PoCS What's The Story?

Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2014
Assignment 9 • code name: Heard it through the grapevine

Dispersed: Thursday, November 6, 2014.
Due: By start of lecture, 1:00 pm, Thursday, November 13, 2014.
Some useful reminders:
Instructor: Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 2:30 pm to 3:45 pm on Tuesday, 12:30 pm to 2:00 pm on Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2014-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

---

1. Please submit your project's current draft in pdf format via email.

   Please use this file name format (all lowercase after CSYS):

   CSYS300-final-project-firstname-lastname-YYYY-MM-DD.pdf

2. (3 + 3)

   Using Gleeson and Calahane's iterative equations below, derive the contagion condition for a vanishing seed by taking the limit $\phi_0 \to 0$ and $t \to \infty$. In lectures, we derived the discrete evolution equations for the fraction of infected nodes ϕ_t and the fraction of infected edges θ_t as follows:

$$\phi_{t+1} = \phi_0 + (1-\phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} \binom{k}{j} \theta_t^j (1-\theta_t)^{k-j} B_{kj},$$

$$\theta_{t+1} = G(\theta_t; \phi_0) = \phi_0 + (1-\phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} {\theta_t}^j (1-\theta_t)^{k-1-j} B_{kj},$$

   where $\theta_0 = \phi_0$, and B_{kj} is the probability that a degree k node becomes active when j of its neighbors are active.

   Recall that by contagion condition, we mean the requirements of a random network for macroscopic spreading to occur.

To connect the paper's model and notation to those of our lectures, given a specific response function F and a threshold model, the B_{kj} are given by $B_{kj} = F(j/k)$.

Allow B_{k0} to be arbitrary (i.e., not necessarily 0 as for simple threshold functions).

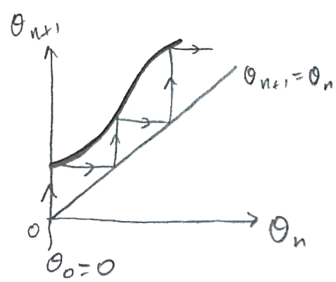
We really only need to understand how θ_t behaves. Write the corresponding equation as $\theta_{t+1} = G(\theta_t; \phi_0)$ and determine when

(a) $G(0; \phi_0) > 0$ (spreading is for free).

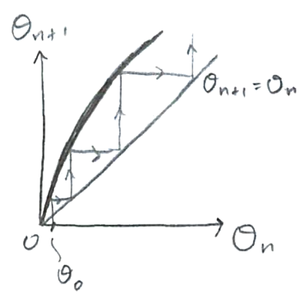
(b) $G(0; \phi_0) = 0$ and $G'(0; \phi_0) > 1$ meaning $\phi = 0$ is a unstable fixed point.

Here's a graphical hint for the three cases you need to consider as $\theta_0 \to 0$:

Success:



Sucesss:



Fail: