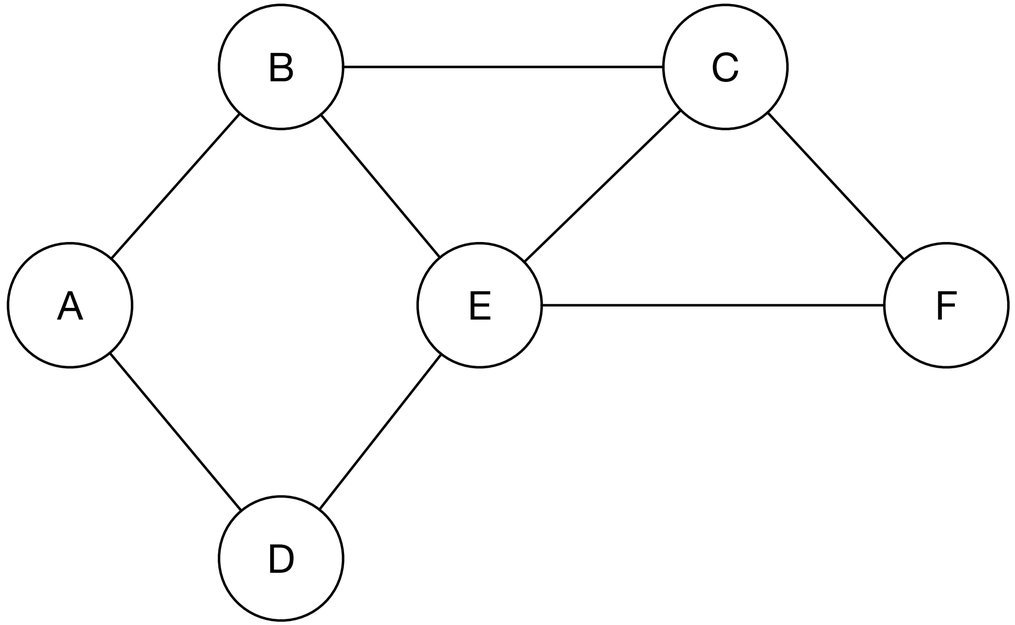


Space: O (|V| + |E|)
Query: O (|V|)

Adjacency list

A	B, D
B	A, C, E
C	B, E, F
D	A, E
E	B, D, C, F
F	C, E

Introduction to graphs

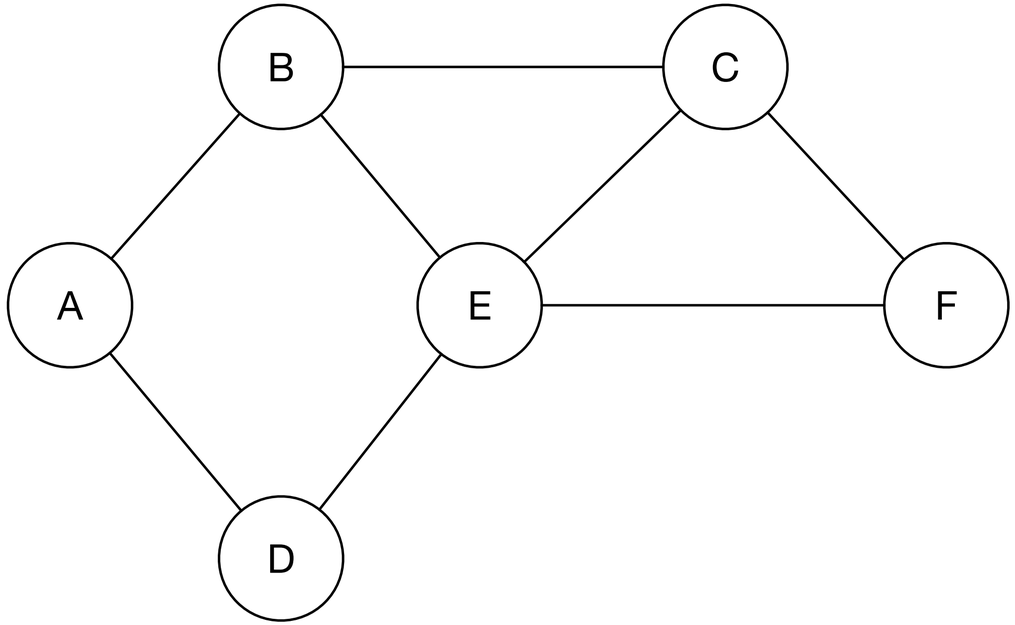


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

Introduction to graphs

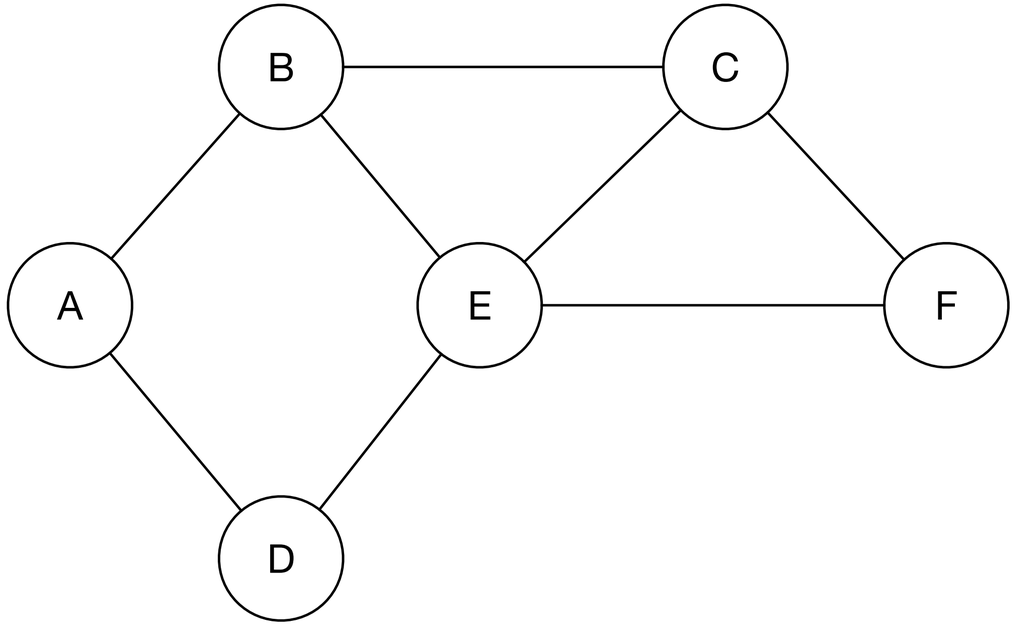


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B						
C						
D						
E						
F						

Introduction to graphs

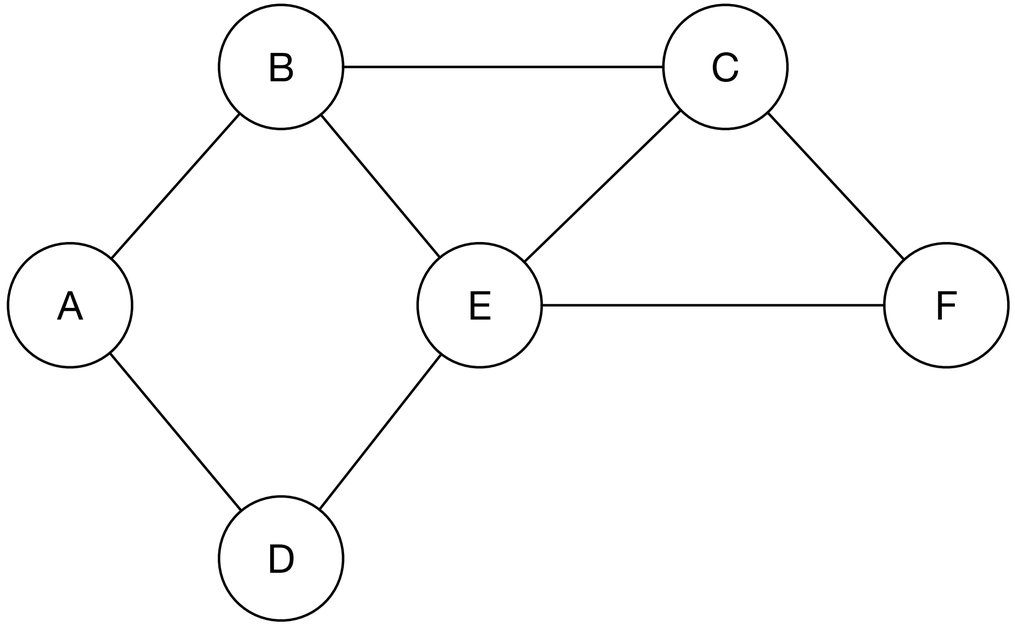


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C						
D						
E						
F						

Introduction to graphs

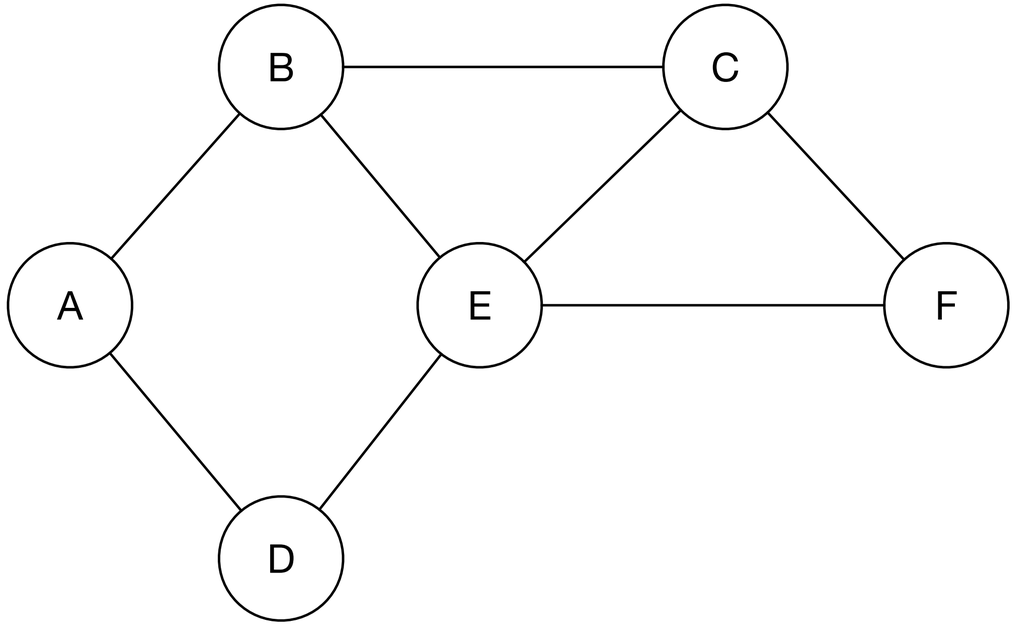


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	0	1	1
D						
E						
F						

Introduction to graphs



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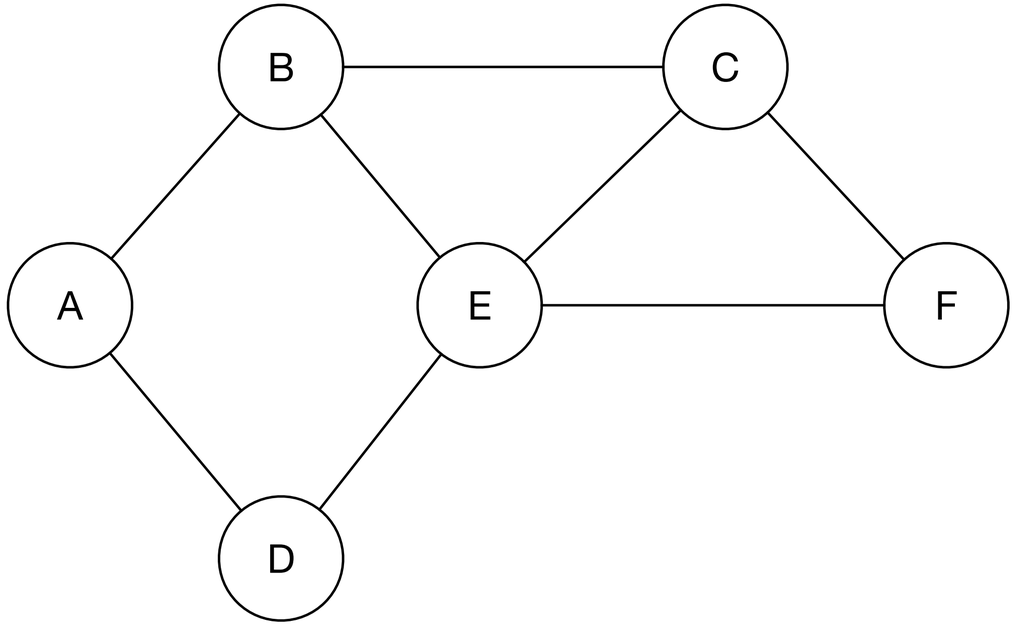
Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	0	1	1
D	1	0	0	0	1	0
E						
F						

Introduction to graphs

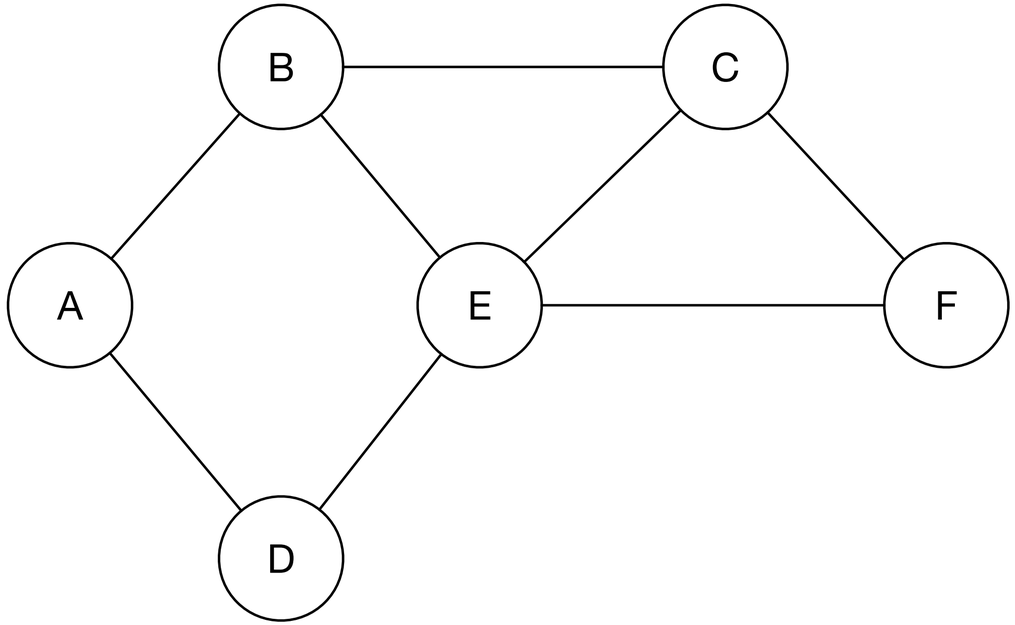


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	0	1	1
D	1	0	0	0	1	0
E	0	1	1	1	0	1
F						

Introduction to graphs

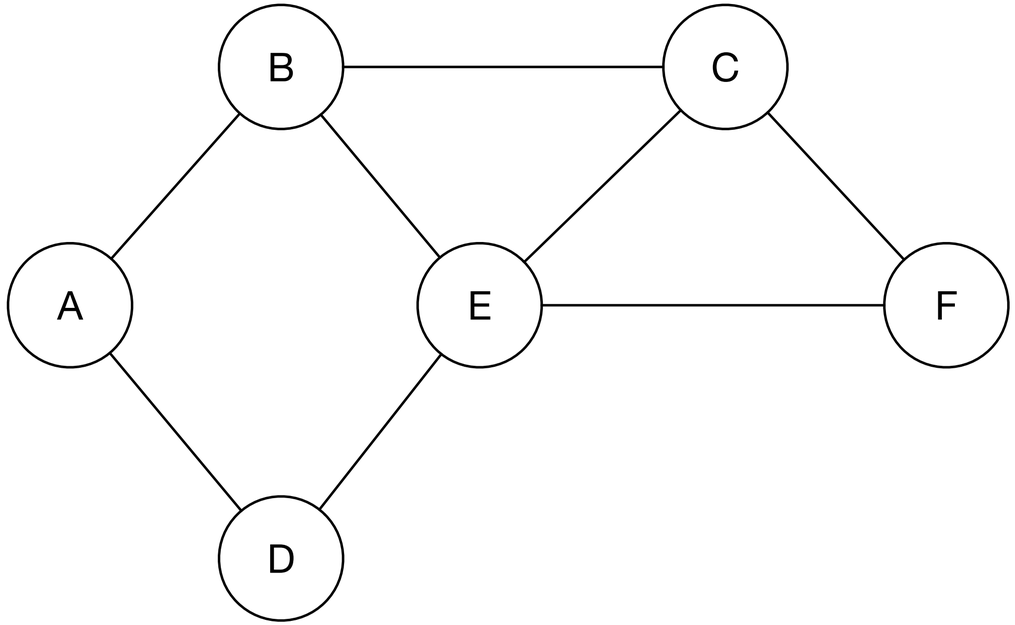


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	0	1	1
D	1	0	0	0	1	0
E	0	1	1	1	0	1
F	0	0	1	0	1	0

Introduction to graphs

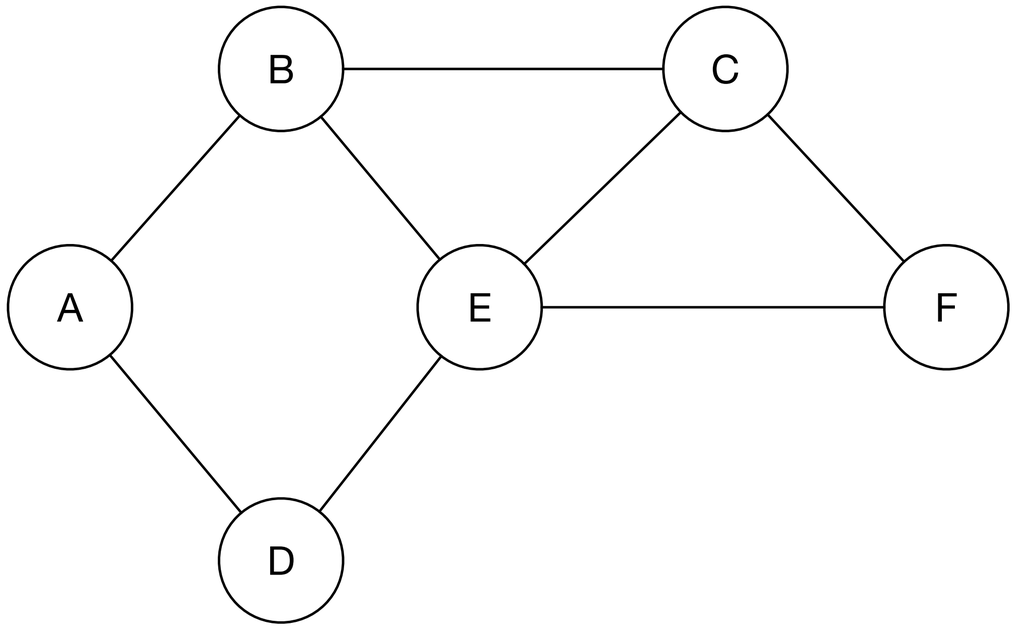


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	0	1	1
D	1	0	0	0	1	0
E	0	1	1	1	0	1
F	0	0	1	0	1	0

Introduction to graphs

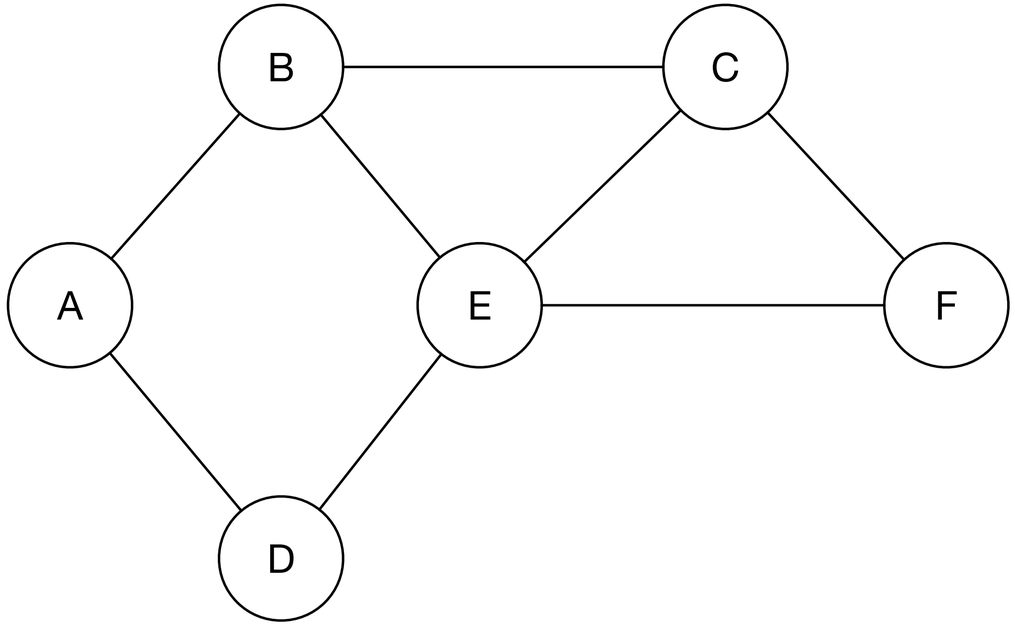


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	0	1	1
D	1	0	0	0	1	0
E	0	1	1	1	0	1
F	0	0	1	0	1	0

Introduction to graphs

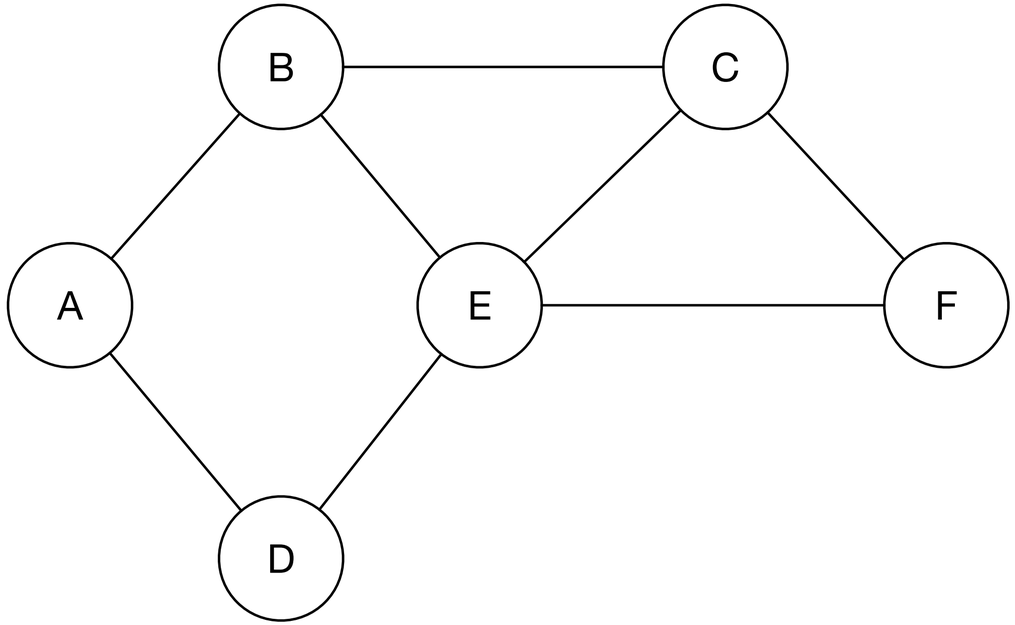


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B		0	1	0	1	0
C			0	0	1	1
D				0	1	0
E					0	1
F						0

Introduction to graphs

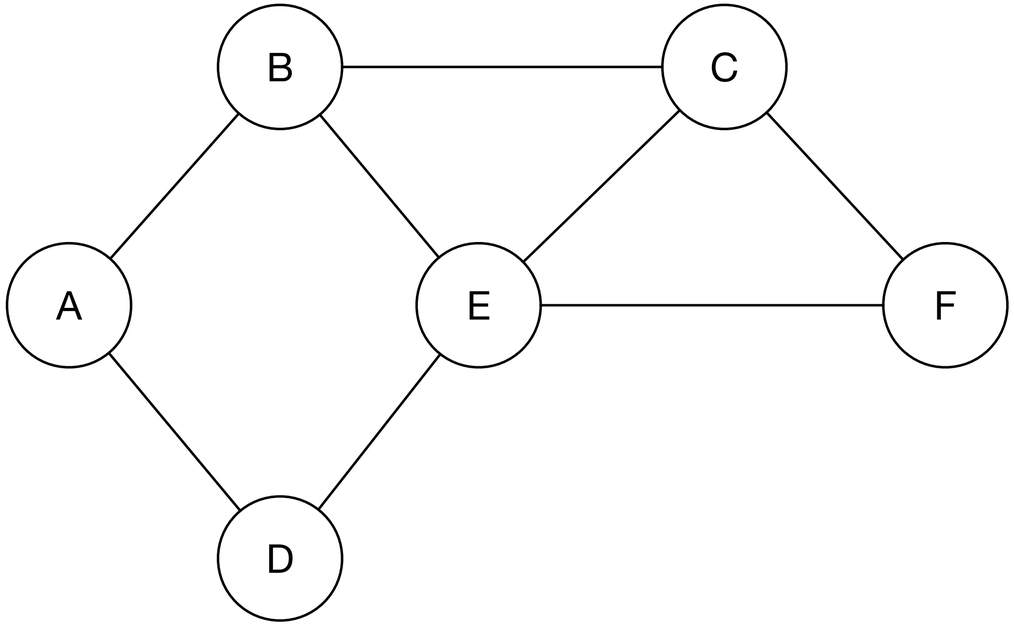


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	0	1	1
D	1	0	0	0	1	0
E	0	1	1	1	0	1
F	0	0	1	0	1	0

Introduction to graphs

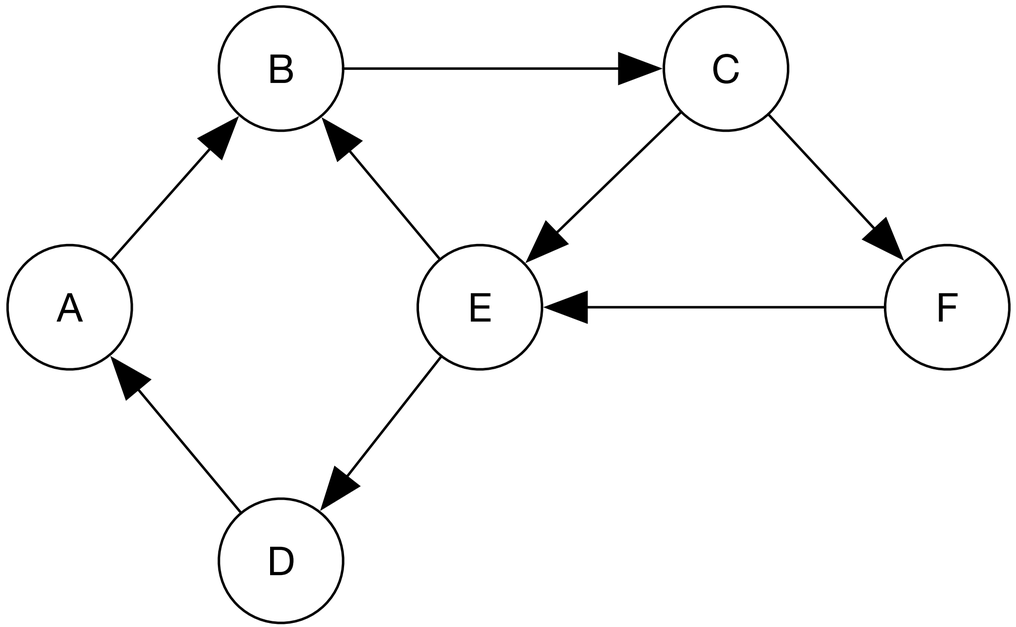


Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	1	0	0
B	1	0	1	0	1	0
C	0	1	0	0	1	1
D	1	0	0	0	1	0
E	0	1	1	1	0	1
F	0	0	1	0	1	0

Space: $O(|V|^2)$
Query: $O(1)$

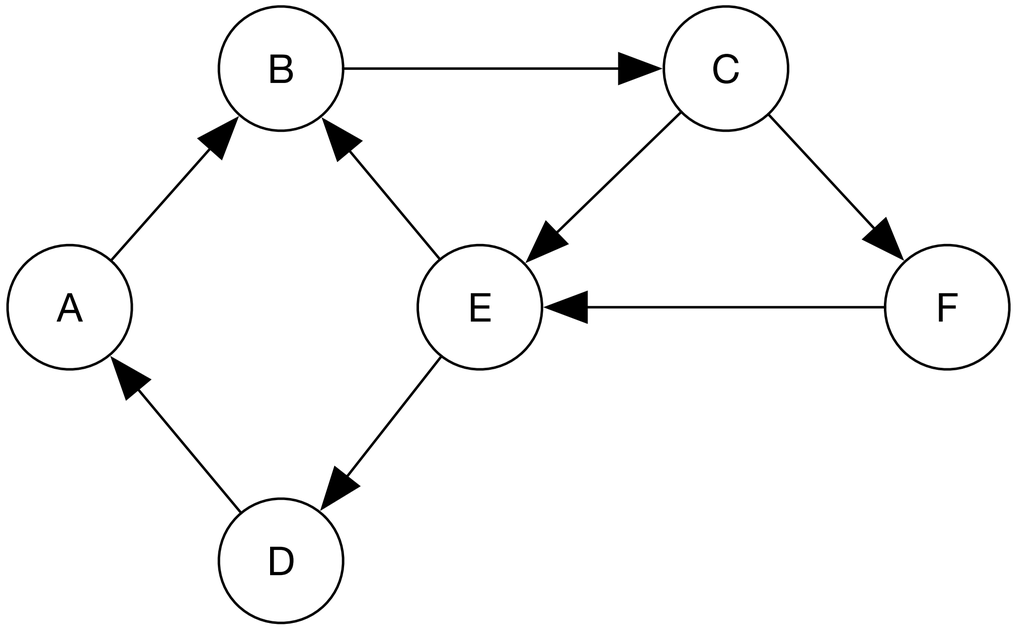
Introduction to graphs



Adjacency list

A	
B	
C	
D	
E	
F	

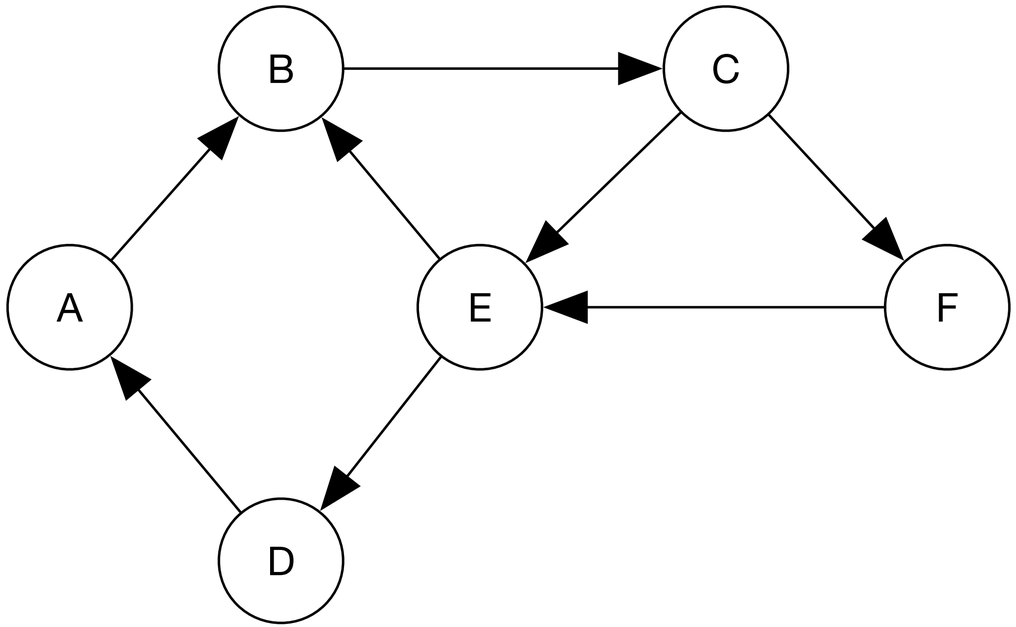
Introduction to graphs



Adjacency list

A	B
B	
C	
D	
E	
F	

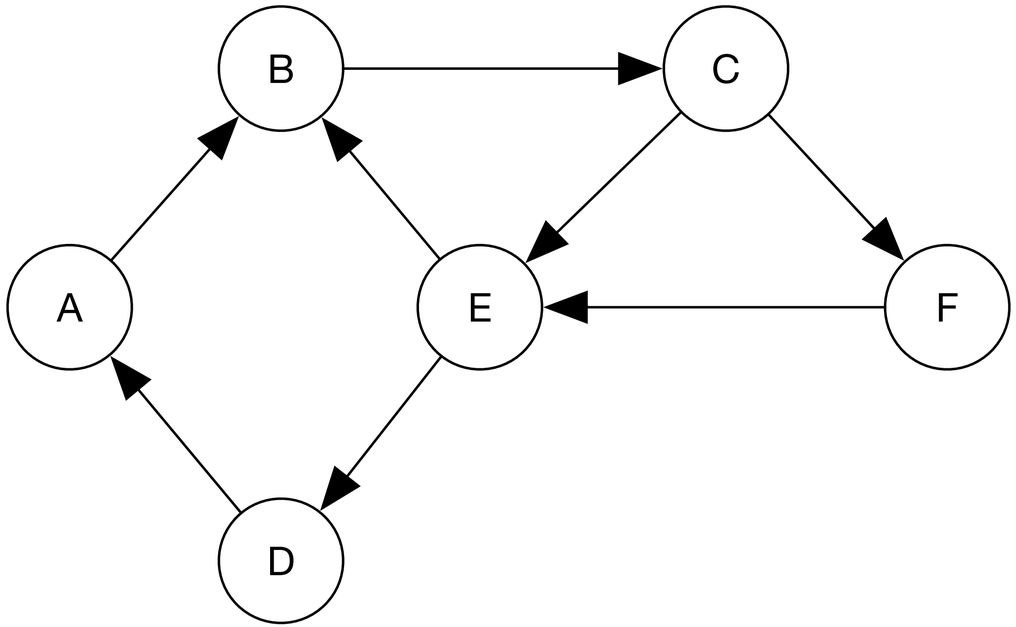
Introduction to graphs



Adjacency list

A	B
B	C
C	
D	
E	
F	

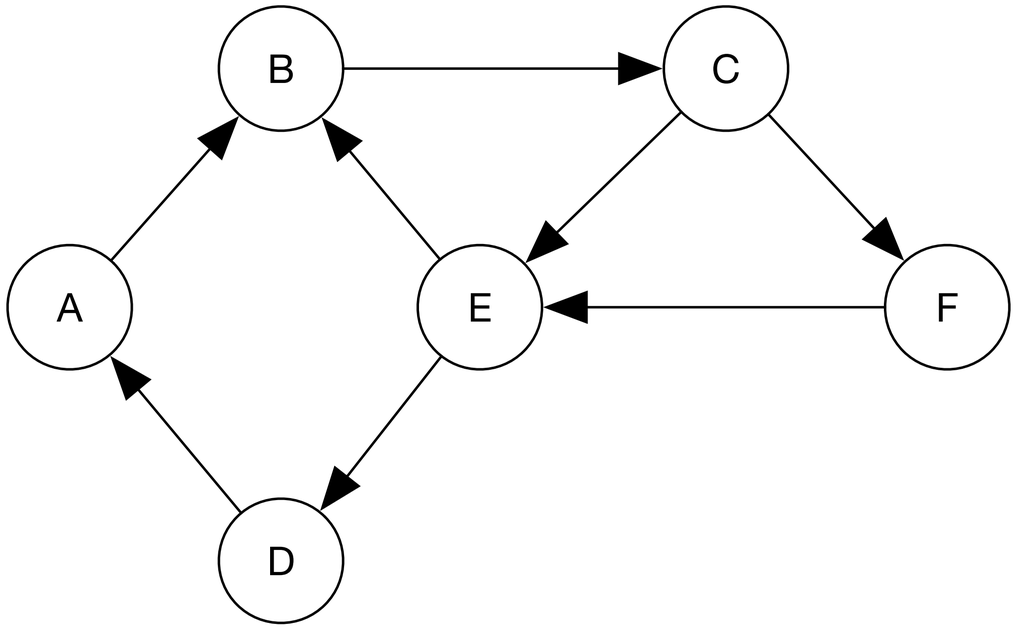
Introduction to graphs



Adjacency list

A	B
B	C
C	E, F
D	
E	
F	

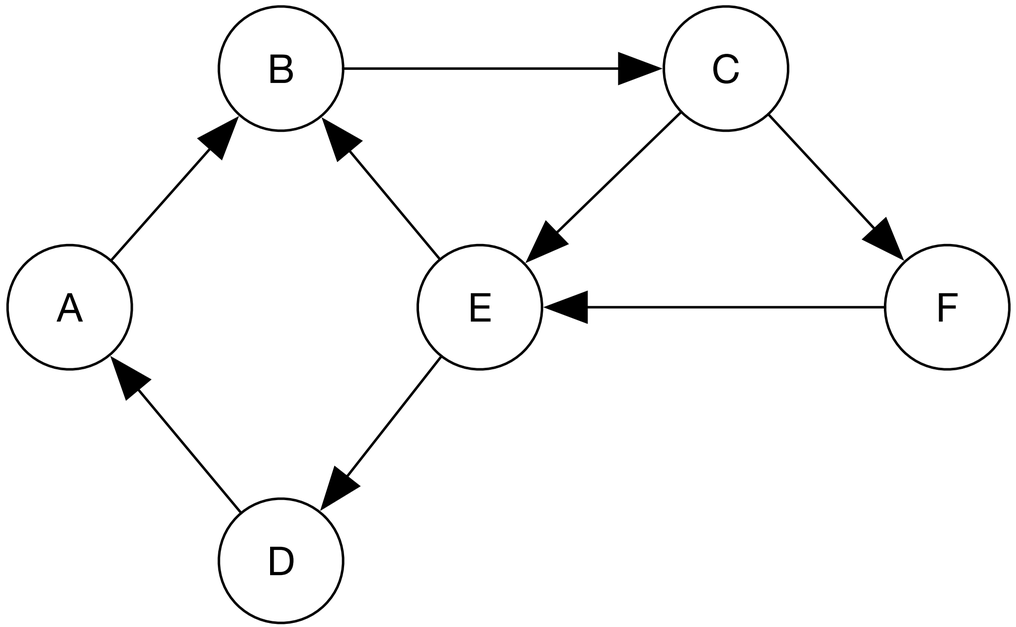
Introduction to graphs



Adjacency list

A	B
B	C
C	E, F
D	A
E	
F	

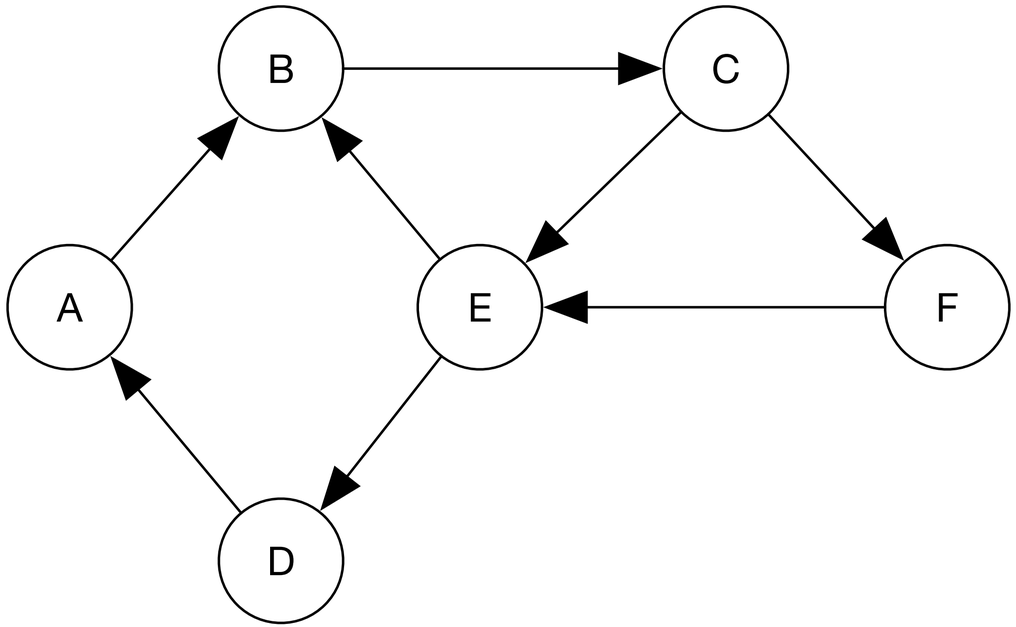
Introduction to graphs



Adjacency list

A	B
B	C
C	E, F
D	A
E	B, D
F	

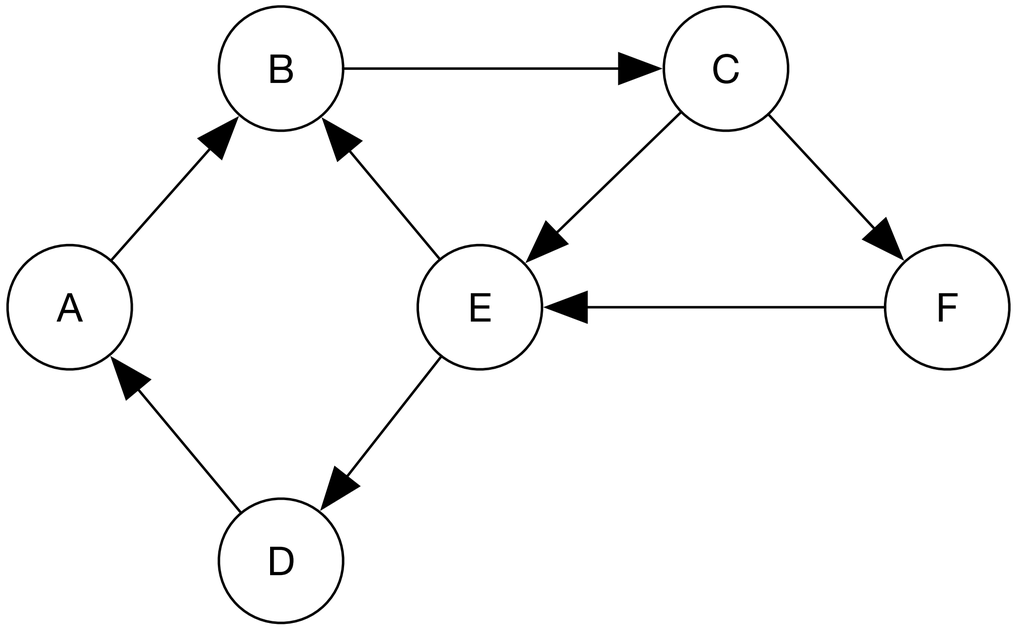
Introduction to graphs



Adjacency list

A	B
B	C
C	E, F
D	A
E	B, D
F	E

Introduction to graphs

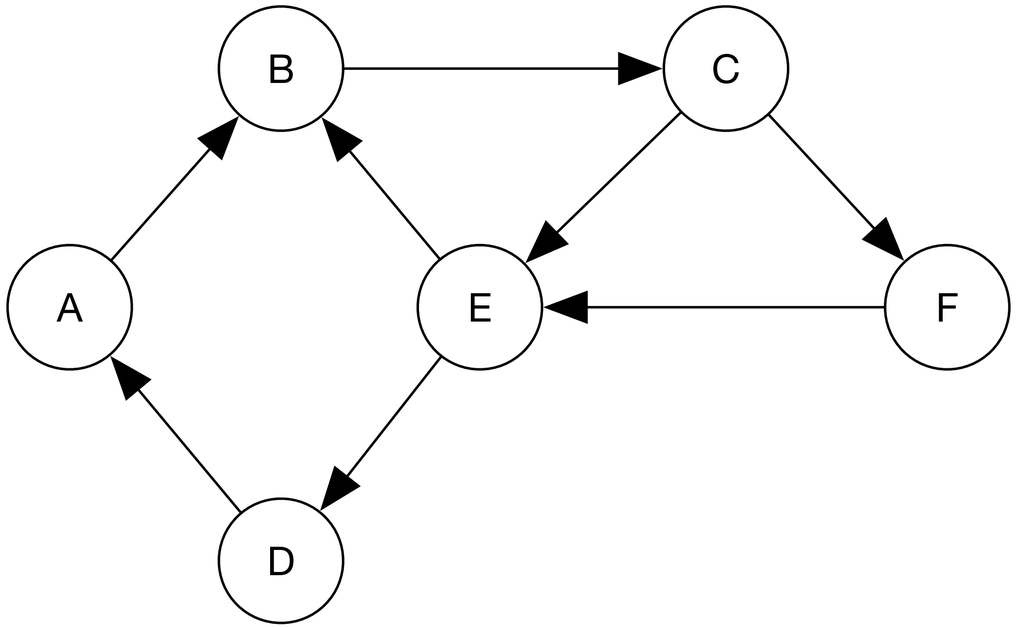


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

Introduction to graphs

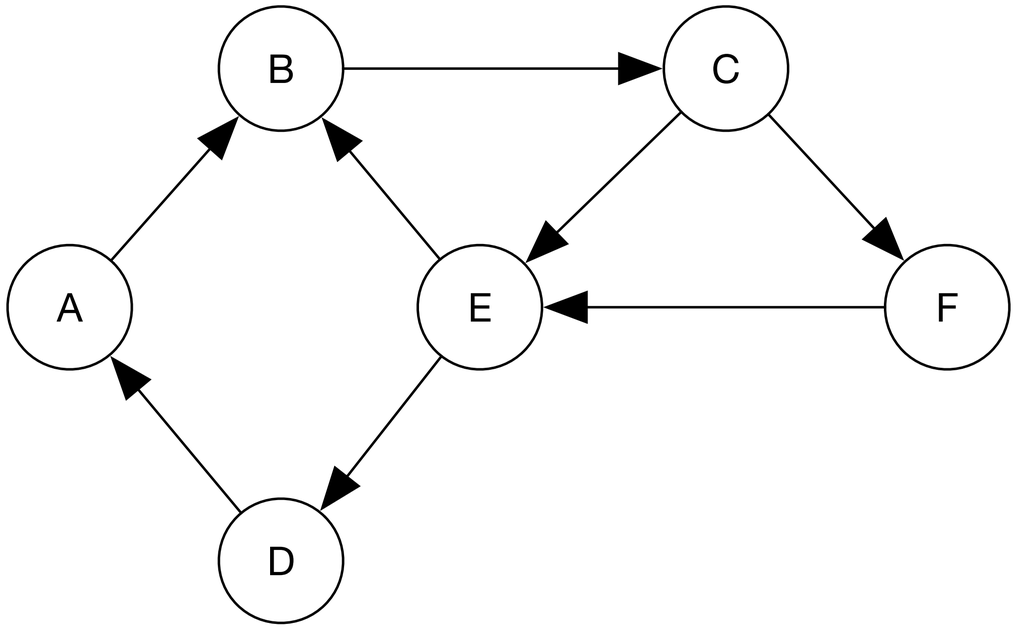


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	0	0	0
B						
C						
D						
E						
F						

Introduction to graphs

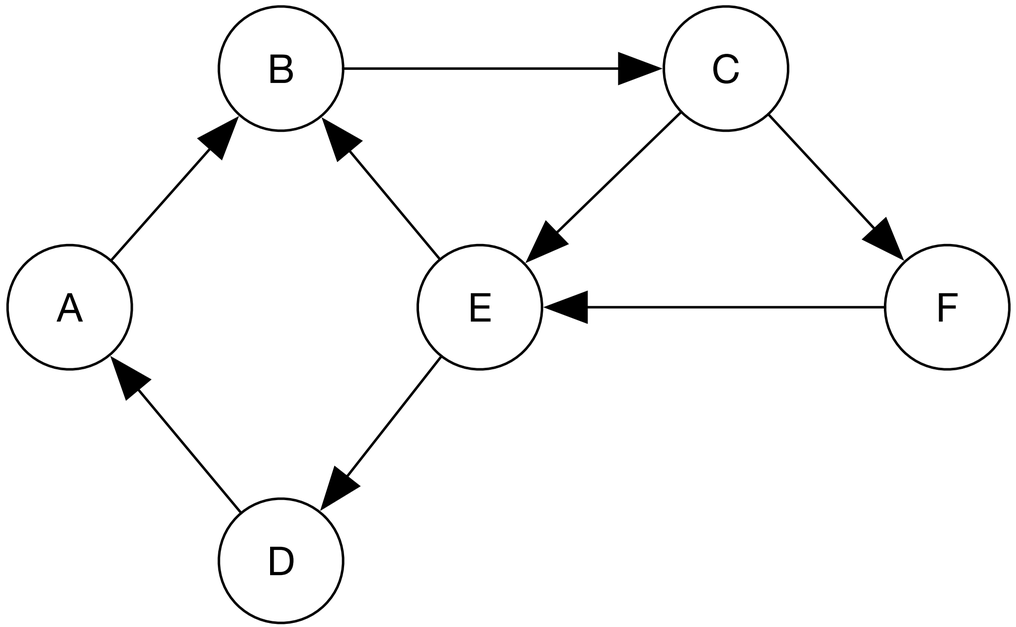


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	0	0
C						
D						
E						
F						

Introduction to graphs

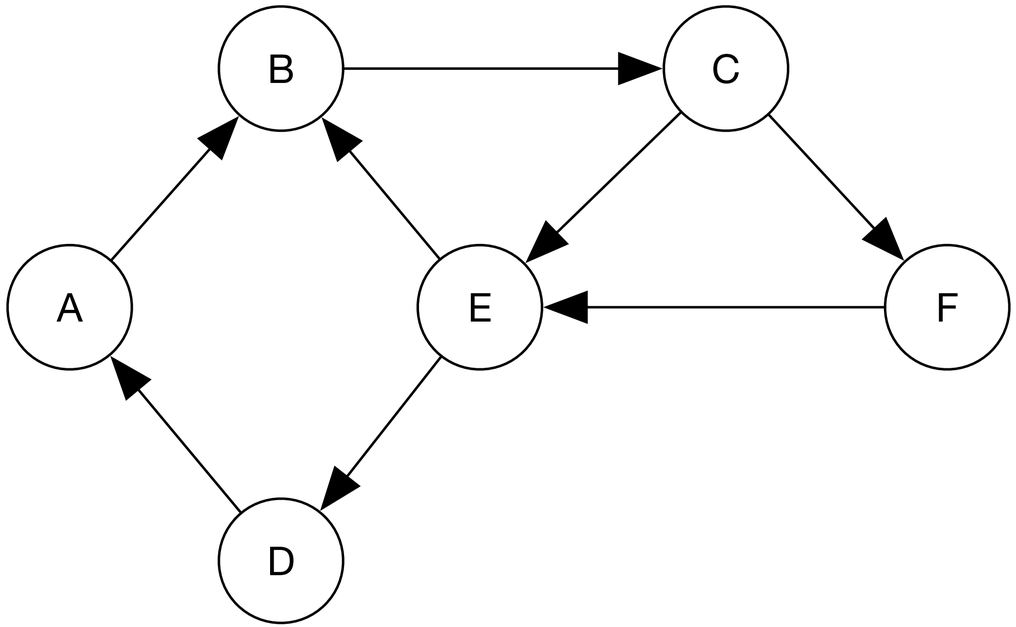


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	0	0
C	0	0	0	0	1	1
D						
E						
F						

Introduction to graphs



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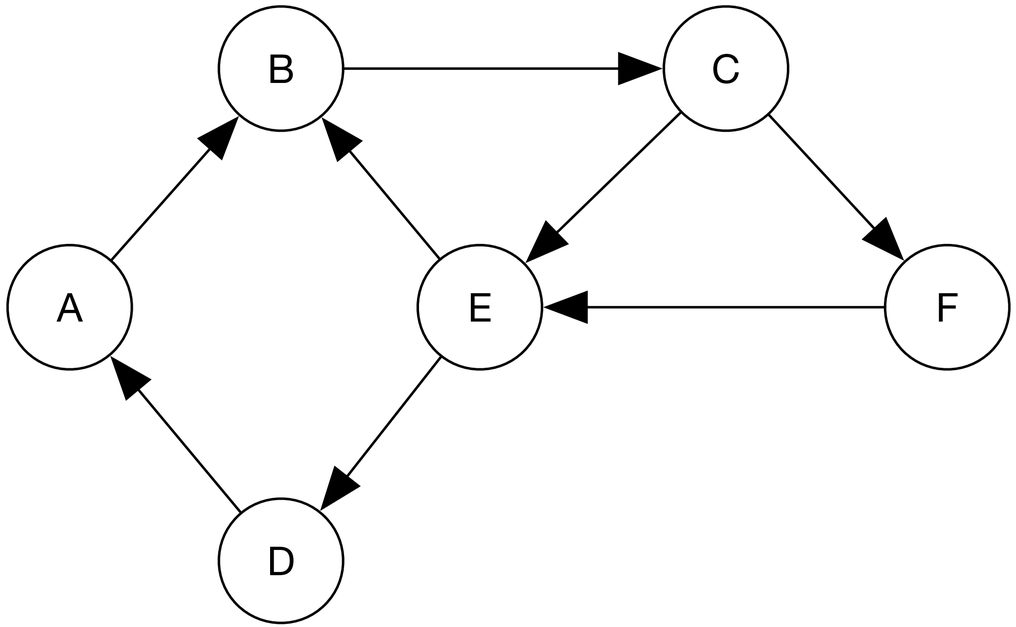
Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	0	0
C	0	0	0	0	1	1
D	1	0	0	0	0	0
E						
F						

Introduction to graphs



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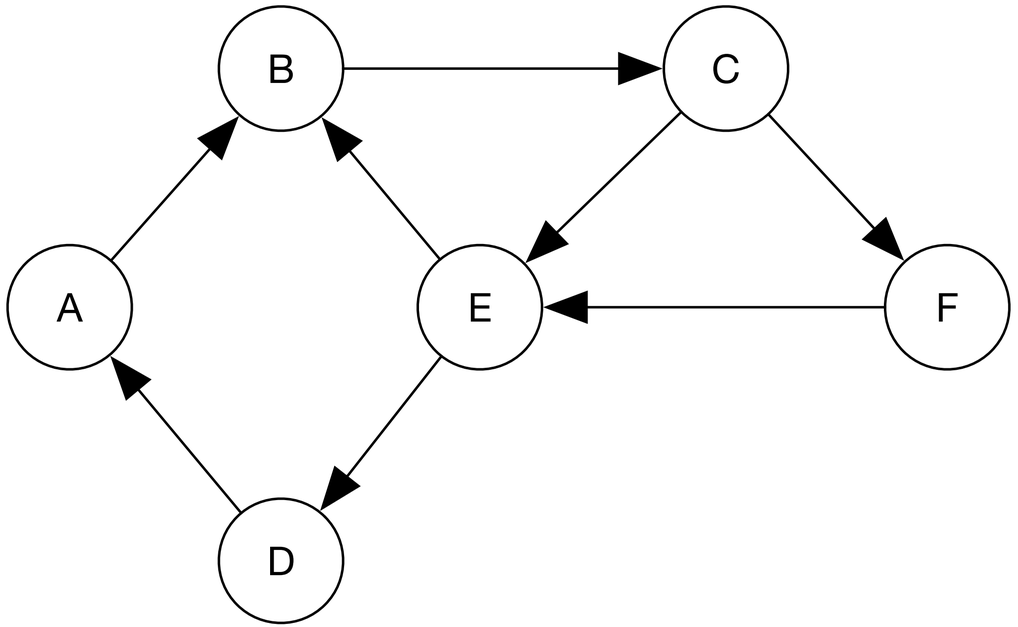
Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	0	0
C	0	0	0	0	1	1
D	1	0	0	0	0	0
E	0	1	0	1	0	0
F						

Introduction to graphs



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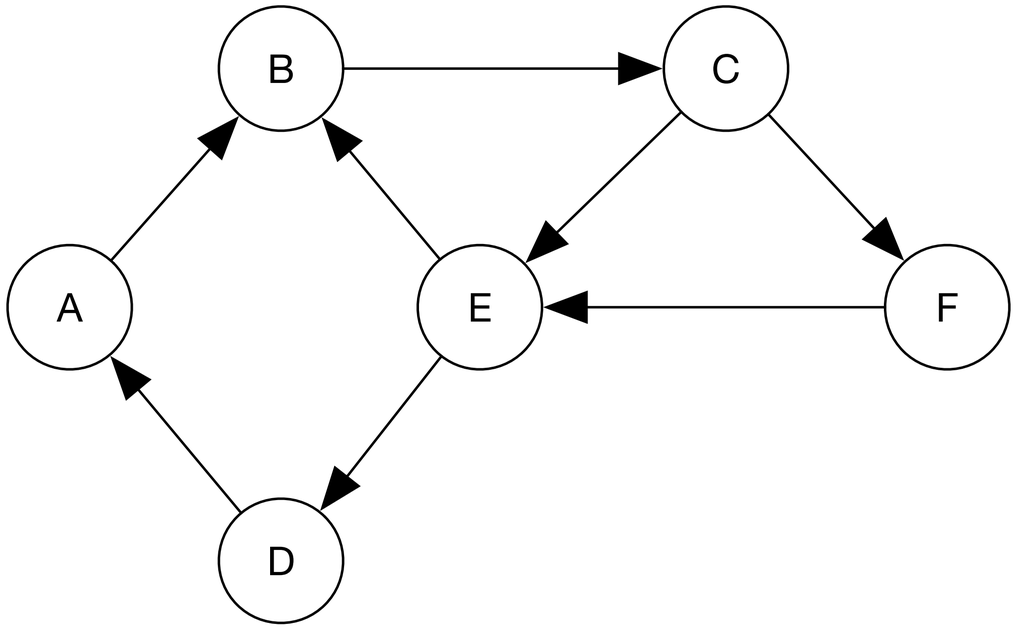
Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	0	0
C	0	0	0	0	1	1
D	1	0	0	0	0	0
E	0	1	0	1	0	0
F	0	0	0	0	1	0

Introduction to graphs

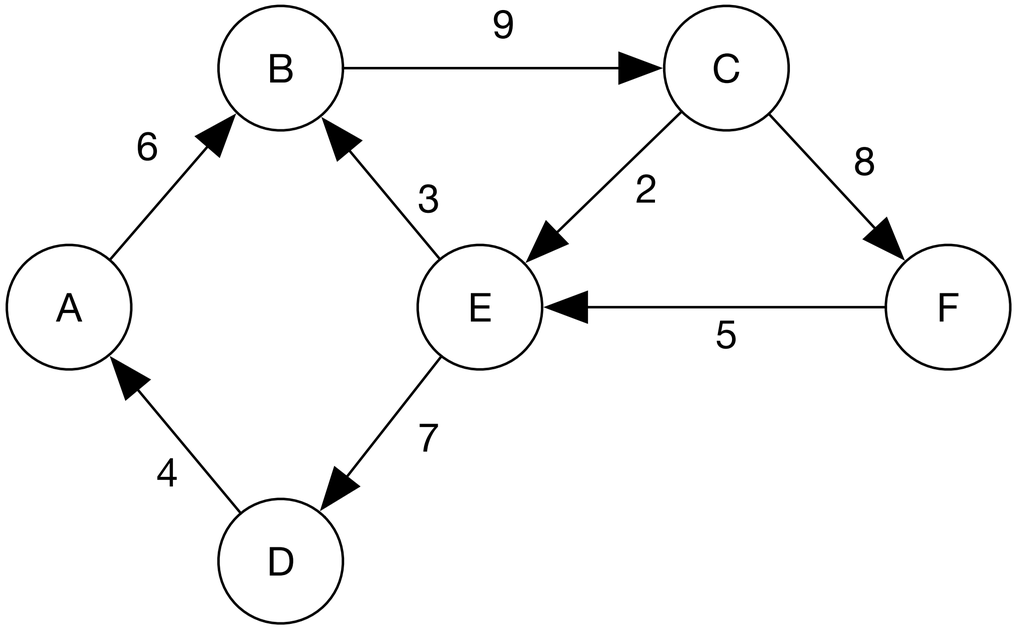


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ 1 & \text{if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	1	0	0	0	0
B	0	0	1	0	0	0
C	0	0	0	0	1	1
D	1	0	0	0	0	0
E	0	1	0	1	0	0
F	0	0	0	0	1	0

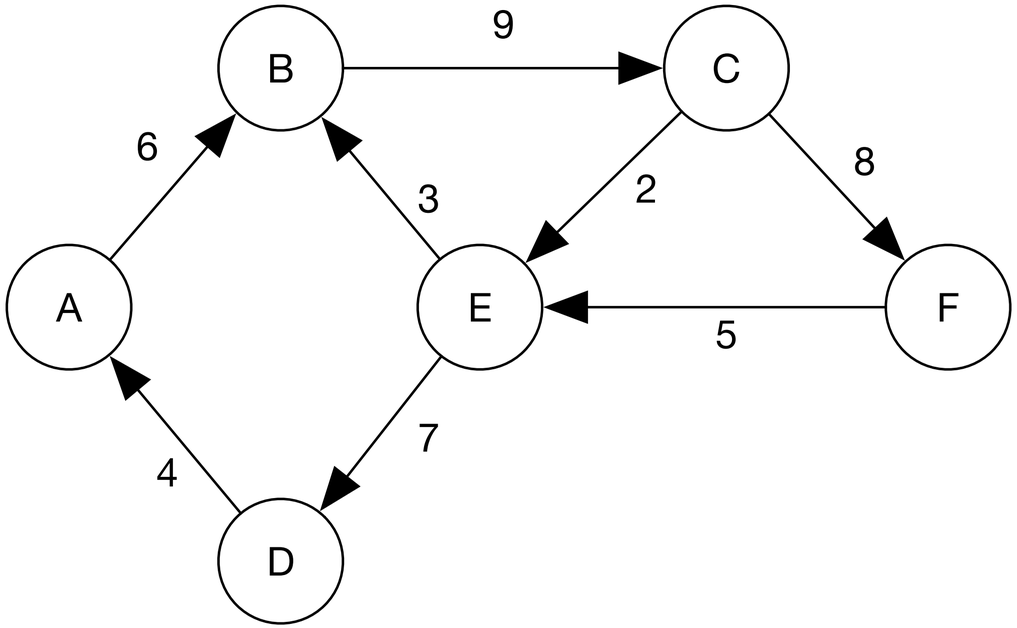
Introduction to graphs



Adjacency list

A	
B	
C	
D	
E	
F	

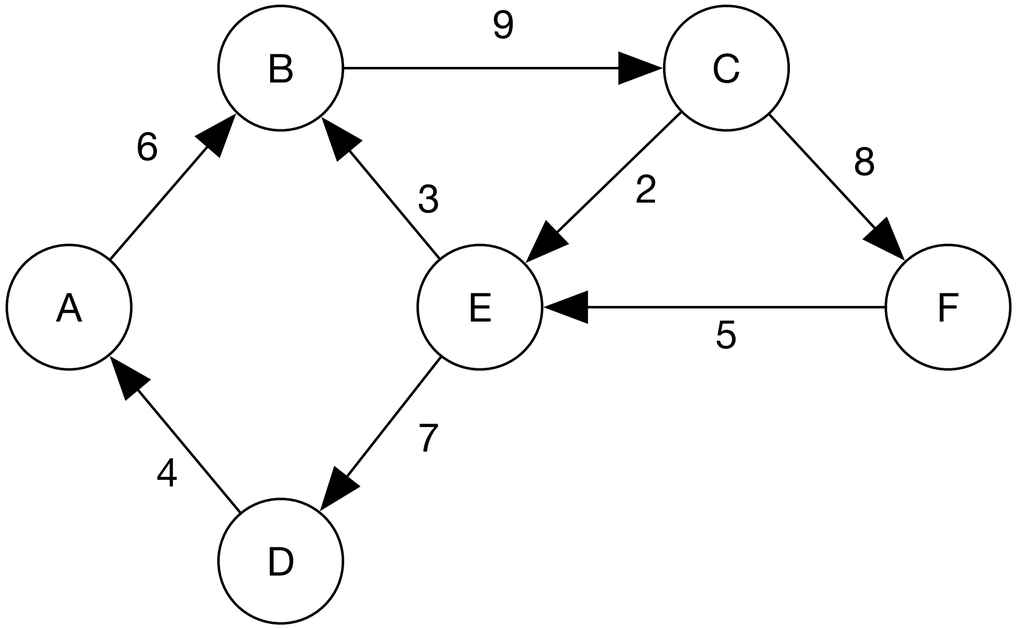
Introduction to graphs



Adjacency list

A	{B, 6}
B	
C	
D	
E	
F	

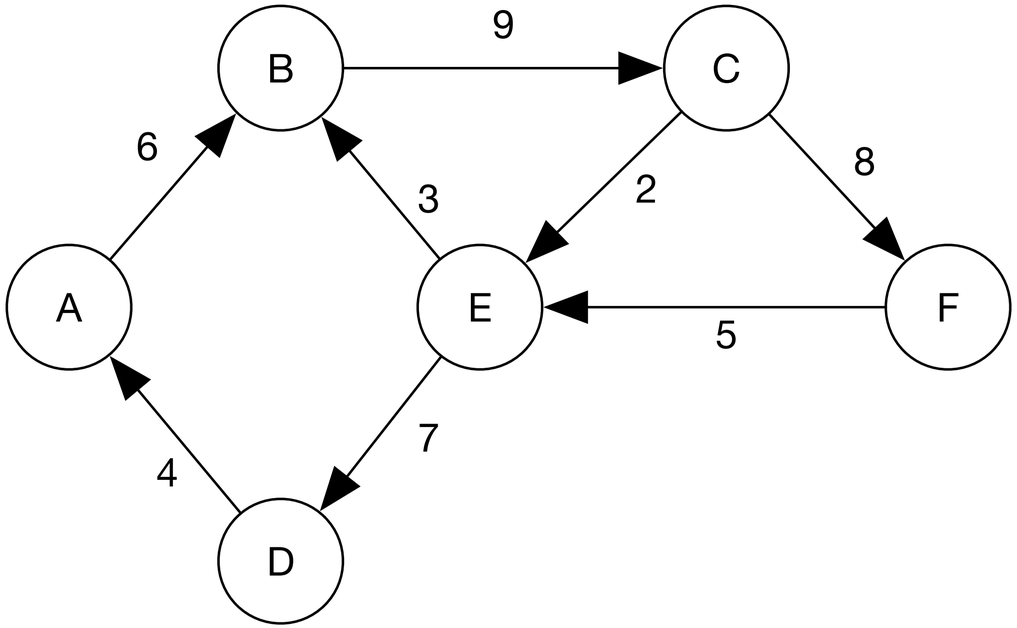
Introduction to graphs



Adjacency list

A	{B, 6}
B	{C, 9}
C	
D	
E	
F	

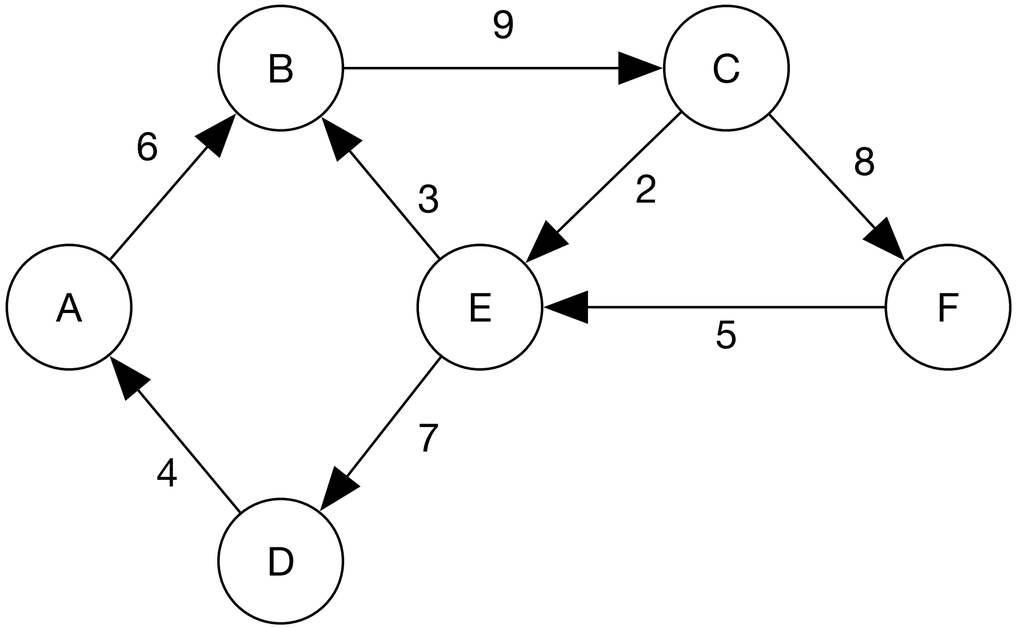
Introduction to graphs



Adjacency list

A	{B, 6}
B	{C, 9}
C	{E, 2}, {F, 8}
D	
E	
F	

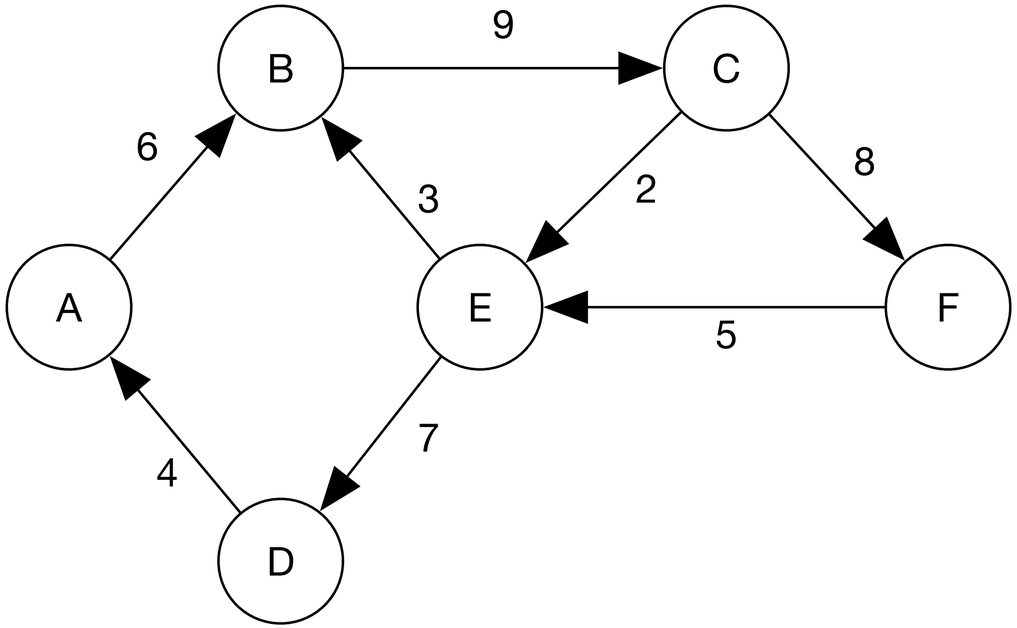
Introduction to graphs



Adjacency list

A	{B, 6}
B	{C, 9}
C	{E, 2}, {F, 8}
D	{A, 4}
E	
F	

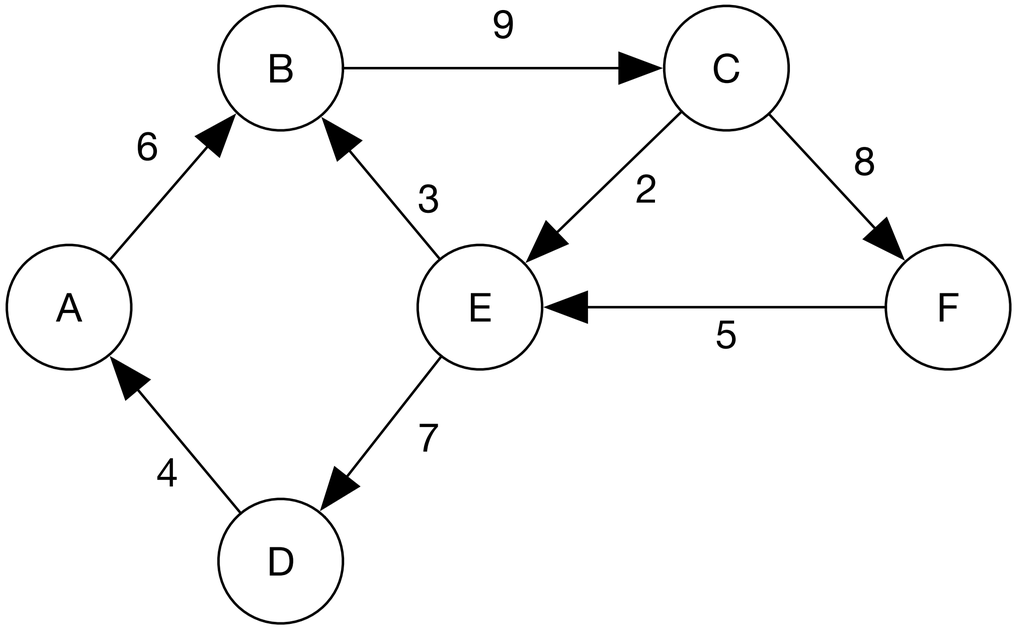
Introduction to graphs



Adjacency list

A	{B, 6}
B	{C, 9}
C	{E, 2}, {F, 8}
D	{A, 4}
E	{B, 3}, {D, 7}
F	

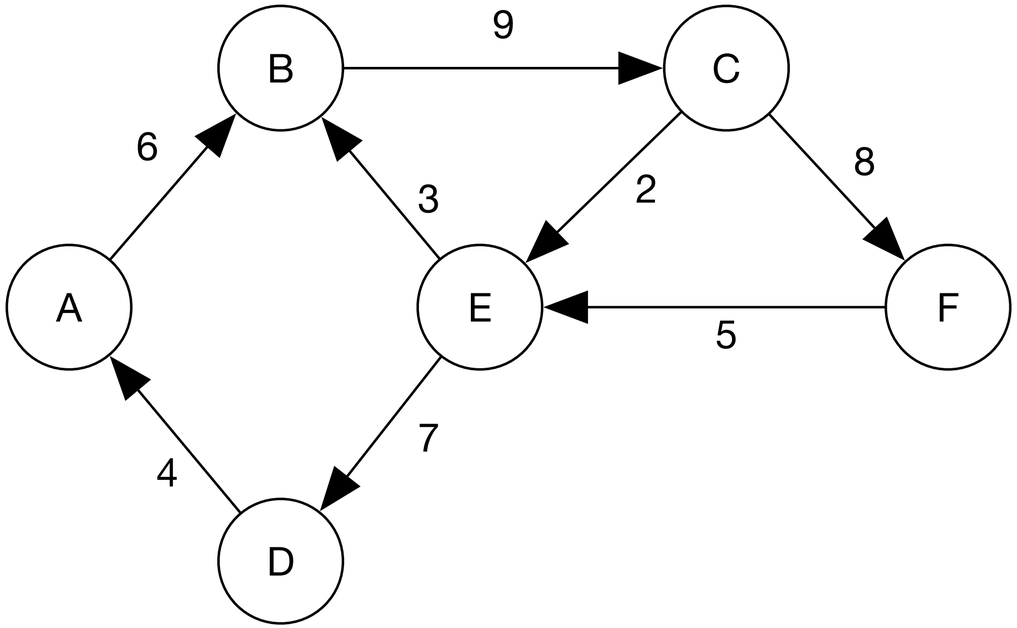
Introduction to graphs



Adjacency list

A	{B, 6}
B	{C, 9}
C	{E, 2}, {F, 8}
D	{A, 4}
E	{B, 3}, {D, 7}
F	{E, 5}

Introduction to graphs

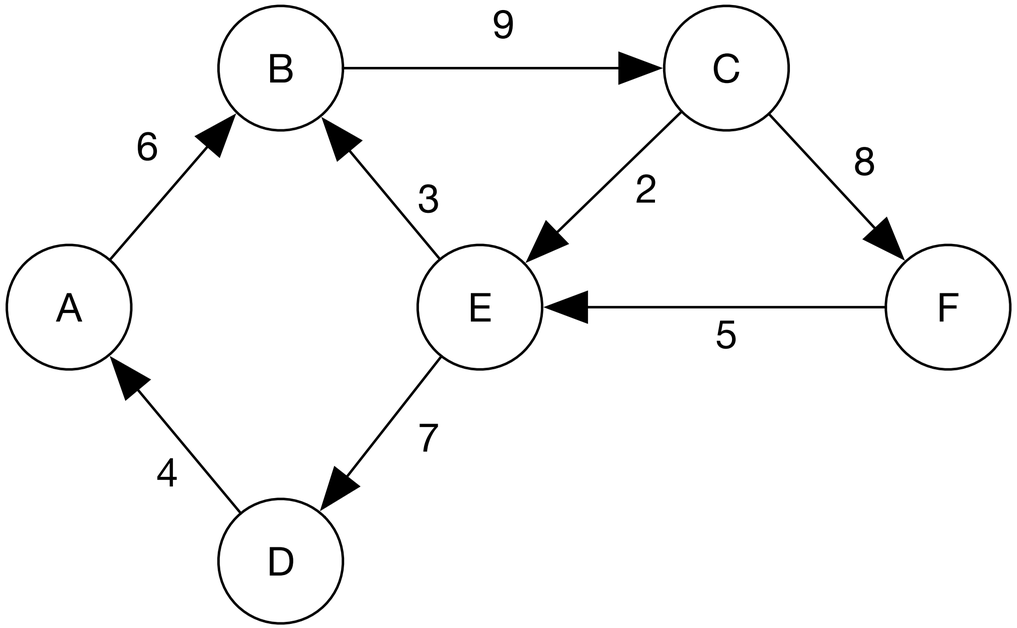


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ w(i,j) & \text{the weight, if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A						
B						
C						
D						
E						
F						

Introduction to graphs

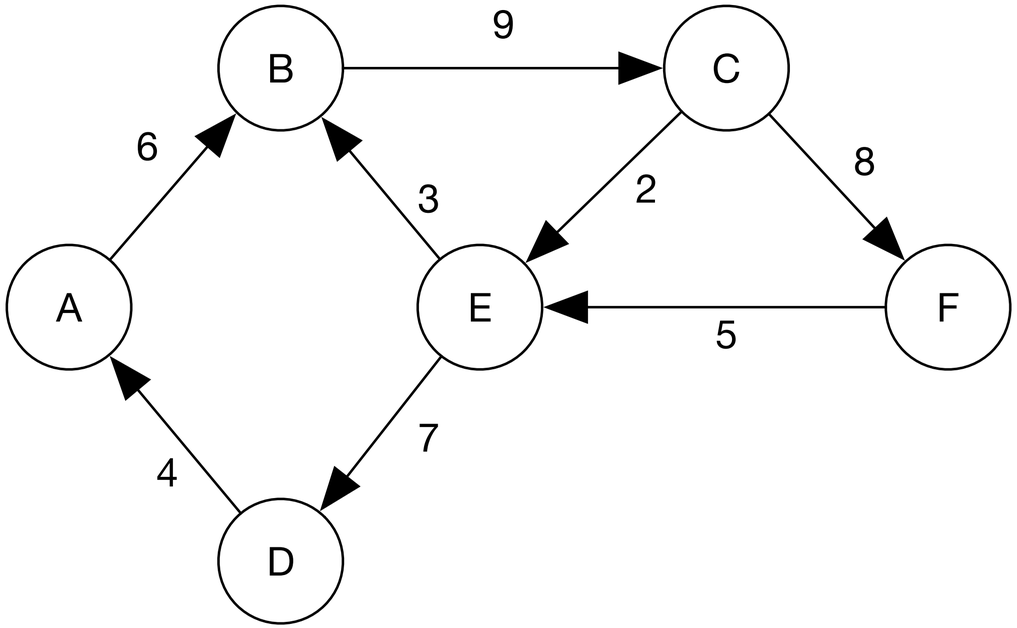


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ w(i,j) & \text{the weight, if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	6	0	0	0	0
B						
C						
D						
E						
F						

Introduction to graphs

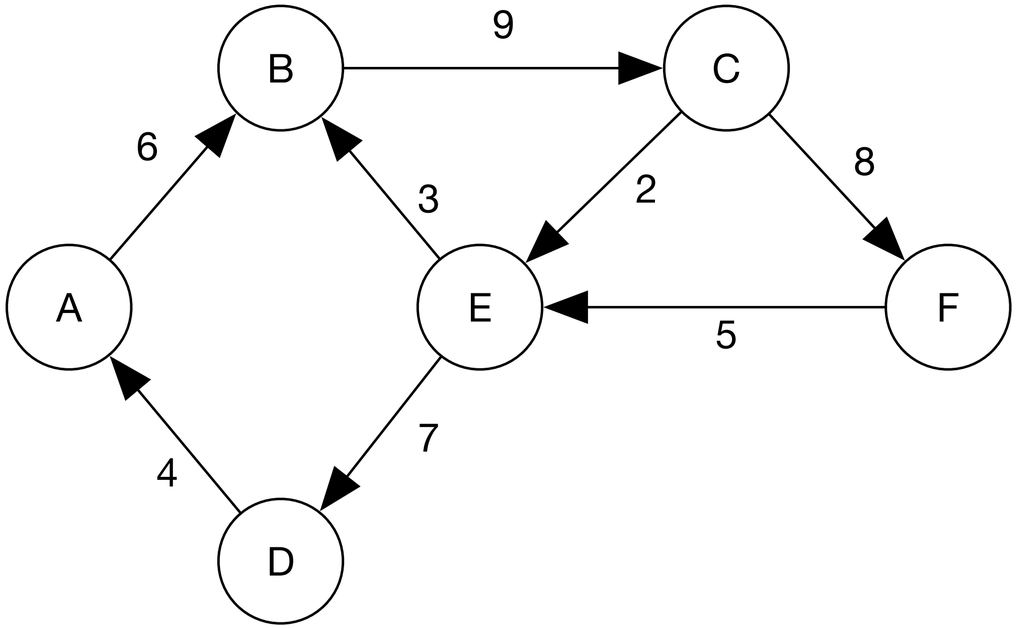


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ w(i,j) & \text{the weight, if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	6	0	0	0	0
B	0	0	9	0	0	0
C						
D						
E						
F						

Introduction to graphs

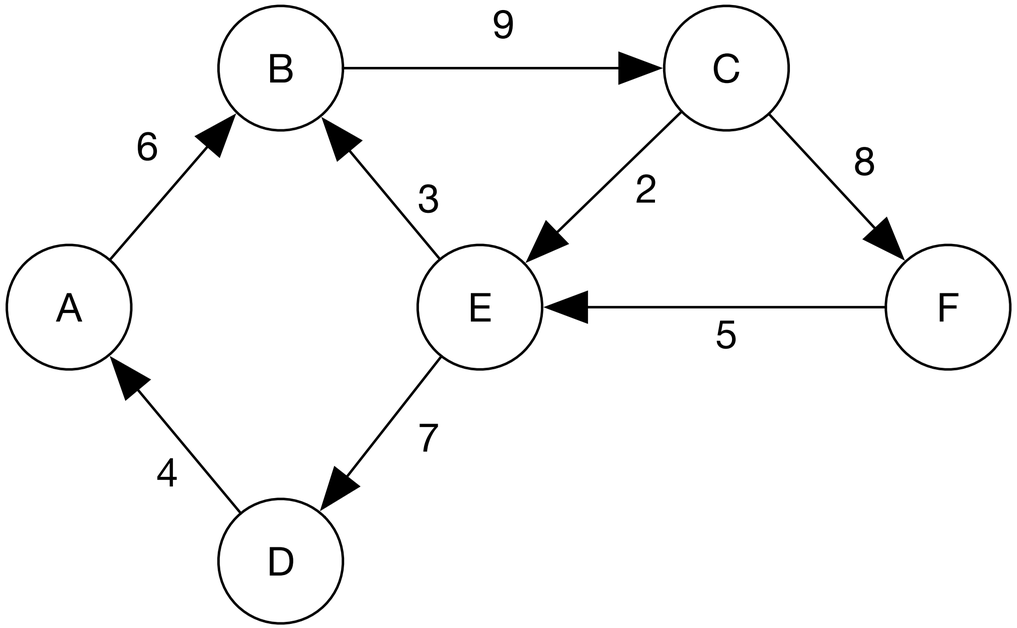


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ w(i,j) & \text{the weight, if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	6	0	0	0	0
B	0	0	9	0	0	0
C	0	0	0	0	2	8
D						
E						
F						

Introduction to graphs

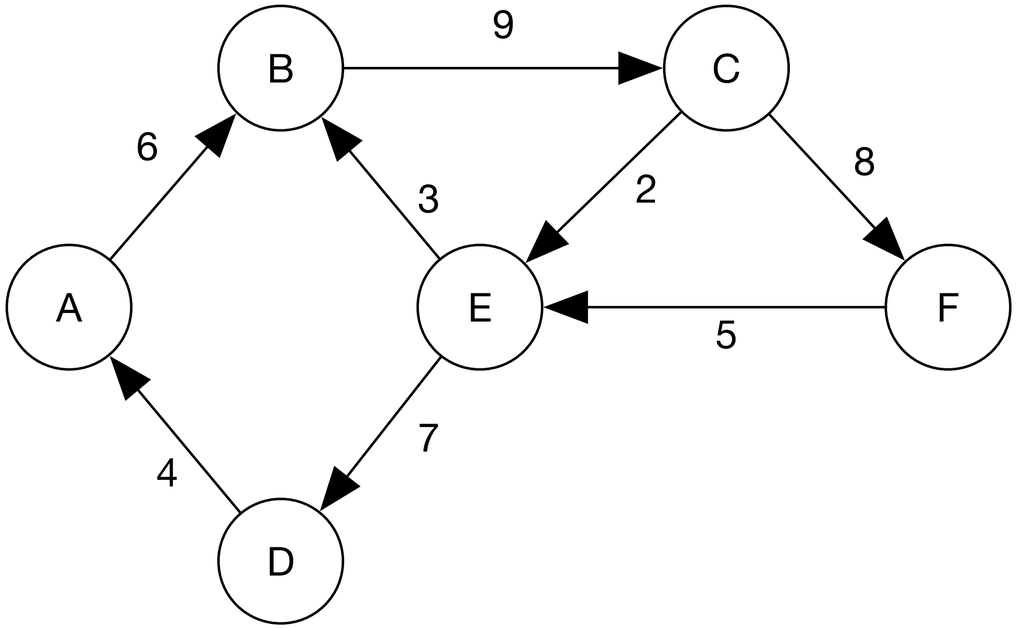


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ w(i,j) & \text{the weight, if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	6	0	0	0	0
B	0	0	9	0	0	0
C	0	0	0	0	2	8
D	4	0	0	0	0	0
E						
F						

Introduction to graphs

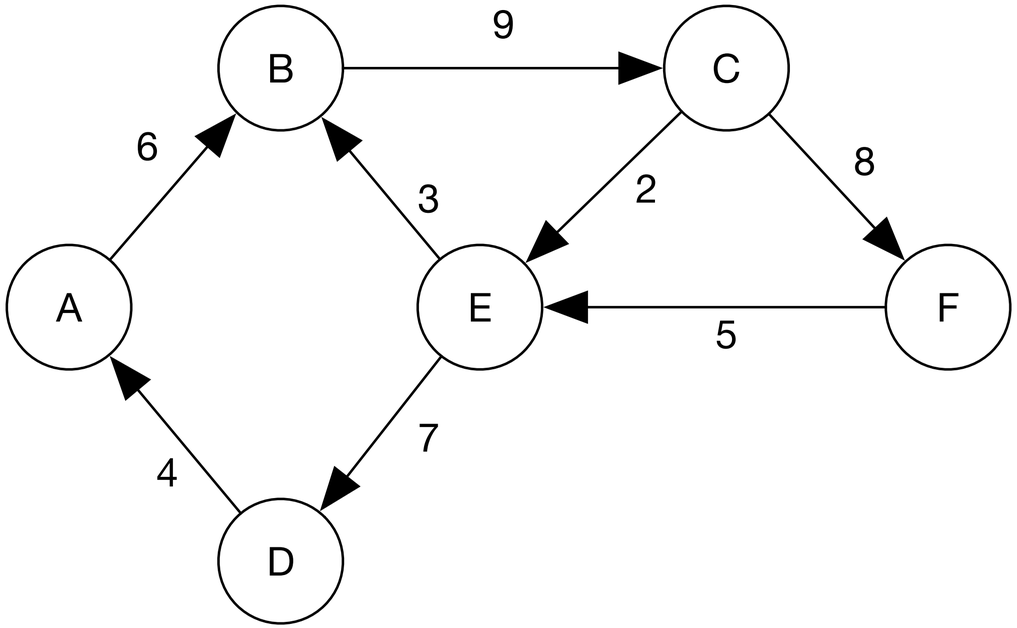


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ w(i,j) & \text{the weight, if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	6	0	0	0	0
B	0	0	9	0	0	0
C	0	0	0	0	2	8
D	4	0	0	0	0	0
E	0	3	0	7	0	0
F						

Introduction to graphs

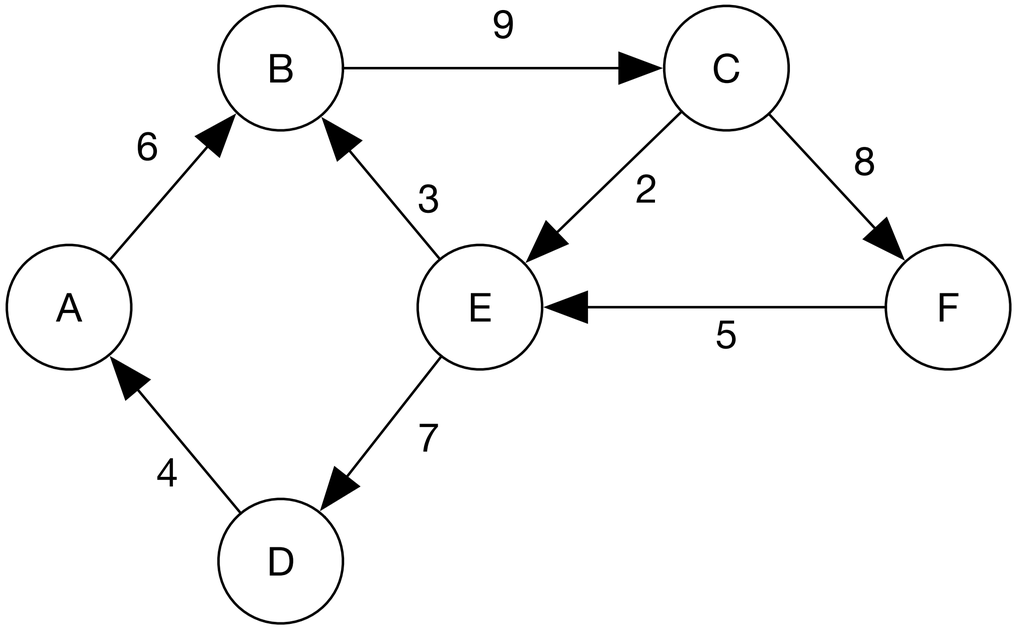


$$M_{i,j} = \begin{cases} 0 & \text{if not adjacent} \\ w(i,j) & \text{the weight, if adjacent} \end{cases}$$

Adjacency matrix

	A	B	C	D	E	F
A	0	6	0	0	0	0
B	0	0	9	0	0	0
C	0	0	0	0	2	8
D	4	0	0	0	0	0
E	0	3	0	7	0	0
F	0	0	0	0	5	0

Introduction to graphs



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Adjacency matrix

	A	B	C	D	E	F
A	0	6	0	0	0	0
B	0	0	9	0	0	0
C	0	0	0	0	2	8
D	4	0	0	0	0	0
E	0	3	0	7	0	0
F	0	0	0	0	5	0

Introduction to graphs

Which data structure is "best"?

Introduction to graphs

Which data structure is "best"? It depends!

Introduction to graphs

Which data structure is "best"? It depends!

	Adjacency list	Adjacency matrix
Space	$O(\|V\| + \|E\|)$	$O(\|V\|^2)$
Add vertex	$O(1)$	$O(\|V\|^2)$
Add edge	$O(1)$	$O(1)$
Adjacency query	$O(\|V\|)$	$O(1)$